ERRORS AND MISCONCEPTIONS OF GRADE 6 LEARNERS IN COMMON FRACTIONS: A CASE STUDY OF TWO PRIMARY SCHOOLS IN THE QUEENSTOWN EDUCATION DISTRICT

BY

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ABSTRACT

The objective of this study was to investigate and explore the errors and misconceptions associated with the concept of fractions which are displayed by Grade 6 learners in two primary schools in the Queenstown Education District of the Eastern Cape Province. The research was done specifically to identify errors and misconceptions made by these learners in the addition and subtraction of common and mixed fractions. The study further tried to establish the causes of these errors and misconceptions. The research explored whether Cognitively Guided Instruction (CGI) can assist in the improvement or reduction of these errors and misconceptions. The study was conducted at Mlungisi and Xhalanga primary schools (pseudo – names) of the Queenstown Education District in the Eastern Cape Province.

This was a mixed method research design, where 100 learners from two purposively identified primary schools were selected and subjected to a pre-test. From the 100 learners, 10 purposively selected learners were identified for interviews. The learners were also subjected to direct observations and documents (such as the Caps document, ANA examination papers, learners' home and classwork books, November/December Grades 5 & 6 mark sheets and schedules, etc.) which were analysed and a post-test was administered. Two mathematics teachers teaching at the two primary schools were interviewed. Triangulation was used for data analysis. A Cognitively Guided Instruction was used to investigate whether the errors and misconceptions can be reduced or improved.

The findings of the study were that learners do make errors when learning common fractions and that the types of errors were careless mistakes, conceptual errors, procedural errors and application errors which were identified and discussed in the study. The study further established that the Cognitively Guided Instruction approach recommended by the Curriculum and Assessment Policy Statement (CAPS) can be used to develop the learners' conceptual understanding of common fractions. The CGI approach advocates the use of concrete materials for conceptual learning to take place. The findings of the study confirmed well documented research on the learners' difficulties when dealing with fractions.

DECLARATION

I declare that this research is a product of my sole endeavour and it is my own unaided effort. It is being submitted in fulfilment of the requirements for the degree of Masters of Education to the University of Fort Hare. It has never been submitted for any other degree or examinations to any university.

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BABINI PRECIOUS LIBAZI

DEDICATION

This thesis is dedicated to my late father, Dlangamandla Christopher Libazi, my late mother, Jane Fundiswa (both of whom were inspirational and loved lifelong learning and education), my wife, Nontandazo, my daughter, Unathi, my son, Zuko, and my granddaughter, Mbasa.

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LIST OF ACCRONYMS

ANA	Annual National Assessment
CAPS	Curriculum and Assessment Policy Statement
CGI	Cognitively Guided Instruction
DBE	Department of Basic Education
ECDoE	Eastern Cape Department of Education

HOD	Head of Department
NCS	National Curriculum Statement
NCTM	National Council of Teachers of Mathematics
NSC	National Senior Certificate
PIRLS	Progress in International Reading Literacy Study
RNP	Rational Number Project
SACMEQ	Southern and Eastern Consortium for Monitoring Education Quality
SES	Senior Education Specialist
SGB	School Governing Body
SMT	School Management Team
TIMMS	Trends in International Mathematics and Science Study
ZPD	Zone of Proximal Development

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CHAPTER 1

INTRODUCTION AND BACKGROUND TO THE RESEARCH

1.1. Introduction

Mathematics as suggested by Erlwanger (1973, p. 54) is a rational and logical subject in which one has to reason, analyze, seek relationships, make generalizations and verify answers, and it consists of different sets of rules for different types of problems and these rote rules often result in various procedural errors and misconceptions. This study investigates the errors and misconceptions which Grade 6 learners make in the learning of common fractions, and the reasons for these errors and misconceptions. It also examines the effectiveness of the Cognitively Guided Instruction (CGI) in the teaching and learning of common fractions.

Riccomini (2005) stated that learners need to acquire proficiency in mathematics so as to enable them to understand and apply concepts because without proficiency in mathematics learners are likely to experience difficulty in completing other more advanced branches of mathematics like algebra. He defines proficiency in mathematics as gaining control over the mathematical register so as to talk like a mathematician. He further argues that learning mathematics is far more than knowing mathematical symbols and their meanings, algorithms and their ordered sequences, and being able to transform them to gain further knowledge and learning of the mathematics concepts.

In this introductory chapter the researcher presents the general introduction to the topic, the background of the study, the statement of the problem, the research questions, the purpose and significance of the study, and the delimitations of the study, ending with the explanation of some of the important terms and concepts used in the study.

Boulet (2000) expounded on the importance of students' mathematical ideas and what would result from lack of them. She wrote:

“Understanding is certainly the goal of learning, and teachers generally believe that their pupils understand their lessons. Without understanding, the learning of mathematics is reduced to the memorization of formulae and the rules governing them. Mathematics thus learned cannot be meaningful, but much less useful” (p. 119).

Mathematics and science are the key areas of knowledge for the development of individuals and for the social and economic development of any developing country like South Africa (DBE, 2012). Fractions are one of the basic concepts essential for further study of mathematical concepts like trigonometric functions, probabilities, algebraic functions, measurements, etc. Ma (2010), states that the above concepts demand a conceptual and deeper understanding of common fractions.

There is growing consensus among mathematics educators that rational number concepts, in particular the common fraction concepts, are among the most ubiquitous, multifarious and significant mathematical ideas that children encounter before reaching high school (Smith, 2002). The wide use of fractions in everyday life makes information and understanding about fractions necessary as early as the primary grades. Smith further argued that learners’ experiences with the fraction concept begins even before formal schooling and extends well into the high school years. However, even with this early introduction, students still have trouble conceptualizing fractions, probably forming one of the most critical barriers to the mathematical maturation of children (Aksu, 2000).

Behr et al. (1983) attributed many of the trouble spots in algebra to an incomplete understanding of earlier fraction ideas. The learners’ inability to perform basic operations on fractions has resulted in error patterns in the successful completion of algebraic exercises and problems. Wu (2001, p.58) underscored the idea by suggesting that “no matter how much algebraic thinking is introduced in the early grades, and no matter how worthwhile this might be, the failure rate in algebra will continue unless teaching of fractions and decimals is radically revamped”. He further stated that the proper study of fractions provides a ramp that leads students gently from whole number arithmetic up to algebra.

Ohlsson (1988) believed that the difficulty associated with fractions is semantic in nature: “The complicated semantics of fractions is, in part, a consequence of the composite nature of fractions. How is the meaning of 2 combined with the meaning of 3 to generate a meaning of $\frac{2}{3}$"? He further purported that the difficulty encountered in fractions is partially due to the result of the "bewildering array of many related but only partially overlapping ideas that surround fractions".

The importance of understanding fractions for mathematics learning, coupled with the poor performance on fraction items in internal and external assessments, have prompted various researchers to take a keen interest in this particular area of mathematics. Most of the prominent work on fractions dates as far back as the early 1980s with one of the most significant studies done on rational numbers - the Rational Number Project (RNP) – starting in 1979 and officially ending in August 2000.

Since 1980, the RNP has graced the research literature with reports on many investigations into teaching and learning fractions among fourth and fifth graders. Careful scrutiny of the research done on fractions to date reveals an attempt by various researchers (Charles & Nason, 2000) to focus their studies where fraction use and/or instruction begins, whether formally or informally, that is, in fifth grade or lower, with sparse research on how middle school or junior high school children make sense of this concept. Various studies on fractions have focused on middle school students' misconceptions or on the instructional approach undertaken by the instructor to lead students to a functional understanding of the topic (Mack, 2000).

Ashlock (2002) outlined a number of the misconceptions plaguing the fraction concepts and stressed the importance of students understanding the underlying concept of a fraction. These misconceptions include:

- Students equating the number of shaded parts in a diagram to the numerator and the number of unshaded parts to the denominator.
- Students associating the denominator of the fraction with the total number of parts indicated in the diagram and the numerator with the number of shaded parts, disregarding the fact that the associations apply only when all parts are equal in area.

- When asked to express a fraction in its lowest term the students will choose a specific factor of the numerator and one for the denominator thus $\frac{3}{8}$ will be reduced to $\frac{1}{4}$ by dividing the numerator by 3 and the denominator by 2.

A research review reveals that most studies done on fractions at the middle school level have focused primarily on students' misconception of computational procedures as the ones described above or on other representations of rational numbers such as percent, ratio, and proportional reasoning (Singh, 2002). Consequently, only a small portion of studies focused on the conceptual issues related to the understanding of fractions, how students make sense of the concept and the role, if any, that physical referents, including real-life problem solving activities, play in the students' understanding of fractions.

Because of the complexity of different subconstructs (part-whole relationship, measure, operator, quotient, and ratio) that can be assigned to a fraction, more in-depth and ongoing research needs to be done in the quest to enhance conceptual and functional understanding of this vital topic in the primary school (Stewart, 2005).

Common fractions can be socially used to emphasize the importance of sharing in the countries' resources. Therefore, an in-depth study and analysis of the pattern of learning errors and misconceptions which affect learners in these concepts can unlock a greater potential within the human capital of our communities with the production of scientists and engineers, thereby opening opportunities for self-reliance and employment within the labour market resulting in the country's economic growth (DBE. 2012).

However, the current picture painted by local and global educational critics, the Annual National Assessment (ANA) and the National Senior Certificate (NSC) results in South Africa, is a cause for concern. From the early seventies various research studies have been undertaken to improve the learning and teaching of common fractions but the status quo remains the same.

1.2 Background

The international and local educational commentators, including electronic and print media, have lamented and rated the quality of South African mathematics education below international standards. The Business Day (2013) reported, "The standard of mathematics teaching in South Africa is at rock bottom." The World Economic Forum (2014) reported that the quality of South Africa's mathematics and science education comes in the last place, behind the countries like Haiti, Lesotho, Chad, Zimbabwe, Nigeria and Kenya. According to this report, South Africa was at 146th place out of 148 countries. The Mail and Guardian (2012) observed that research conducted by Trends in International Mathematics and Science Study (TIMSS) between 1997 and 2011 showed that learners from rural public schools in South Africa performed worse than their urban counterparts in mathematics and science, with South Africa, Botswana and Honduras at the bottom. According to TIMSS (2011), there was a slight improvement in 2002 and 2006, with South Africa gaining 285 and 352 points out of 1000 points respectively in the two subjects. Learners in the Western Cape, Gauteng and Northern Cape performed better in the TIMSS assessment. The three lowest performers were KwaZulu Natal, Limpopo and the Eastern Cape.

In response to this challenge of improving the quality and levels of educational outcomes, the Department of Basic Education intervened by administering an Annual National Assessment (ANA) programme to monitor the extent to which these educational outcomes are achieved. The main purpose of this assessment was to strengthen the foundational skills of Literacy and Numeracy in line with the experiences from various international assessment programme in which South Africa actively participated. These included, among others, the regional Southern and East Africa Consortium for Monitoring Education Quality (SACMEQ), the global Progress in International Reading Literacy Study (PIRLS) and TIMMS.

The first ANA was written in February 2011 (DBE, 2012), involving about six million (6 000 000) primary school learners from Grades 1 – 6. The ANA results for Grades 3 and 6 were independently moderated by the Human Sciences Research Council. The Grade 6 results were as follows: In 2011, 2012 and 2013 the results for Grade 6 nationally were 25%, 27% and 39% respectively. In the same years the Eastern Cape results were 23, 7%, 24, 9% and 33%. In the Queenstown Education District during the same period the results were 30%, 23, 7% and 35, 4% respectively.

The diagnostic report for Grade 6 ANA results (DBE, 2012) identified gaps in specific skills, knowledge and competencies that displayed either lack of, or inadequate understanding of, common fractions. It was pointed out that the learners' performance in pure sums, where they had to perform basic mathematical operations without any distracting language components, was not different from their performance where they had to interrogate textual information and translate it into mathematical expressions and computations. Gross misunderstanding of basic mathematical operations was evident, regardless of the format in which these questions were crafted, i.e. pure sums, word problems and even multiple-choice questions (DBE, 2012). The report (DBE, 2011-2013) for Grade 6 further revealed that learners in this grade have serious computational and conceptual challenges in apparently all content areas in the mathematics learning area. The basic conceptual skills which they were expected to have in order to achieve the required minimum standards for progression were non-existent. The content knowledge and skills that characterize these areas determine the extent to which learners will make a success of their experiences in mathematics.

The qualitative analysis (DBE, 2012) suggested that most learners had not mastered knowledge and skills that were appropriate to calculate common fraction problems. Learners lacked competency in the addition and subtraction of mixed numbers (whole numbers and fractions). Learners had more procedural knowledge and understanding than conceptual skills to compute problems involving common fractions. This procedural approach to mathematics leads to memorization of theories and formulae without understanding. In a similar study conducted by Mdaka (2011) with a group of 49 purposively selected learners from within two classes of Grade 5, learners had to write a class work, homework and a test on addition and subtraction of common fractions. Interviews of the learners and their teachers were also conducted. The findings were that learners made a number of errors including conceptual errors, careless errors, procedural errors and application errors. Mdaka stated that the concern for her study has been that learners acquired misconceptions and made errors irrespective of the different school contexts. The data for research was collected from learners' written class and homework and tests, as well as from follow-up probing interviews, and teachers' responses to a questionnaire.

In a similar study undertaken by Dhlamini (2014) in the Gauteng Province on Grade 9 learners' addition of fractions and algebraic algorithms, the findings were that learners produced errors and revealed some misconceptions when they were adding algebraic fractions as a result of short cuts that they perceived as short and easy methods. This is consistent with the theory that learners invent imaginary rules which might not be correct. A further concern registered by Dhlamini was that learners performed badly in algorithm questions that involved fractions and suggested that knowledge of fractions is crucial for success in algebra and this was confirmed by Newton & Twiss-Garrity, 2014; Norton, 2013; & Siegler, Thompson & Schneider, 2011). They stated that fractions are most difficult to teach, most cognitively challenging and most essential for advanced mathematics. Duzenli-Gokalp & Sharma (2010) stated that the difficulty that the learners face when adding fractions is as a result of improper connections to everyday examples and mathematical problems on the addition of fractions.

Luneta & Makonye (2010) pointed out that errors and misconceptions may relate but are different in form and meaning. They identified two types of errors: systematic and non-systematic errors. According to them, non-systematic errors are exhibited without the intention of learners; learners may discover and correct them independently. Learners are likely not to repeat those errors. Watanabe (2002) attributed errors to short cuts done by learners to solve mathematical problems which result in errors. Systematic errors may be repeated. These errors are constructed or reconstructed over a period of time due to grasp of incorrect conceptions of solving a particular problem (Idris, 2011). Nesher et al. (2000) described misconceptions as a display of an already acquired system of concepts and algorithms that have been wrongly applied.

Suhrit & Roma (2010:170) are of the opinion that understanding the causes of errors and misconceptions can improve learner performance in fractions. Jamilah & John (2010) concur that the teaching of fractions is both important and challenging at the lower levels of schooling and that teachers should provide experiences that involve other mathematical concepts, including number, length, weight and money, as these experiences should be set in meaningful situations to which learners relate. Hansen (2006) stated that, despite these experiences, learners still experience difficulties with learning fractions because teachers tend to forget about the analysis of the errors displayed by the learners which they can use as sources of learning and teaching.

Yetkiner & Capraro (2009) state that middle school teachers need to possess a conceptual knowledge and understanding of fractional operations to deliver a sense-making curriculum. They further argued that teachers need to focus on students' attention to multiplicative reasoning at the fraction equivalency and that conceptually based instruction of fractions requires teachers to have a complete understanding of the subject matter. Jamilah & John (2010) further concurred that, although the concept of fractions is often complex in character, fractions provide learners with an important prerequisite for a conceptual foundation for the growth and understanding of the other number types of operations, including algebraic ones, in the later years of their school experience.

Most teachers within the Queenstown Education District generally acknowledge poor performance of learners in common fractions and related tasks and even some learners concur with this assumption that fractions are hard to grasp. Teaching of fractions seems to be the most worrying part of the mathematics curriculum to educators. Hence, the focus in this study is on how best can we teach and analyze misconceptions and errors in fractions and learners can effectively learn common fractions.

1.3 Statement of the problem

The Grade 6 ANA report (DBE, 2012) identified gaps in specific mathematics computational skills, knowledge and competencies which demonstrated either lack of, or inadequate understanding of, common fractions. Learners were unable to correctly add or subtract common fractions with the same denominator, common fractions with denominators which are multiples of each other and mixed fractions with the same denominators and those with denominators which are multiples of each other.

According to the Curriculum and Assessment Policy Statement (CAPS) published by the Department of Basic Education (DBE, 2011), learners who are in Grade 6 should have done and understood addition and subtraction of common fractions with the same denominators and some simple calculations on mixed fractions with the same denominators in Grade 5. The difference between Grade 5 and Grade 6 in this particular work (addition and subtraction) is that the learners should add or subtract common fractions with denominators which are multiples of each other (DBE, 2012). These trends seem to create more challenges for learners in the further study of mathematics in later years as confirmed by teachers in the higher grades

For example, in Grades 7 to 12, learners are expected to extend the general laws of exponents to include $a^{-m} = \frac{1}{a^m}$, they are expected to solve problems in contexts involving common fractions, mixed numbers and percentages, equivalence of fractions, estimation of place values of decimal fractions, to solve problems of higher order such as fractions and identity, factorization of fraction expressions, solve equations in trigonometric functions, probability, Euclidean geometry and so on. The above concepts usually constitute about 40% to 60% of the National Senior Certificate (NSC) final exam paper (depending on the examiner).

Going through learners' books and past papers in Grades 10 to 12, specifically in topics which deal with fractional concepts, including algebraic expressions, learners' poor performance is revealed. These problems can be attributed to learners lacking basic conceptual and computational skills in the addition and subtraction of common and mixed fractions. They lack conceptual understanding of the meaning of a fraction, the relative size of fractions and equivalence of fractions and the operations in common fractions.

1.4 Research questions

The researcher through this research sought answers to the following questions:

1.4.1What is the common errors made by Grade 6 learners in common fractions?

1.4.2 What are the reasons for these errors made by the learners in Grade 6?

1.4.3 How can Cognitively Guided Instruction (CGI) are used to improve learning and understanding of common fractions?

1.5 Purpose of the study

The purpose of this study was to investigate and determine the types of common errors and misconceptions made by Grade 6 learners in the Queenstown Education District in the learning common fractions. It further intends to find possible reasons for and causes of these errors and misconceptions made by Grade 6 learners when they learn common fractions. The study also investigated how Cognitively Guided Instruction (CGI) can be used to improve the learning and understanding of common fractions in Grade 6.

1.6 Significance of the study

This study will help to gain insights into the critical challenges that contribute to the commission of errors and misconceptions by learners in common fractions. It will further assist learners to identify these errors and be able to use effective strategies in calculating common fractions and eliminating such errors. It further seeks to assist teachers to effectively use CGI as a recommended approach in mathematics teaching in Grade 6. The findings could help teachers to identify and to do a critical analysis of the errors and misconceptions or reasons for these errors and misconceptions made by learners when learning common fractions.

The study can further help teachers to identify gaps in the learners' mastery or construction of their own knowledge and understanding of the common fraction concept. It is also hoped that the study will enable learners to learn better with understanding and confidence, using various strategies to calculate fractions and develop their own conceptual understanding in the topic.

Common fractions are basic foundations for future learning of mathematical concepts like trigonometric functions, construction of graphs, ratios, probability, algebraic expressions and equations, sequences and series, to name but a few. These concepts are critical in scientific innovations and research. The findings of this study might benefit learners in identifying and using effective mathematics learning strategies that could be used to improve their computational and conceptual skills in common fractions when furthering their studies. Furthermore, the lessons learnt from this study will be able to indicate patterns of teaching and learning which will result in improved learning of common fractions in the two schools in particular and in the Queenstown Education District and the province in general.

1.7 Delimitations of the study

This study was carried out in only two primary schools in the Queenstown Education District. Participants were comprised of 100 Grade 6 learners from the two primary schools and of these 100 learners only ten (10) learners, five from each of the two primary schools were interviewed. The two mathematics teachers of the two selected primary schools teaching mathematics in Grade 6 were also interviewed. The researcher focused only on those lessons in which learners add and subtract common and mixed fractions as these are the focus of the study.

1.8 Definition of terms

Approach/strategy – Means a method or form of instruction one uses to reach one's intended objectives, in this case teaching and learning common fractions.

Assessment **–** An act of collecting evidence to assess the achievement of outcomes by learners.

Authorities – People, who are in power or those who are managers or governors, e.g. district manager, SGB and/or SMT of a school.

Cognitive **–** Having to do with mental processes including thinking and reasoning.

Concept – A thought or idea formed mentally.

Errors **–** Mistakes made by learners as a result of carelessness, misinterpretation of symbols and texts, lack of relevant experience or knowledge related to a mathematical topic, learning objective or concept, lack of awareness or inability to check the answer given (Hansen, 2006 quoted in Mdaka, 2010).

Fraction - Fraction is a Latin word (*fractus*), meaning broken, which represents a part of a whole or, more generally, any number of equal parts, when spoken in everyday English; it describes how many parts of a certain size there are, e.g. one half ($\frac{1}{2}$), eight fifths ($\frac{8}{5}$), etc.

Learners **-** Pupils in a school (those who are being taught).

Learning Theories - Are conceptual frameworks that describe how information is absorbed, processed and retained by learners.

Misconceptions **–** Defined by Drew (2005) as the misapplication of a rule, an over-generalization or under-generalization or an alternative conception of the situation.

Policy brochures **-** Policy documents issued by the national and provincial departments of education. These include Curriculum and Assessment Policy Statement **(**CAPS); National Curriculum Statement (NCS), and policy documents which prescribe what has to be taught in Grade 6.

1.9 Conclusion

The results of assessments done locally, nationally and internationally on learners' knowledge and understanding of common fractions have displayed sufficient evidence to suggest that the fraction concepts are difficult to learn for the majority of learners. Some learners confuse fractions with whole numbers. This has caused real concern in the mathematics education arena, thus prompting individuals to try to understand, explain and/or offer suggestions and recommendations to deal with this challenge.

Clearly, as indicated above, the common fractions as mathematical concepts do pose a very big challenge in our schools and need greater attention from both teachers and learners.

The ANA results and the computational errors and misconceptions reported about Grade 6 learners' performance in common fractions motivated me to undertake this study so as to look in-depth at the errors made by learners in the addition and subtraction of common fractions and also to try to find a solution to eliminate the errors and misconceptions learners have with regard to this concept.

1.10 Chapter outline

CHAPTER 1 - Contains the introduction to the study, the background of the study, the statement of the problem, the research questions, the purpose of the study, significance of the study, delimitations of the study and the definition of terms.

CHAPTER 2 – The literature review includes the theoretical framework and the general review of literature related to the topic.

CHAPTER 3 – The research methodology contains the research paradigm, population and sampling strategy, data collection, ethical considerations, reliability and validity.

CHAPTER 4 - Research findings.

CHAPTER 5 - Data analysis, recommendations and conclusion.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

This chapter presents the review of literature related to the errors and misconceptions Grade 6 learners have or display when learning common fractions. The researcher reviews the opinions, arguments, suggestions and recommendations other authors have presented about errors and misconceptions of Grade 6 learners when learning fractions. The theoretical framework on which the study was based is discussed, followed by a discussion on the forms of fractions and the errors made by learners. This discussion includes explanations and illustrations of the meaning of the fraction concept as well as a discussion of the Cognitively Guided Instruction as an approach which teachers can use in assisting learners to learn common fractions effectively.

The researcher will further look at remedial action which can be taken by teachers to improve the commission of these errors and investigate the effectiveness of using physical models or manipulatives in the teaching and learning of common fractions. The researcher will also consider as a matter of importance the connection between the physical modelling and the symbolic notation, where learners are required to outgrow the manipulative stage of the actual algorithm. This is because some learners tend to remain in the physical model stage.

Teaching fractions for understanding is one of the main goals or objectives of education (Behr et al., 1983). It is therefore imperative for the researcher to look at the various forms of understanding to find the best form to define his instructional practices and how learners benefit from each model of understanding. It must be remembered that constructivist researchers who are scholars of cognitively guided instruction are of the opinion that the teachers plan or design their instructional strategy based on the learners' thinking and understanding, hence the researcher has got to look at these different models of understanding.

2.2 The theoretical framework

In this section the researcher examines two kinds of theories, the social constructivist theory by Vygotsky (1978) and the Hodes & Nolting theory (1998) focusing on the types of common errors and misconceptions of learners when learning common fractions.

2.2.1 The social constructivist theory

This study is grounded in the social constructivist theory of learning in order to understand how learners construct their meaning, including how learners' misconceptions and errors associated with learning common fractions are displayed.

Social constructivism refers to how learners socially construct knowledge linguistically through interacting with signs and symbols (Vygotsky, 1978). Vygotsky introduced two important concepts to constructivism: the first one being the Zone of Proximal Development (ZPD). He described the ZPD as the distance between two placeholders: the potential and the actual development. According to Vygotsky (1978), the potential development is the learner's ability to solve mathematical problems with the help of a more knowledgeable peer or adult and the actual development is the complete developmental cycle which is shown when learners solve mathematical problems independently. The second concept raised by him was scaffolding, which refers to the help given to learners by a teacher or a more knowledgeable peer at the beginning of a learning activity and is gradually withdrawn to allow the learner to build mental structures in collaboration with signs and symbols (Vygotsky, 1978). In the context of this, the researcher viewed errors and misconceptions that learners made independently when responding to the given tasks.

Bruner (2000) further concurred with Vygotsky by defining constructivism as the process whereby learners actively construct their understanding and knowledge of the world through experiencing things and reflecting on those experiences. He further stated that when learners encounter new things, they reconcile them with their previous ideas and experiences. Changes or the rejection of new ideas if found to be irrelevant are made with the help of a knowledgeable instructor whose role is to mediate the learners' construction of knowledge.

Watts and Bently (2003) are of the opinion that constructivism is a theory brought about by perception, language, memory and the ability to enquire beyond given information. Olivier (2001) described constructivism as the resultant knowledge of interaction between learners' learning experiences and previous knowledge. He further stated that the quality of existent knowledge drives learning and understanding in constructivism. Ausubel (2000) also agreed that constructivism refers to the construction of meaning through hypothesis generation, testing and generalizations.

This constructivist view is incorporated in this study in order to examine and determine the underlying factors that cause learners to commit errors and the type of misconceptions discussed below. The theory is relevant because it deals with how learners think when they are constructing their own knowledge, their reasoning and learning.

2.2.2 Hodes and Nolting theory

A study undertaken by Hodes and Nolting (1998) identified four types of errors made by learners in their manipulation of common fractions. These are:

- **Careless errors**: which they explained as mistakes made during the calculation process and can be easily detected when learners review their work.
- **Conceptual errors:** the second type of errors defined as mistakes made when a learner does not understand the properties or principles covered in the textbook and/or lecture.
- **Application errors**: are explained as those mistakes learners make when they know the concept but cannot apply it to a specific situation or question.
- **Procedural errors**: are the fourth type of errors which occur when learners skip directions, steps or misunderstand directions, but answer the question or the problem anyway.

When analyzing data in this study, the researcher used Vygotsky's social constructivist theory to examine how the Grade 6 learners constructed their knowledge, understanding and meaning in common fractions. The researcher further made use of Hodes and Nolting's findings on the various forms of errors and misconceptions to classify and analyze the errors and misconceptions that were displayed by learners in the study.

2.3 Why do we teach common fractions?

One of the reasons we teach fractions is that they are part of our daily lives and they are also taught to further investigate or explore important mathematical aspects of fractions as a number fraction and as an extension of whole numbers (Park, 2012). From existing studies which are about teaching and learning of fractions, Park identified five subconstructs of fractions as part-whole, measurement, ratio, operator and quotient.

We also teach fractions in order to compare them, to add and subtract them. Skemp (1986) quoted in Hansen (2006) is of the opinion that we learn or teach fractions because we need to measure. Skemp further argued that, if we want to measure the length of an object in centimeters, it may be that the object lies between two unit measures. To measure the object we need to introduce a fractional unit. Sharing an object or sets of objects is another example of a need to introduce a fractional unit.

Laridon et al. (2005) explained the concept of fraction as part of a whole with the top part of a fraction being referred to as the numerator and the bottom part as the denominator. Russell (2001) also explained fractions as part of whole, and stated that the key to teaching fractions to young children was to keep it concrete by using materials like fraction strips, fraction circles and other manipulatives.

Kerslake (2000) explained fractions as numbers and argued that it was ambiguous to explain fractions as part of geometric shapes. The manner in which fractions are taught mainly involves the following skills: naming the fraction, finding a fraction of a whole, discovering equivalent fractions, conversions to mixed numbers, addition and subtraction of fractions. De Turk (2008) is of the opinion that without a foundation in fractions, students who come to the study of rational expressions in Algebra will be severely handicapped.

Denise (2007) stated that elementary students need to know how to read the fraction and how to work with fraction families. He further said that learners need to be taught fractions because fractions are important to learners' test scores, and scores are important to college admissions officers, and that fractions are a necessary foundations underlying algebra and algebra is the gateway to all higher fractions.

Hansen (2006) observed that the effects of teaching fractions for understanding help children to be able to solve problems involving more complex fractions. Lamon (in

Couco, 2011) argued that a common error is to consider a fraction as two separate natural numbers rather than as one number. This becomes explicit in fraction comparison problems, when discussions are based on comparing components separately rather than on comparing the holistic fraction values. In such problems learners frequently get biasedconfused by the natural numbers that make up the fractions (Meert, Gre'gorie and Noel, 2010b; Van Hoof, Lijnen, Verschaffel, & Van Dooren, 2013). For example, learners believe that $\frac{1}{4}$ is larger than $\frac{1}{3}$ because 4 is larger than 3.

2.4 Sources of errors in common fractions

Cockburn (1999) suggested the following model to explain some of the commonest sources of mathematical errors:

- Experiences which learners bring to school differ remarkably and according to him mathematical errors may occur when teachers make assumptions about what the learners already know.
- The expertise of the learners with the basic rules for completing a given task: Misconception may occur when a learner lacks ability to understand what is required from the task.
- Lack of understanding of strategies or procedures: Learners may make mistakes or errors maybe due to a lack of understanding of which strategies or procedures to apply and how those strategies work.
- Imagination and creativity: He is of the opinion that mathematical errors may arise from a learner's imagination or creativity when deciding upon an approach using past experience as this may contribute to a mathematically incorrect answer.
- Mood of the learners: The mood with which the task is being tackled, may affect the learners' performance. If the learner is not in the right mood for working or rushes through a task, careless errors may be made.

2.5 The task and mathematical errors

Cockburn (1999) is of the view that avoiding errors in the learning of mathematics is impossible; however, he argued that these errors may be identified during the teaching and learning process. He outlined some common errors associated with mathematical tasks that teachers may take note of such as the following:

- Mathematical complexity – Learners may make errors if the task they are doing is too difficult.
- Presentational complexity – If a task is not presented in an appropriate way, a learner may become confused about what is required from her.
- Translational complexity – This requires the learners to read and interpret problems and understand what mathematics is required as well as understanding the language used.

2.6 The teacher and learner's mathematical errors

- Attitude and confidence: As with the learner, if a teacher lacks confidence or dislikes mathematics, the number of errors made within the teaching may increase.
- Mood: With the pressures of teaching today, teachers may feel under pressure or rushed for time and not perform to the best of their ability.
- 2.6.3 Imagination and creativity: Where a teacher is creative, she may teach concepts in a broader manner, looking for applications and alternative approaches, thereby reducing probability of error in learning.
- Knowledge: Too much teacher knowledge could result in a teacher not understanding the difficulties learners have whereas too little knowledge could result in concepts being thought in a limited way.
- Expertise: Expertise not may only be in the subject matter but also in how the teacher communicates with children and creates an effective learning environment. Without teacher expertise, some learners' mathematics may suffer.

- Experience that the teacher has: Knowledge can be gained from mistakes learners do and the teachers may learn about learners' misconceptions when they are teaching.

2.7 The meaning of the concept 'fraction' and how it is being taught

Common fractions are sometimes called proper or simple fractions and are written in the form $\frac{a}{b}$ or $a/_b$, where *a* and *b* are both integers. Common fractions can be classified as proper and improper. Proper fractions have smaller numerators and bigger denominators $\left(\frac{2}{3}\right)$ and improper fractions have bigger numerators than denominators $\left(\frac{3}{2}\right)$. Other forms are mixed fractions $4\frac{2}{3}$, ratios (2:3) and proportions, reciprocal $\frac{1}{7}$ x $\frac{7}{1}$ and the invisible denominator, complex fractions, compound fractions, decimal fractions, percentages and equivalent fractions.

Fraction is a Latin word (*fractus*), meaning 'broken', which represents a part of a whole or, more generally, any number of equal parts. When spoken in everyday English, it describes how many parts of a certain size there are, for example one half $\frac{1}{2}$, eight fifths $\frac{8}{5}$ etc. (Scholarly Google).

Chan et al. (2004) define fractions as a key foundation of elementary mathematics. They not only serve as a gate to real world situations but also form a basis for many mathematics concepts introduced in primary school (e.g. ratios, proportions, decimals, percentages and rational numbers). According to them, although the importance of fractions is generally recognized with a multiple year programme, they are traditionally considered one of the most difficult and troublesome mathematics domains. Fractions can be assigned various meanings according to the context in which the concept is used. Ohlsson (1988, p. 54) argued that in order to understand the meaning of fraction one has to "pay attention to the mathematical theory in which fractions are embedded, to which fractions are applied, and to the referential mapping between the theory and those situations".

Fractions form a part of a subset of rational numbers which form a subset of a larger set of real numbers. Freudenthal (2000) viewed fractions as the phenomenological source of rational numbers. Kieren (1976) divided the concept of fractions into five major subconstructs: part-whole relationship, measure, operator, quotient, and ratio.

Each subconstruct will be briefly described and the fraction $\frac{2}{3}$, where 2 represent the numerator and 3 the denominator, will be used to illustrate this concept for clarity.

- Part-whole relationship: This "depends directly on the ability to partition either a continuous quantity or a set of discrete objects into equal sized subparts or sets" (Behr et al., 1983). In this study the part-whole relationship is considered to be the ratio of the part to the whole. The fraction $\frac{2}{3}$ representing two equal slices out of a cake cut into three equal pieces.
- Measure: This refers to the position of a number on the number line or a ruler.
- Operator: In this context, the fraction is considered as a transformational operator which uses multiple operations to solve a problem e.g. changing from division to multiplication.
- Quotient: The quotient may sometimes be defined as the number of times the divisor may be subtracted from the dividend without the remainder becoming negative.
- Ratio: Contextually, the fraction is a symbol of comparison between two phenomena.

2.8 Misconceptions in common fractions

Askew and Williams (2001) suggested that one of the most significant findings in mathematical research carried out in Britain over the last twenty years has been that all learners constantly invent rules to explain patterns they see around them. They argue that, although many of these invented rules may be correct, they may apply only in a limited domain. When learners systematically apply incorrect rules or use a correct rule beyond their proper domain of application, we have a misconception. They further observed that to teach in a way that can avoid learners creating any misconceptions or errors is not possible and that we have to accept that learners will make some generalizations that are not correct and many of these errors and misconceptions will remain hidden unless the teacher makes specific efforts to uncover them.

Ashlock (2002) outlined a number of the misconceptions learners have in learning common fractions such as learners equating the number of shaded parts in a diagram to the numerator and the number of unshaded parts to the denominator. The second misconception is where learners associate the denominator of the fraction with the total number of parts indicated in the diagram and the numerator with the number of shaded parts, disregarding the fact that the associations apply only when all parts are equal in area. The third error is that when learners are required to express a fraction in its lowest term the students will choose a specific factor of the numerator and one for the denominator, thus $\frac{3}{8}$ will be reduced to $\frac{1}{4}$ by dividing the numerator by 3 and the denominator by 2.

Swan (2001), states that frequently a misconception is not wrong thinking but is a concept in embryo or a local generalization that the learner has made. He further suggests it to be natural stages of development. He is of the opinion that we should try to steer clear of activities and examples that might create space for misconceptions or errors. According to him, misconceptions cannot be simply avoided. The teacher therefore must have strategies for remedying as well as for avoiding misconceptions.

Sarra (2003) attributed learners' errors and misconceptions to lapse in concentration, hasty reasoning, memory overload, or a failure to notice important features of a problem. According to her, other mistakes may indicate alternative ways of reasoning and such mistakes or misconceptions should not be dismissed as wrong thinking as they may be necessary stages of conceptual development.

Misconceptions cannot be easily identified in the learners' written tasks, but can be understood by probing further into their supplied responses. Nesher (2000) stated that misconceptions can also produce correct contributions whilst Hatano and Smith (2005) argued that when learners' misconceptions are appropriately coordinated with other ideas they can and do provide points of continuity for the restructuring of current knowledge into new knowledge.

Hansen (2006) concurred with Hodes and Nolting (1998) in that errors or mistakes made by learners are as a result of carelessness, misinterpretation of symbols and texts, lack of relevant experience or knowledge related to a mathematical topic, learning object or concept, lack of awareness, or inability to check the answer given. She found out that misconceptions lead to errors.

Drew (2005) defined misconceptions as the misapplication of a rule, an overgeneralization or undergeneralisation or an alternative conception of the situation. Mistakes are displayed due to misconceptions learners have about a topic and they indicate incorrect interpretation of a mathematical idea as a result of a learner’s personal experience or incomplete observation. Luneta and Makonye (2010) in their study defined an error as a mistake, a slip, a blunder or inaccuracy and a deviation from accuracy.

Swan (2001) stated that a misconception is not wrong thinking but is a concept in embryo or a local generalization that the learner has made. According to him, the misconception may in fact be a natural stage of development. Hart (1981) viewed a very common error in the addition of fractions as the use of “add tops and bottoms rule”. According to Hart, this rule was more prevalent in examples where the two denominators were different. It was also interesting to note that this particular error occurred more when the question was posed in computation form than in word problem form. Askew and William (2001) are of the view that addressing misconceptions during teaching does actually improve achievement and long term retention of mathematical skills and concepts. They believe that drawing attention to a misconception before giving the example was less effective than letting the learners fall into a trap and then having the discussions.

Gabriel (2013) pointed out that the wrong conception resulted in learners facing difficulty in working with fractions because of the multifaceted nature of fractions which are: ratio, operation, quotient and measure. They further stated that the procedural and conceptual nature of knowledge that fractions demand poses confusion in learners’ choice of the correct notion that needs to be applied to a specific problem.

Watanabe (2002) argued that learners perform easy methods which are alternatives and end up making errors. He stated that it is vital to study these alternatives and correct them with the learners as they will become learners’ previous knowledge. He called these alternative methods short-cuts and he pointed out that the short-cuts are easier attempts by learners to solve the problems. Lukhele et al. (2002) stated that teaching fractions involves showing fractions as tools for equivalence and sharing. They further argued that when dealing with fractions the notion of equivalence is used to share wholes equally.

Another cause of errors is that learners often apply algorithms from other procedures in new concepts. For example, learners presume that fractions with big whole numbers are bigger fractions because they are employing incorrectly the algorithm of whole numbers (Lukhele et al., 2002). Siegler et al. (2013) state that fractions are difficult for many learners because learners assume that algorithms, procedures and properties of whole numbers are also properties of all other numbers. Bush and Karp (2013) explained that such concepts come as a result of learners not paying attention to procedures of fractions such as comparing and ordering which is the identification of different forms of numbers. They stated that fractions are rational numbers and does not hold place values as whole numbers. They revealed that between the numbers 1 and 2 there are two whole numbers but numerous fractions between the two numbers. The learners' focus is on the whole numbers in disregard of conceptual knowledge that comes as a result of the amalgamation of concepts.

Bush & Karp (2013) gave the following example where learners may add fractions in this manner $\frac{1}{4} + \frac{1}{4} = \frac{2}{8}$. According to them, this is a misconception caused by a misinterpretation of a concept. Had the learner understood that $\frac{1}{4}$ represents a quarter, then they would know that a quarter plus a quarter equals to a half, thus $\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$. In the above example, the learner mistook the numbers on the fraction as separate whole numbers and then added the numerator and denominators using the algorithm of whole numbers, i.e. using the 'tops add and bottoms add' approach.

Oliver (2001) carried out a study on how learners used the lowest common denominator (LCD) to add fractions. Learners listed multiples of two numbers, and used a correct common denominator to add the fractions. Findings of the study revealed that learners could correctly reach the required simplification.

Battista (2001) states that the way in which learners construct knowledge depends on the cognitive structures learners have previously developed. This means that there are conceptions and preconceptions that learners of different ages and backgrounds bring with them to mathematics classrooms, and if preconceptions are misconceptions, teachers need knowledge of strategies most likely to be fruitful in reorganizing the learners' understanding.

Higgins (2002) found out that possible causes of mistakes learners make may be attributed to lapses in concentration, hasty reasoning, memory overload or failure to notice important features of the problem. Bell (1993) found that the main cause of many learners' errors to be that they appear to understand a concept at the end of the unit, but do not retain it after a few months; in other words, the lack of long term learning. In contrast, learners with long term learning do not forget the acquired knowledge, and are able to apply it in real life situations. Askew and Williams (2001) suggest that a diagnostic approach as a teaching strategy can help promote long term learning and transfer from the immediate topic to wider situations.

Bell (1993) argued that students see scores and not weaknesses because they often want to know if their answer is correct or what score they got in a test, but don't want to go beyond scores to look into why they got the score. Yet paradoxically, this is development. Piaget (1972) explained development as consisting of four stages: the sensory motor, pre-operational, concrete operational, and formal operational stages. He stated that it is through these stages that we can understand the development of knowledge. For instance, the formal operation stage is when learners are able to hypothesis, reason and generalize. Piaget talked about three developmental processes of how children progress conceptually from one stage to another, namely: assimilation referring to the manner in which learners transform incoming information so that it fits within their way of thinking, accommodation as a stage where the learner receives new information which is quite different from the existing knowledge which then they try to reconstruct and re-organize ideas, and lastly equilibration referring to the keystone of developmental change between the learner's cognitive system and the external world. In other words, equilibration is the stage in which learners begin to realize the errors and misconceptions they have developed and further use these mistakes to restructure their existing knowledge.

2.9 Strategies to teach fractions for better understanding

Shulman (1986) argued that the ability to identify learners' misconceptions is based on teacher pedagogical skills or teacher competence. In other words, the teachers' main focus is not mainly on classroom management, preparing good lessons and presenting well-structured tasks, but also on the quality of questions about the content of lesson, and explanations given to learners.

2.9.1 The use of cognitively guided instruction as a strategy

Hansen (2006) stated that the learning of mathematics concepts requires a conducive learning environment where learners are free to interact and share their views. Learners' opportunity to communicate their views, including the errors they make when using mathematical concepts, are as a result of, among others, the creation of a conducive learning environment.

According to Carpenter, Fennema, Franke, Levi & Empson (2000) cognitively guided instruction (CGI) is a professional development programme based on an integrated programme of research focused on (a) development of students' mathematical thinking; (b) instruction that influences that development; (c) teachers' knowledge and beliefs that influence their instructional practices; and (d) the way that teachers' knowledge, beliefs and practices are influenced by their understanding of students' mathematical thinking. In order for teachers to distinguish what level each student is currently working at, problems are classified into different categories. The concrete exploration stage in CGI is called 'direct modeling'. Direct modeling is when the student literally does what the problem is asking with manipulating materials.

Constructivists believe that effective learning takes place when learners build their own knowledge. In the Curriculum and Assessment and Policy Statement (CAPS), the Department of Basic Education (DBE, 2012) proposed that the teachers use the CGI in their classrooms for instructional practices where learners use manipulatives to make sense of their learning and promote learner mathematical thinking.

2.9.2 What is cognitively guided instruction?

Carpenter et al. (2000) described CGI as an inquiry-based approach to teach mathematics which was developed at Wisconsin Center for Education Research. According to them this approach provides teachers with knowledge about the developmental stages of children and their mathematical reasoning. This knowledge enables teachers to plan mathematics instruction based on their learners' understanding and guide them towards greater mathematical reasoning and concept mastery.Their research was cyclic starting with explicit knowledge about the development of learners' mathematical thinking. They used their research as a context to study teachers' knowledge of learners' mathematical thinking. They found out that, if teachers were expected to plan instruction based on their knowledge of learners thinking, they needed some coherent basis for making instructional decisions. To address this problem Carpenter et al. (2000) designed CGI to help teachers construct conceptual maps of the development of learners' mathematical thinking in specific mathematical domains. After a series of studies (Carpenter et al. 1989, 1993, 1996) found that learning to understand the development of learners' mathematical thinking could lead to fundamental changes in teachers' beliefs and practices and that these changes were reflected in their learning.

The studies provided sites for examining the development of learners' mathematical thinking in situations where their intuitive strategies for solving problems were a focus for teachers' reflection and discussions. Other studies (Carpenter et al., 1993, 1996, 1998) provided new perspectives on the development of learners' mathematical thinking and the instructional contexts that support that development, which in turn have led to revisions in their approach to teacher development.

Watson (2003) described CGI as an approach to teaching mathematics that uses a learner's own mathematics thinking as the basis of instruction. He describes this method as having its premise from the underlying assumption that learners already possess an informal knowledge of mathematics. He further states that the teacher's role is to build from this prior knowledge so that learners can eventually make connections between situational experiences and the abstract symbols typically found to represent them in mathematical equations. He reveals that this approach is quite different from the traditional method of teaching symbolic computation first, and then expecting the learners to apply the concepts to problem solving situations.

Watson was of the opinion that, for CGI to be successful, teachers must have a thorough understanding of the grade level curriculum and the distinctions between the different problem types with that curriculum. Teachers must also have a working knowledge of the processes that are typically used to solve those problems, and the various stages that learners go through in developing their own knowledge.

2.9.3 Word problems

Hub (2012) also concurred with Watson (2003) in that learners in a CGI math's classroom spend most of their time solving word problems. Learners are encouraged to use and develop a variety of self-selected strategies and models to solve the problems. They are also held accountable for explaining exactly how they solve the problem. The teachers then use this information to guide the learners' learning and identify any breakdowns or misconceptions that might occur. Engaging in this type of ongoing learner assessment provides the teacher with the opportunity to:

- Identify what base knowledge the learner has.
- Understand different strategies employed for solving problems.
- Understand what those strategies tell us about the learners' thinking.
- Understand and demonstrate the ways that this thinking evolves.

2.10 Physical models and the learning of the fraction concept

Cramer et al. (2002) revealed that representations are useful when used to communicate mathematical approaches, algorithms of adding fractions and to support learners' understanding of the concepts for adding and subtracting fractions and their relationships. Watanabe (2002) showed that representations were meaningful to those who created them; therefore, it was critical that teachers allow learners to create their own representations and only guide them to use representations correctly to add fractions or subtract fractions. Siegler et al. (2011) stated that in constructivism representations are scaffolds that are essential in the addition of fractions. Results from a study by Gabriel et al. (2013) revealed that primary school learners master the part-whole and proportion categories of fractions and struggle to understand fraction as numbers, which resulted in learners applying procedures that they do not understand.

The issue of the use of manipulative materials in the mathematics classroom is a worthy debated. Some research articles reported on the ineffectiveness of physical models in helping the students understand the underlining mathematical concept involved within a fraction, and the difficulty children have in moving from using physical models to using symbolic notation (Bezuk and Bleek, 1993; Thompson and Saldanha, 2003). The majority of the research is conditional on the use of manipulatives in a mathematics classroom. However, some research has shown that the active use of manipulatives in a mathematics classroom may assist in the understanding and in the development of the fraction concept (Hall, 2008).

These concrete analogs are used by teachers with the conviction that they will facilitate the construction, understanding and retrieval of knowledge verifying the truth. They mediate transfer between task and situation, and indirectly facilitate transition to higher levels of abstraction (Boulton-Lewis, 1998).

Physical referents play a pivotal role in the acquisition and retention of vital mathematical concepts (Stewart, 2005). The NCTM (2000) in its presentation of its study on representations by Grades 6-8 learners stated that representation is central to the study of mathematics. Learners can develop and deepen their understanding of mathematical concepts and relationships as they create, compare and use various representations.

The “representations such as physical objects, drawings, charts, graphs and symbols also help students to communicate their thinking” (Stewart, 2005, p. 280). From the learners’ perspective, the representations could be their reconstruction or drawing of a contextualized problem situation or their interpretations or use of a representation that was structured or designed by the teacher/researcher. Building on the theories of Piaget, Bruner, and Dienes, the Lesh’s translation model (Cramer, 2002, p. 325) suggested that “a deep understanding of mathematical ideas can be developed by involving students in activities that embed the mathematical ideas to be learned in five different modes of representation with an emphasis on translations within modes.” These five modes are manipulatives, pictures, real life contexts, verbal symbols and written symbols. Cramer stressed that understanding is reflected in the ability to represent mathematical ideas in multiple ways, plus the ability to make connections among the different embodiments.

The translations that are within and between various modes of representations make ideas meaningful for students. The Lesh’s translational model shows the connections among different external representations including real world situations and manipulatives or aids. This instructional model necessitates the active involvement of students, as well as the extensive use of manipulatives and other learning aids such as pictures, diagrams, students’ verbal interactions and real life situations, and simulations modeled in the classroom. Each model is connected to every other model, performing a translation such as picture to symbolic. Each connecting link is bidirectional. This signifies the interconnection with the different modes evidenced during problem solving.

2.10.1 Effectiveness of physical models

Although physical models are recommended for helping learners in learning about mathematics, the benefits of using physical models for instructions have not been clearly demonstrated (Wenrick, 2003). Reviewing previous research on the effectiveness of using base ten blocks for developing learners' understanding or ability with computation, Thompson et al. (2003) found conflicting results. He proposed that these different conclusions may be accounted for by how learners utilized concrete materials, how concrete materials were connected to symbolic representation, and the overall goals of the studies. In addition to learning about relationships between the physical model and the symbolic notations, learners needed to identify and use relationships included in the construction of the physical model. These relationships impacted what learners understood so they could solve the word problems and justify their answers.

The Rational Number Project (RNP) has investigated the learners developing understanding of fractions through curriculum which includes a variety of manipulatives or physical models (Behr et al., 1983). RNP is a major research project that examined the use of physical models in the development of learners' conceptual understanding of rational numbers, including comparing and ordering fractions. Results from a small group teaching experiment indicated that fourth grade learners were developing a quantitative notion of fractions that provided a foundation for order and equivalence (Behr et al., 1983).

For learners to develop this understanding of fractions, they had to find and build on relationships between the numerator and the denominator of a fraction and relationships to make comparisons to referents points. Learners used whole number relationships some of the time when they solved the problems; however, it caused problems when the denominators were not equal.

Researchers have proposed multiple approaches for how physical models should be used. Many recommended specified steps that should be taken to introduce physical materials (Behr et al., 1983), whereas others provided a framework for considering how the physical materials should be utilized. Gravemeijer (2001) described both a "top-down and a bottom- up" approach for using concrete models in the classroom. According to him, the concrete models can be used as tools for thought or as tools to get an answer. He further distinctly provided a useful framework for analyzing the approaches other researchers used with physical models. He named it the Physical Model Framework.

2.10.2 The physical model framework

The 'top-down and bottom-up' strategies are different ways to connect formal mathematics and physical models (Gravemeijer, 2001). In a top-down approach the physical models are developed by experts who then introduce the materials to learners. Since the experts understand the underlying mathematics concepts, the physical models embody relationships the expert values and wants students to learn. Learners are taught how to use the physical models to solve a problem.

The intention is to use the physical materials so that learners will understand the relationships the expert intended, which will aid in connecting the physical models with the symbolic notation. An example cited by Gravemeijer was that base ten blocks are chosen because they illustrate the relationships between ones, tens, and hundreds in the place value system. Learners are shown how to subtract using base ten blocks by starting with the ones and trading as needed and then they learn how to record these steps in the traditional algorithm form (Thompson et al., 2003).

Note that learners are using physical materials to solve a problem, but the steps they follow are not connected to their informal knowledge or their previous learning. To connect learners' informal knowledge with formal mathematics, Gravemeijer advised that learners needed to begin by developing their own models using a bottom-up approach. By presenting problems in real life contexts builds an inductive knowledge that learners have before instruction (Behr et al., 1983) and requires learners to find and build on relationships they understand.

In a study Streefland (1991) gave learners a problem, with learners sitting at a table and sharing pancakes and they had to determine each learner's fair share. Then learners were given the opportunity to work with physical models, including using drawings, to help figure out the problems. Frequently learners used repeated halving to share the pancakes, which was similar to the strategy Brinker (1997) reported.

Learners were developing their own physical representations, which Gravemeijer (2001) called models of the mathematical phenomena. As learners had opportunities to explore similar problems further and understand relationships, they translated their drawings into the ratio tables later in the curriculum unit (Streefland, 1991).

In using a model for mathematical reasoning, learners may use physical materials to help them figure out an answer or as a way to think about mathematical relationships. However, in a top-down approach where there is a focus on the procedure learners are likely to use the physical models as tools to get an answer (Gravemeijer et al., 2001). If learners' explanations for how they solved the problem are based on procedures they followed, they are using an analytic argument to support their answer.

When learners are primarily concerned about getting an answer, they will not pay attention to relationships in the physical models (Wenrick, 2003). In a bottom-up approach, learners may use the materials either to get answers or as tools for thought. To encourage the use of the materials to further thinking and understanding about concepts, problems must go beyond the available physical materials and require learners to provide a substantial argument to justify their answer. Unlike analytical arguments that follow the procedures, a substantial argument is based on relationships and generalizations. By providing numbers beyond the use of physical materials, learners must extend their thinking and use relationships to solve problems.

2.10.3 Recommendations for instruction

In the summary of the literature, Bezuk and Bieck (1993) emphasized the importance of using a variety of physical models to develop fraction concepts including: fraction kits either prefabricated or made by the teacher or learners; Cuisenaire rods, pattern blocks, colored chips, colored tiles, and number lines of different sizes with different patterns. They say that by using a variety of models, learner's develop different understandings of fractions as part of a whole, part of a group, and a place on a line. Bezuk and Bieck urged only using fractions that could be represented using the manipulatives which was contrary to Gravemeijer's (2001) notion to include problems that could not be represented using manipulatives.

Since learners could use the manipulatives to solve problems, it might lead them to use models as tools to get an answer. Bezuk and Bieck (1993) also indicated that learners should use more than one mode of representation in instruction. After learners have understood a concept using different materials which required students to use as tools for thought approach to the manipulatives. Through this process, learners could compare and contrast the physical materials, which should bring out relationships both within specific models and across different models.

In addition Bezuk et al. (1993) explained that the verbal and written forms had to be connected to the physical materials, always building upon learners' understanding developed through the use of the manipulatives. The recommendations could be interpreted in both a bottom-up approach and a top-down style for using physical materials. Thompson et al. (2003) rejected the top-down approach, focusing instead on what learners should do with or without physical materials when planning for instruction. They emphasized that the teachers' first concern must be "What do I want my students to understand?" When teachers were solving mathematical problems, the physical materials provided hands-on ways for learners to find ans xwers and a concrete way to describe their solutions to the class. Through working with the materials and talking about both the physical materials and the related mathematical conversations, they focused on constructing strong connections among ways of thinking about concrete situations, conventional mathematical language and notation (p. 558). They further provided some hints for using materials, but always returned to what learners should understand as a guide for making instructional decisions.

In contrast to the general approaches described above, Peck and Jencks (2004) provided a specific set of steps for using physical models that followed a bottom-up approach. They proposed posing problems in the beginning for learners to engage in while working with the physical materials. The instructor does not tell the learner's specific steps for using the manipulatives, rather learners figure out what to do through their active involvement in working with the materials to solve problems. The teacher introduces symbols in relation to the learners' work with the physical models. Peck et al. (2004) Declared, "As the children use these physical materials to solve problems, they actively constructed operations and principles of arithmetic (p. 333). As teachers encourage learners to move beyond the physical materials, learners first draw pictures of the physical materials. Through these steps, they believed that learners could reach the fifth level where learners have strong arithmetic generalization and problem solving skills.

In curriculum, the teaching and learning materials are developed for research purposes and for classroom use, where the instructor introduces learners to a variety of concrete manipulatives such as fraction circles, rectangles, Cuisenaire rods, paper folding, number lines and counting chips during instruction (Cramer et al., 2002). Most of the lessons required learners to use the manipulatives.

Similar to the proposal by Bezuk and Bieck (1993), the Rational Number Project emphasized making translations with concrete manipulatives (Cramer et al. 2002). There are two types of translations learners were expected to learn. One kind of translation was between different manipulatives. This type of activity could lead to a bottom-up approach for using physical models, yet the descriptions indicated the translations were taught in a top-down manner (Wenrick, 2003).

Behr et al. (1983) suggested that after learners became skilled with one particular manipulative, the teacher should introduce a second one and guide the class in a discussion about how the teacher used this manipulative, how they could use it themselves, and how it was similar to and different from the previously used manipulatives. When learners are using a procedure modeled by the teacher, they may be inclined to use manipulatives as tools to get an answer and not identify or use relationships.

At the same time, answering questions about the similarities and differences may encourage learners to use the manipulatives as tools for thought as they look for relationships between and within the material. The other type of translation requires moving between the five forms of representations: manipulatives, pictures, real life examples, written symbols and spoken language (Cramer, 2002). This emphasis on translations encouraged learners to use the manipulatives as tools for thought as they made identified relationships or connections across a variety of representations and moved fluently between the different representations.

When learners solved problems without manipulatives, Cramer recommended that learners should not be instructed to use them. These are a variety of ways in which learners use manipulatives to develop their understanding of the fraction concepts.

Making connections between physical materials and symbolic notation is a part of developing fraction concepts. The RNP emphasized translating between real-life examples, physical representations including manipulatives and pictures, the oral words and written symbols. This is similar to the approach described by Bezuk and Bieck (1993).

In a more limited manner, Peck et al. (2004) stressed that symbolic notation must always be connected to representations. Though there are many opportunities for learners to seek out and extend relationships in the physical models, not all learners make these connections.

2.10.4 Concerns about connecting physical models to symbolic notations

Even though the intention in many of the previously mentioned studies was to make connections between physical models and symbolic notation, researchers have documented the disconnection between materials and symbols in learners' learning (Brinker, 1997).

Learners were sometimes satisfied with two different answers that they found using manipulatives and symbols (Brinker, 1997) or learners switched to using symbols to solve problems when they recognized the manipulatives and symbols or learners switched to using symbols to solve problems when they recognized the manipulatives only gave an approximate answer. Furthermore, learners did not try to connect their symbolic approach to the fraction strips, and a learner who relied on the fraction strips was unable to use symbolic strategies to find exact answers. As described by Hope and Owens (1986), these learners see little connection between the worlds of the physical and symbolic. Each is viewed as a different system operating independently.

Some physical models allow learners to work without making connections to the representations. In another study where learners were using pattern blocks, they did not refer to the fractional relationships on their descriptions of the various ways to make hexagons (Mewborn, 2001). Instead, learners talked about the different pieces by colour, such as: "I used two reds or one red, one blue and one green". Mewborn (2001) hypothesized that the pattern block did not fit the teacher's ways of discussing fractions using a part-whole approach, where the denominator referred to the number of pieces and the numerator referred to how many pieces were shaded.

Although the researchers intended for certain fractional relationships to be explored through these materials, the teacher missed the opportunity to make connections between the representations with pattern block and terminology.

The National Council of Teachers of Mathematics (NCTM) in its publication of principles and standards for school mathematics (2000) outlined one of the next principles for school mathematics. Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge. They (NCTM, 2000) went on to say that mathematics must be learnt by all students.

Whilst the National Research Council and its (2000) national report stated that only in the United States do people believe that learning mathematics depends on a special ability, in other countries learners, parents and teachers all expect that students can master mathematics if only they work hard enough. The records in their countries and in some intervention programmes in the United States show that most students can learn much more mathematics than is commonly assumed in this culture. It is therefore the teacher’s responsibility to provide the opportunity for all learners to learn this subject. This includes mathematical concepts that are consistently difficult for students to understand such as fractions, ratios and prepositions. One aspect of the teacher’s role is to provoke students’ reasoning about mathematics. Teachers do this through the task they provide and the questions they ask.

Teachers are responsible for the quality of mathematical tasks that are likely to promote the development of students’ understanding of concepts and procedures in a way that also fosters their ability to resolve problems, to reason and to communicate mathematically. There is documented evidence that direct instruction may not provide an adequate base for students’ development and for students’ use of high cognitive skills (Stewart, 2005). Burns (2000) believed that for students to learn mathematics it becomes imperative for them to create and recreate mathematical relationships in their own minds. When providing proper instruction teachers can be seduced by the symbolism of mathematics. Children need direct and concrete interaction with mathematical ideas. Continuous interaction between a child’s mind and concrete experiences with mathematics in the real world is necessary (Burns, 2000).

Defining quality instruction has been a goal of researchers from the beginning of formal schooling. Over the last forty years data have accumulated showing that students who receive quality instruction demonstrate more successful school learning than students who do not (Joyce, Weil, and Calhoun, 2003). While most individuals value student learning, a major problem arises when quality instruction is discussed outside the context of specific educational objectives or desired educational outcomes. Gardner (2002) made this point emphatically: "I do not think it is possible to talk intelligently about how to teach unless one has taken a stand on what one should teach and why (p. 72).

Huitt (2009) was of the opinion that the curriculum objectives, the form and instruction of assessment, and the standards used for evaluation come first; discussion of quality instruction should follow. Therefore the analysis of the design of instruction must give consideration to the product of instruction. "While there is a national discussion regarding desired outcomes for a successful adulthood in the 21st century, at this time the most widely used measures of student learning are standardized tests of basic skills" (Huitt, 2009 p. 65).

2.11 The use of learners' errors in teaching fractions

Melis (2008) argued that in order for learners to shift from routine and factual knowledge to more emphasis on developing competencies such as solving mathematics related problems, reasoning and communicating mathematically, learners should be encouraged to explore, verbalize ideas, and build confidence within them in order to learn mathematics. According to him this can be done by accommodating their mistakes. Learners' mistakes are considered to be a source of their learning, and research shows that learners learn best if they are involved in the construction of their own knowledge (Suffolk, 2008). Therefore, they are free to show their reasoning and understanding knowing that their errors are the stepping stones to stimulate their conceptual development and effective learning.

Luneta et al. (2010) argued that mistakes are legitimate attempts to understand mathematics. This can be evident when learners actively attempt to make sure of their experience by trying to link everyday knowledge and school knowledge, for example, when learning fractions and relating it to their concept of charity at home. However, Olivier (2001) stated that concepts are not taken directly from experience, but that a person's ability to learn from and what they learn from experience depends on the quality of ideas that he or she is able to bring to that experience.This means that a person's experiences in learning depend on his/her interpretation and other thoughts about such experience. Research by William and Ryan (2000) and Suffolk (2008) continued to support the idea that knowledge of common mathematics errors and misconceptions of children can provide teachers with forces for teaching and learning. Hansen (2006) argued that the most effective teachers in mathematics cultivate an environment where pupils do not mind making mistakes, because making mistakes or errors is seen as part of learning. Hansen, drawing from Swan (2001), further argued that trainee teachers needed to be taught to recognize pupils' common errors and misconceptions in mathematics, understand how these arise, how they can be prevented and learn how to remedy them.

This means that, although mathematics teachers should not teach to hide or avoid learners' mistakes, at some stage they need to minimize them, and to be careful of how they handle them. This can be done by applying various teaching and assessment strategies that take cognizance of the value of misconceptions in teaching mathematics. Hansen (2006) suggests that placing children in situations where they feel in control of identifying mathematical errors and misconceptions leads to greater openness on the part of the learners to explore and discuss their own misconceptions.

Jansen (2001) argued that instead of teachers becoming the dominant force in the classroom, they become a guide on the side rather than the sage on the stage. This suggests that a teacher's image should slowly but deliberately move from the centre stage into an invisible position in the classroom in order to facilitate conditions where learners take charge of their learning. Thus, with the help of the teacher as a facilitator, learners' mathematical understanding is likely to be developed because they are in a position to compare their thinking with that of fellow learners in the context of error analysis.

2.12 Conclusion

In the studies that have been analyzed and concluded above, fractions have been proved and seen as one of the difficult concepts of mathematics. They are not only part of the curriculum of mathematics but components of our daily lives and therefore it is the duty of the teachers to develop instructional strategies which are conducive to the effective learning and understanding of the fraction concept. The studies have revealed that there is no perfect or specific way of solving a mathematical problem, and there is no correct way either. If educators can keep in mind the latter statement, then mathematics learning and teaching will be fun and rewarding. It is therefore the duty of the teacher and learner to come up with their comfortable ways in which they can develop or construct their own conceptual meaning of mathematical signs and symbols which will lead to their effective computation in future.

Although the top-down and bottom-up strategies are very different, these approaches provide opportunities for learners to connect physical materials with formal mathematics. Examining whether learners use the physical materials as tools to get an answer or as tools for thought provides another framework to view approaches for using manipulatives (Gravemeijer, 2001). By using a bottom-up approach to introduce physical models and expecting learners to use them as tools for thought, students are required to focus on relationships.

The CGI is an approach to teaching mathematics that uses a student's mathematical thinking as the basis for instruction. This method is the result of research conducted by Carpenter and Fennema (1989-1999). Its premise is based on the underlying assumption that students can eventually make connections between situation experience and the abstract symbols typically found to represent them in mathematics equations. This method is quite different from the traditional method of teaching the symbolic computation first and then expecting students to apply the concepts to problem solving situations.

For CGI to be successful teachers must have a thorough understanding of the grade-level curriculum and the distinctions between problem types within that curriculum. They must also have a working knowledge of the processes that are typically used to solve these problems and those the various students go through in developing their own concrete knowledge and understanding. Most accomplished teachers use a variety of practices to extend children's mathematics. The research on learners' mathematical understanding and learning upon which CGI is based shows that leaners are able to solve problems without direct instruction by drawing upon informal knowledge of everyday situations (Bezuk and Bieck, 1993).

CHAPTER 3

METHODOLOGY

3.1. Introduction

All research is based on some underlying philosophical assumptions about what constitutes a “valid” research and which research methods are appropriate for the development of knowledge in a given study. In order to conduct and evaluate any research, it is therefore important to know what these assumptions are (Myers, 2009). The methodology chapter is where the philosophical assumptions and the design strategies are discussed. It further discusses the research methodology and the research design incorporated in it, including strategies, instrumentation for data collection and methods of data analysis.

The study begun with a quantitative approach in the strategic sampling stage in which learners who were going to be interviewed were selected. A pre-test, post-test, document analysis, interviews and direct observations were the instruments used to collect quantitative data where learners’ work was marked and quantified using a memorandum. After the quantitative approach, a purposeful selection of ten learners who were to be interviewed was performed, and the qualitative approach was then undertaken using the following instruments to collect qualitative data: observation, interviews and document analysis.

3.2 The research paradigm

A paradigm is a set of beliefs and assumptions about how something works and can be seen as a framework for how we interpret things we observe or learn (Guba & Lincoln, 1994, quoted in Henning, 2004). Terreblanche and Durrheim (2001) stated that the research process has three major dimensions which are ontology, epistemology and methodology. They further said that a research paradigm encompasses a system of interrelated practices and thinking that define the nature of the investigation along these three dimensions. Kuhn (1977) defined a paradigm as an integrated cluster of substantive concepts, variables and problems attached with corresponding methodological approaches and tools. He argued that the term ‘paradigm’ refers to a research culture with a set of beliefs, values and assumptions that a community of researchers has in common regarding the nature and conduct of the research.

A paradigm therefore implies a pattern, structure and framework or system of scientific and academic ideas, values and assumptions (Olsen, Lodwick and Dunlop, 1992). They argued that ontological and epistemological aspects concern what is commonly referred to as a person's worldview which has significant influence on the perceived relative importance of aspects of reality. Olsen et al. concurred with Lather (1986) in that they also viewed a research paradigm as inherently reflecting on our beliefs about the world we live in, hence the researcher employed mixed methods research.

3.3 The research methodology

Myers (2009) argued that the research method is a strategy of inquiry and data collection. According to him, there are other distinctions in the research modes, of which the most common classification of research methods is the quantitative and the qualitative. On the one hand qualitative and quantitative refers to distinctions about the nature of knowledge, i.e. how one understands the world and the purpose of research. On the other hand, on the level of discourse the terms refer to research methods that are the way in which data are collected and analyzed and the type of generalizations and representations derived from data.

3.3.1. What is qualitative research?

Qualitative research is naturalistic; it attempts to study the everyday life of different groups of people and communities in their natural settings, and it is particularly useful to study educational settings and processes. "… qualitative research involves an interpretive, naturalistic approach to its subject matter; it attempts to make sense of or to interpret phenomena in terms of meaning people bring to them" (Denizen and Lincoln, 2005). Domegan and Fleming (2007) viewed qualitative research as aiming to explore and to discover issues about the problem on hand, because very little is known about the problem. Myers (2009) viewed qualitative research as designed to help researchers understand people, and the social and cultural contexts within which they live.

In qualitative research, different knowledge claims, enquiry strategies and data collection methods and analysis are employed (Creswell, 2003). He further argued that qualitative data sources include observation, participant observation, interviews and questionnaires, documents and texts, and the researchers' impressions and reactions. Data is derived from direct observation, interviews, from written opinions or from public documents. Written descriptions of people, events, opinions, attitudes and environments, or combinations of these can also be sources of data. An obvious basic distinction between qualitative and quantitative research is the form of data collection, analysis and presentation.

3.3.2 What is quantitative research?

Quantitative research makes use of questionnaires, surveys, and experiments to gather data that is revised and tabulated in numbers, which allows the data to be characterized by the use of statistical analysis (Hittleman et al., 2003). They further stated that quantitative researchers measure variables using effective statistics such as correlations, relative frequencies, or differences between means, and their focus is to a large extent on the testing of theory. Fischer (2000) defined quantitative research as a type of educational research in which the researcher decides on what to study; asks specific, narrow questions; collects quantifiable data from a large number of participants; analyzes these numbers using statistics; and conducts the inquiry in an unbiased, objective manner. He further explained it as ‘postpositivist’ as it is anchored in singular realism as an objective and a deductive inquiry. Quantitative inquiry generally attempts to quantify variables of interest and research questions to be answered must be measurable. It involves collecting numerical data that can be subjected to statistical analysis. Quantitative inquiries use performance tests, personality measures, questionnaires and content analysis to collect ‘hard’ data.

3.3.3 Mixed methods research

When defining mixed methods research, Fischer (2000) said that a mixed method research is a procedure for collecting, analyzing and “mixing” both quantitative and qualitative research and methods in a single study to understand a research problem. He further stated that to utilize mixed methods effectively, a researcher must understand both quantitative and qualitative research. Mixed methods can always be used when both quantitative and qualitative data, together, provide a better understanding of your research problem than either type by itself. It can further be used when one type of research is not enough to address the research problem or answer the research questions.

This type of inquiry addresses pragmatism with practicality. It has multiple viewpoints and it can also be used to build from one phase of a study to another. Onwuegbuzie and Leech (2005) and Johnson (2007) argued that the mixed methods approach works beyond quantitative and qualitative exclusively or in affiliation, and in a pragmatic paradigm. According to them, the mixed method draws on and integrates both numeric, and narrative approaches of data collection. Johnson was of the opinion that, in order to answer the research questions fully, both the qualitative and quantitative methods are necessary and desirable; hence the mixed methods. This approach was suited to my study in that it addressed the researcher's research questions and the purpose of the study without bias. The method also addressed both the qualitative (narrative) of the study in terms of what actually took place and the quantitative (numeric) achievement of the learners.

3.4 The research design

Research designs are procedures for collecting, analyzing, interpreting, and reporting data in research studies. They represent different models for doing research, and those models have distinct names and procedures associated with them. Research designs are useful, because they help guide the methods discussions that researchers must make during their studies and set the logic by which they make interpretations at the end of their studies (Patton, 2002).

The research design for this study was multiple case studies as it was conducted in two primary schools. The design sort to understand, explain and interpret data that was collected and to compare it against the existing knowledge to "… understand, interpret and explore the causes of these phenomena with a view to make the teaching and learning of common fractions more effective" (Thomas, 2011 p. 105). The study was exploratory and interpretive in nature as it was framed by the expectation that the research would be looking to evaluate how the errors and misconceptions made by learners contribute to their understanding and performance in the learning of common fractions.

3.4.1 What is a case study?

Baxter and Jack (2008) defined a case study as an approach to research that facilitates exploration of a phenomenon within its context using a variety of data sources. They argued that this ensures that the issue is not explored through one lens but rather a variety of lenses which allows for multiple facts of the phenomena to be revealed and understood. There are multiple definitions and understandings of a case study. Bromley (1990) defined a case study as a systematic inquiry into an event or a set of related events which aim to describe and explain the phenomenon of interest. Feagan et al. (1991) defined a case study as an ideal methodology when a holistic, in-depth investigation is needed. In concurring with Feagan and others, Stake (2005 p. 97) argued that, "… whether a study is experimental or quasi-experimental, the data collection and analysis methods are known to hide some details …" and he is therefore of the opinion that case studies are designed to bring out those details from the viewpoint of the participants by using multiple sources of data. The unit of analysis can vary from an individual to a corporation.

Yin (2003) identified some specific types of case studies which he called 'explanatory' – which are cases considered to be a prelude to social investigation. Explanatory case studies may be used for doing causal investigations, while descriptive cases require a descriptive theory to be developed before starting the project. Stake (2005) added three other forms of case studies: the intrinsic case study is when the researcher has an interest in the case; the instrumental is defined as when the case is used to understand more than what is obvious to the observer and, lastly, the collective is when a group of cases are being studied. Data is largely derived from documentation, archival records, interviews, direct observation, participant observation and physical artifacts (Yin, 2003).

The key features of a case study are its scientific credentials and its evidence base for professional applications (Zuicker, 2009). Citing an example of an individual case study, he argued that these studies often involve in-depth interviews with participants and key informants, review of the medical records, observation, and excerpts from patients' personal writings and diaries. Mariano (2000) stated that the purpose of a case study research may be exploratory, descriptive, interpretive, and explanatory.

This study is a multiple case study as it takes place in two primary schools. The reason for this form of study is that the researcher wants to collect more reliable and rich data for the study from multiple sites. This will help in exploring and interpreting the data as well as advocating for credibility, trustworthiness as well as rigour in the study.

3.5 Population

Population is defined as a total quantity of things or cases of the type which is subject to one's study (Walliman, 2006). The study was conducted in the

Queenstown Education District of the Eastern Cape Province, South Africa. The two schools in which the study was carried out are public schools which have a total enrolment of 203 Grade 6 learners, both boys and girls. Thus the total population of Grade 6 learners in this study is 203. One of the primary schools is Mlungisi Primary School (pseudo name), and the second school is Xhalanga Primary School (pseudo name). There are only three teachers who are teaching mathematics in Grade 6 in these two primary schools, i.e. one mathematics teacher in Xhalanga Primary School and two mathematics teachers in Mlungisi Primary School teaching Grade 6 classes.

3.6 Sampling strategy

Two primary schools from the Queenstown Education District were purposively selected for this study based on the following reasons:

- Consistent performance in the Annual National Assessment (ANA) during the period 2011-2013.
- Their proximity within the District.
- Their large enrolments to provide a reasonable and reliable sample.

These two primary schools have common characteristics. Most of their learners come from poor backgrounds and their parents have very little education if any at all, most families depend on government grants, the schools have large learner enrollments, and their learners and communities come from previously disadvantaged groups. However, for the past two consecutive years, Mlungisi Primary School's learners have been obtaining more than 78% pass mark in ANA whilst learners from Xhalanga Primary School have been achieving 50% and below.

From the 203 Grade 6 learners the researcher purposively selected only two classes from the two primary schools. The researcher selected Class Grade 6A in Xhalanga Primary School and Class Grade 6B in Mlungisi Primary School. The reason for this selection was that the mathematics periods alternated between both schools during the week, which gave the researcher ample time to switch schools and classes when needed. The other advantage of this sampling was that the proximity to the District Office helped the researcher to get back to the office to undertake his day to day duties. This selection of the two classes caused the total sample to be 100 learners for both the primary schools, with Mlungisi Primary School having 47 learners and Xhalanga Primary School having 53 learners.

A further sampling was done to select learners who were going to be interviewed. A pre- test test was conducted and marked by the researcher. From the test, ten (10) Grade 6 learners from the two primary schools (5 learners from each primary school) were selected purposively. These learners were selected according to their performance in terms of marks, nature of responses, and errors or misconceptions found in their answers. This selection was done irrespective of gender.

To distinguish between the two primary schools and their classes, coding was used. For Mlungisi Primary School, the subject teacher in Grade 6A was Mrs Jobodwana (pseudo name) and therefore the code for her class was Grade 6J. To comply with the ethical considerations in terms of learner anonymity, the researcher coded the names of the learners in 6J with a number (e.g. the first learner in the class next to the door was named 6J1, meaning first learner or learner number one in Mrs Jobodwana’s class). These code names were memorized by learners so that when the researcher selected learners for interviews they would be able to recognize themselves. For Xhalanga Primary School, only Mrs. Xabendlini taught mathematics in Grade 6 and the researcher did not code the class and the teacher’s name was not coded except that the first learner in Grade 6 B was number 48, continuing the counting to 100 since the last learner in Mrs. Jobodwana’s class was 6J47.

3.7 Data collection

Yin (2003) suggests six sources of evidence and three principles in the collection of valid and reliable data in case studies. According to him, the six primary sources of evidence are documents (known as document analysis), archival records, interviews, direct observation, participant observation, and physical artifacts. The researcher used the following data collection instruments and methods:

3.7.1 Collecting data from documents

Yin (2003) suggested the following documents as examples but not limited to letters, memoirs, emails, agendas, minutes of meetings, reports, and other evaluations or studies. The researcher accessed these documents by requesting them from the subject teachers. For this study, the researcher used learner’s written class and homework books, ANA examination papers, mark sheets and mark schedules, Curriculum and Assessment Policy Statement (CAPS) documents, pre-test and post-test scripts, and ANA diagnostic reports and analysis.

Yin (2003) believes that documents are valid and reliable because they give correct and corroborative evidence and can be confidently used for inferences. According to him, documents are literal records of what happened and are written for a specific audience. For this study, the documents from which data was collected were the mathematics CAPS document for the Intermediate Phase (IP), the learners' Grade 5 final exam marks in mathematics, their final mathematics exam answer sheets, and the schools' records of performance in ANA since 2011 to 2013. The researcher also analyzed the learners' written work (classwork, tests, homework, projects, assignments, investigations) on common fractions.

3.7.2 Collecting data using interviews

Creswell (2000) defined an interview as a conversation where questions are asked to elicit information usually pertaining to a product or service, as a means of getting better understanding about the phenomenon under study. An interview is basically a qualitative form of data collection.

Yin (2003) suggested that to have a rewarding interview the researcher must first obtain permission to conduct an interview more especially when dealing with minors. The researcher must follow the line of inquiry and make 'sure' to capture the responses to the questions. The researcher should always ask the actual and relevant questions while maintaining a friendly climate by asking the 'How' questions more than the 'Why' questions.

Fielding and Thomas (2001) argued that qualitative interviewing aims to delve deep beneath the surface of superficial responses to obtain true meanings that individuals assigned to events, and the complexity of their attitudes, behaviours and experiences. They proposed two types of interviews, unstructured and semi-structured, their use depending on the aims of the study. Unstructured interviews allow the respondent to talk in their own words with prompting by the interviewer. Lofland (1971 p. 76) summarized the objectives of the unstructured interview as being, "to elicit" rich, detailed materials that can be used in qualitative analysis. According to him, the objective of the interview is to find out what kind of things are happening rather than determine frequency of predetermined kinds of things that the researcher already believes can happen. In an unstructured interview, the researcher simply has a list of topics that they want the respondent to talk about. But the way the questions are phrased and which order they come will vary from one interview to the next as the interview process is determined by the responses of the interviewees. Fielding and Thomas (2001) further suggested that semi-structured interviews as characterized by topic guides containing major questions that are used in the same way as every interview, although the sequence of the questions might vary as well as the level of probing for more information by the interviewer. According to them, semi-structured interviewing is suitable when the researcher already has some grasp of what is happening with the sample in relation to the research topic. However, the researcher should ensure that there is no loss of meaning as a consequence of imposing a standard way of asking questions, this could be achieved by conducting pilot interviews before the data is collected. The interviews were conducted with individual learners in the school library on a one-on-one basis. The reason for this was that the learners should not feel under undue pressure and that these interviews were confidential and required to know or understand each learner's action as an individual without embarrassment.

Yin (2003) argued that interviews are used in research to capture data that has not been recorded; they also give an opportunity to capture the opposing or confirming views of a phenomenon. However, he cautions of bias, poor recall or inaccurate articulation as other factors the researcher must be aware of and plan for in advance. He suggested that recording devices can be helpful in collecting more reliable evidence.

For the purpose of this study the researcher used semi-structured interviews, in which the researcher prepared questions based on the findings from the learners' responses from the pre-test and asked further unplanned probing questions depending on the responses or type of errors or misconceptions displayed by the specific learner. Learners are unique and therefore they did display varying responses according to their understanding of the concept. The researcher piloted the questions on other learners and teachers who were not participants in the study, including the GET Band mathematics subject advisor in the District. The questions posed were open-ended and were done privately with the respondents. See questions posed in Chapter 4 in the findings.

3.7.3 Collecting data using direct observation

Yin (2003) is of the opinion that observation as a data collecting tool can be used in the following places but not limited to: meetings, factories, classrooms, and other work places. During direct observation it is common for an observer to be present and sit passively recording behaviour, action and reactions as accurately as possible. Usually it is the behaviour of one or more persons that is recorded, and an advantage of this technique is that a number of people interacting with each other and the same piece of equipment can be observed. According to him, the researcher should decide on the level of formality during the observation; that is, whether there will be any observational protocols, or whether they do direct observation by taking notes only or participant observation and/or lastly doing the less formal observations. Observation can be of facts, and can also focus on events as they happen in a classroom. Furthermore, it can focus on behaviour or qualities.

Yin (2003) argues that observation as a tool is used to capture the silent actions and reactions of the participants. He further explained it to be useful in that it provides additional information and understanding of the case. However, he cautions that capturing events with cameras, etc. may require written permission and that single observers might miss important events, more especially in big classrooms. Cohen (2011) concurs with Yin by suggesting that the use of immediate awareness, or direct cognition, as a principal mode of research has a potential to yield more valid or authentic data than would otherwise be the case with the mediated or inferential method.

Other attractions in favour of observation is that what people do may differ from what they say they do, and observation provides a reality check, and also enables the researcher to look afresh at everyday behaviour that might be otherwise taken for granted, expected or go unnoticed (Cooper and Schindler, 2001). Observation, with its carefully prepared recording schedules, avoids problems caused when there is a time gap between the act of observation and the recording of the event. Moyles (2002) argues that observational data are sensitive to contexts and demonstrate strong ecological validity. This enables the researcher to understand the context of programmes, to be open-ended and inductive, to see things that might otherwise be unconsciously missed, and to discover things that participants might not freely talk about in interview situations, to move beyond perception-based data and to access personal knowledge.

For this study the researcher used direct observation by taking notes to record the data. The observation varied from document analysis (mark sheets, task performances and learner interactions with each other and the task, behaviour during the pre-test, behaviour after the pre-test, the learner's behaviour during the lesson (engagement with the task, questions and answering, interaction with each other), and behaviour during and after the post-test.

3.7.4. Collecting data using the pre-test

Cohen (2011) suggested that a test is a powerful method of data collection. According to him, they have an impressive array of tests for gathering data of a numerical rather than verbal kind. In considering testing for gathering data, several issues need to be borne in mind: What is being tested? Is it a parametric or non-parametric test? Are they available commercially or will researchers have to develop their own? Are they group or individual tests? Are they norm-referenced or criterion-referenced tests? Callingham (2005) defined norm-referenced tests as tests which compare students' achievement relative to other students' achievement (e.g. a national mathematical performance or a test of intelligence which has been standardized on a large and representative sample between a certain age bracket). He defined a criterion-referenced test as not comparing student with student but, rather, requires the student to fulfil a given set of criteria, a predefined and absolute standard or outcome.

In concurring with Callingham, Cohen (2011) further argued that, a criterion-referenced test provides the researcher with information about exactly what a student has learned, what she can do, whereas a norm-referenced test can provide the researcher only with information on how well one student has achieved in comparison to another, enabling rank orderings of performance and achievement to be constructed. Gronlund (1990) stated that the researcher must identify the purposes of the test, for example, to diagnose students' strengths, weaknesses and difficulties, to measure achievement, to measure aptitude and potential, and to identify readiness for the programme.

Cohen (2011) proposed the following attributes of testing: the purpose, the specification, validity and reliability, form of the test, contents, layout, timing of the test, and piloting the test. According to him, the specification includes the details the programme objectives and student learning outcomes, content areas to be addressed and the total number of items in the test. To address validity in a test he suggested that it is essential to ensure that the objectives of the test are fairly addressed in the test items and it concerns the extent to which the test tests what is supposed to test, and it must measure what it purports to measure. It should demonstrate several forms of validity and reliability. Construct validity is the extent to which the test measures a particular construct, trait, or behaviour evidenced through convergent validity. Concurrent validity is the extent to which a test correlates with other tests in a similar field. Predictive validity is validity which can accurately predict the final scores. Criterion related validity measures the extent to which the performance on the test enables the researcher to infer individuals' performance on a particular performance on a particular criterion of interest, often calculated as a correlation between the score on a test and a score in another indication of the item that the test was intended to measure. The cultural validity is the validity which measures fairness with regard to the language and culture of the individual test-takers and avoidance of cultural bias. Finally, Cohen (2011) referred to consequential validity which ensures that the results of the test are used fairly and ethically and only for the purpose of, and the ways in which, the test was constructed. He argued that reliability concerns the degree of confidence that can be placed in the results and the data, which is often a matter of statistical calculation and subsequent test redesigning. According to him, reliability is addressed through the following forms and techniques: test/re-test, parallel forms and internal consistency. For the reasons mentioned above, a pre-test for this study was conducted so as to understand the learner's misconceptions and errors in work on common fractions.

The pre-test contained four questions which were coded (A, B, C and D) and all the questions, except question A which had four (4) sub-questions, had five (5) sub-questions. Each question was designed such that it allowed the learners to display some form of competency and skill in understanding and knowledge of the concept of common fractions in two operations (addition and subtraction only). The design further intended to investigate the learners in converting shaded diagrams to numerical notation and vice-versa. This exercise tested the basic knowledge of learners in the concept. Question A requested learners to give the correct numerical notation of diagrams, Question B required learners to convert fractions to diagrams, Question C instructed learners to solve word problems (convert to numerical notations and simplify) and, finally, Question D requested learners to solve given common fraction problems including mixed fractions.

3.8 CGI lesson presentation

After having marked the pre-test scripts and an observation was made on learners' errors and misconceptions, the researcher prepared lessons over a period of a week to address the learners' errors. A lesson plan with the following activities was planned:

Activity 1

Learners are required to fold A4 size papers and fold them as many times as possible, cut out the figures formed, label or name them. This was to be done as an ice breaker to develop the fraction concept (i.e. recognizing and naming the fraction), e.g. where there are four parts, the learners will label or name them 4 out of 4 or 4 quarters. Then one part or two parts are shaded to form a quarter or two quarters, with learners actually building these fractions and naming them. Learners are instructed to arrange the fraction pieces according to their sizes. This activity was aimed at building the learners' comparing skills of the fraction concept, and lastly, the learners are instructed to add or subtract the fraction pieces formed. Learners thus play a fraction pieces game as explained in the lesson plan.

Activity 2

In **2.1** the learners are instructed to give the numerical notations of given fraction diagrams using fraction pieces and a fraction wall as a guide and in **2.2** the learners are instructed to draw fraction diagrams of given numerical notations, write them in words and add or subtract. In **2.3** the learners use fraction charts to compare, add and subtract fractions. In **2.4** the researcher presents learners with word problems and worksheets to solve. In this activity the aim is to acquaint learners with naming and identifying the fractions and understanding the relative sizes thereof.

Activity 3

In **3.1** the researcher introduces the mixed fraction concept to the learners where common fractions have whole numbers. In **3.2** learners are instructed to add mixed fractions with the same denominators Learners must use the cut-off pieces and a fraction wall chart to discover equivalent fractions. In **3.3**. the learners are assisted to find the common ratio or general rule and discover on their own on how to find the proceeding equivalent fraction.

Activity 4

In **4.1** the researcher introduces denominators of multiples using the cutoff pieces, a fraction wall chart and the equivalent fraction concept. In **4.2** the learners are introduced to calculus where the denominator is raised through multiplication.

Activity 5

Consolidation

Learners play fraction games on a designed chart to alternate the representations of word problems either numerically or diagrammatically. These representations are then added or subtracted. Written work is also done to measure the learners' understanding.

Resources to be used

The following available resources will be used: the chalkboard, fraction pieces, A4 size paper, a pencil and cartridge paper, a ruler to cut paper and fraction bars.

Reflection on the lesson plan

- The lesson plan will cover all the desired outcomes.
- It will cater for skills and knowledge that have to be acquired.
- Improve the learners' attitude towards mathematics.
- Enable learners to discover fraction facts.
- Afforded learners an opportunity to interact and find solutions.
- It will build self-worth and confidence in the individual learner.

3.9 Collecting data using the post-test

The researcher then prepared a post-test to observe the difference made by the intervention. The post-test has the same number of questions as the pre-test. The same pattern was to be followed in the administration of the pre-test. For further clarity see collecting data using pre-test in 3.7.4

3.9.1 Piloting the test

According to Cohen (2011) piloting the test can be done in several ways including: A small group of experts can examine the items of the test, their suitability, validity, relevance, possible cultural bias, and sources of invalidity and unreliability, remoteness from the test-takers' experience. Or this can be done by having a small group of test-takers and asking them to give feedback on: the clarity of the items, instructions and layout, ambiguity or difficulties, readability levels and language problems for the target audience, the type of question and its format, omissions, redundancy and irrelevant items and the time taken to complete the test. For this study, the researcher piloted the test on the mathematics cluster leaders and subject advisor in the district.

3.9.2Administering the post-test

After the pre-test results were captured and analysed. 10 learners were selected for interviews, and after the interviews, the researcher planned a lesson to remedy the learners' errors and misconceptions using the Cognitively Guided Instruction approach for a week. According to the research schedule, the lesson would be done in a week's period and learners would be given a post-test to evaluate the success of the CGI. However, several disturbances occurred to disrupt the programme. The post-test was administered on a Tuesday following a long weekend because of unforeseen school programmes that cropped up and disturbed the whole programme.

The number of learners who wrote the post-test dropped from 47 to 46 and from 53 to 50 for both Mlungisi and Xhalanga Primary schools respectively. The reasons for this absenteeism varied from truancy, to bereavement, to test fear. However, there was a slight improvement shown in the test results. There were some unusual disturbances in the schools' of programmes starting with music competitions at the local and district levels, sport competitions from the local level to the district level, incoming matches which took learners either away from class or out of school as a result, prepared lessons that could not be taught as planned by the researcher because you would either have to cut the period short or skip a day or more. This means that the momentum of learning was hampered. As stated above, the post-test was administered after a long weekend owing to these disturbances.

3.9.3 Ethical issues observed in testing

Cohen (2011) is of the view that a major source of unreliability of test data derives from the extent to which and ways in which students have been prepared for the test. According to him, these can be located on a continuum from direct and specific preparations, through indirect and general preparations to no preparation at all. Preparations can take many forms (Gibbs, 1997).

The examiner must ensure coverage of programme contents and objectives that will be tested, restricting the coverage of the programme content and objectives to those only that will be tested. The tests must be valid and reliable whilst the administration and access to test materials should be controlled and the privacy and dignity of individuals should be respected. The individual participants must not be harmed by the test or its results and, finally, informed consent to participate in the test should be sought.

This study was a mixed method research design study which begun with a pre-test being administered to 100 Grade 6 learners in two primary schools in the Queenstown Education District. The learners had already been taught some fraction concepts in Grade 5. The task was composed of four questions. Question 1 was where learners were expected to give a correct numerical notation of presented shaded diagrams as shown in Appendix B.

Question 2 consisted of a task in which learners were instructed to draw diagrams to represent the given fractions. In Question 3, learners were required to give the correct numerical notations for the word problems and further simplify them. In Question 4 the learners were instructed to simplify common and mixed fractions with the same denominators and denominators which are multiplies of each other.

These tasks were categorized according to the concepts learners should master before they could engage in the actual arithmetic calculations. For example, in Question A learners were expected to draw their diagrammatic representation knowledge of fractions. In Question B, learners were expected to show their knowledge and skills of naming and representing different fractions diagrammatically, in Question C learners were expected to display their knowledge, skills and understanding of the concept ‘fraction’ by writing word problems into numerical notations and in Question D learners were expected to display their calculating skills, including the use of the principle of equivalency and common denominators.

After marking the test, learners who got least marks and those with interesting responses as displayed were selected (see Appendix E for the pre-test questions). Five learners were selected in each school as indicated in the sampling strategy. The learners were interviewed and the results of the interviews are presented in Chapter 4 (Research Findings) (see Appendix F for the interview guide). After the interviews, the researcher prepared a lesson on common fraction using the CGI (direct modelling) approach to teach the learners in order to investigate the effectiveness of the CGI to assist learners to understand and calculate common fractions correctly and minimize errors and misconceptions (see Appendix G – lesson plan).

After four days of intensive teaching using manipulatives, and teaching and learning aids through CGI, the researcher administered a post-test to measure the success of the treatment. All learners including those who were not interviewed wrote the test and the results were recorded (see Appendix H for the post-test). The tests were valid and reliable as the sources used were the Caps Intermediate document which is an official document from the Department of Education. The tests were piloted by using a small group of experts, including the mathematics Subject Advisor in the District, the lead teachers who are teaching mathematics in the District, the class teachers and Heads of Department in both schools.

3.10 Ethical considerations

Ethical issues are important since the study involved human subjects and institutions, and in particular because the study involved the analysis of documents like lesson plans, recording sheets, and/or learners' books including marking tools and strategies. This situation called for more sensitivity and confidentiality towards the identity of the subjects. The identities of the learners and institutions who participated in this study were protected by the use of pseudo names and codes.

The researcher obtained an ethical clearance certificate and asked for a written request from the University of Fort Hare and from the school authorities to undertake this study (Appendix A). These school authorities included the District Director, the principals, teachers and the school governing bodies of the affected institutions. The researcher started the study after obtaining written permission from the District Director (Appendix A) and the schools (Appendix A). No visual or photographic materials of the participants were used as they might have compromised the confidentiality clause and revealed the identities of the subjects under study. Photographs of the resource material used and samples of learners' and teachers' work were used to illustrate factual discussions.

The purpose of the study and how it was going to be conducted was discussed with the participants. Confidentiality aspects of the study were discussed in detail and in advance with the participants and the teachers were requested to sign a form for voluntary participation (Appendix A). Permission for learners to participate in this study was requested and obtained from their parents since they are the lawful guardians of learners who are minors and cannot take certain decisions by themselves, (Appendix A). The researcher acquired this permission through the School Governing Body (SGB), which has a parent representative component and by writing directly to the individual parents. Participants were free to withdraw their participation when and if they felt uncomfortable. Thus some learners decided not avail themselves for the interview. Only six learners presented themselves for the interview.

3.11 Reliability and validity

To develop a credible and quality study the researcher used triangulation. Triangulation is defined by Patton (2002) as combining multiple observers, theories, methods and data sources in order to overcome the intrinsic bias that comes from single methods, single observers and single theory studies. According to Patton, four kinds of triangulations can contribute to verification and validation of qualitative analysis:

For the purpose of this study; the researcher used four forms of triangulations to determine credibility. These are:

- Methods triangulations – here the researcher used two research paradigms (quantitative and qualitative) and varied methods of collecting data, including observation, interviews, document analysis, pre-test and post-test for evidence. All photographic pictures of the physical surroundings including participants and their work was exempted from this procedure.

- Triangulation of sources where data collected from different sources was cross-checked for chances of comparability. Themes might converge or contrast and therefore this form of validation was key to rule out those themes which were irrelevant to what the study wanted to portray.
- Analysts' triangulation was inevitable because this study was constantly under examination from my supervisors and colleagues. Any questionable act was highly scrutinized. The findings of this study were closely examined by internal and external academics.
- Theory perspectives were also engaged, especially when the researcher was dealing with the use of manipulatives as resources to enhance the teachers' approach. Various learning theories were compared against the prevailing findings on the ground.

3.12 Conclusion

In this chapter we have learnt that all research studies require some validity to become reliable and be accepted by the international research community. There are three main research paradigms: the qualitative research paradigm which is narrative in nature, the quantitative research paradigm which is numeric, and the mixed methods research design which combines both narrative and numeric disciplines of research. The research design for this study was a multiple case study which took place in two primary schools of the Queenstown Education District.

Data collection was done through direct observation of participants while taking notes of the observable behaviors and attitudes, pre-test and post-test administration, document analysis and interviews. Validity and reliability were upheld by using the Mathematics CAPS Intermediate Phase document to ensure that the test did measure within the scope of the learners' work and that it measured what it was supposed to measure. The test was piloted to some experts to assure reliability. A lesson plan was prepared using the CGI approach to improve learners' understanding of fractions. Triangulation was recommended as a reliable measure of validity and reliability. Ethical considerations were taken into account as they are important in keeping the study in check for possible human rights violations and unethical conduct.

CHAPTER 4

RESEARCH FINDINGS

4.1 Introduction

In this chapter the researcher presents the findings of the data collected in chapter 3. The researcher then details the findings of the study, and presents them in the form of tables, graphs and narratives as this study was both quantitative and qualitative. The researcher had administered a pre-test and analysed the findings from the test. Based on the findings of the pre-test, a Cognitively Guided Instruction (CGI) lesson was prepared, taught and then a post- test was administered. The observed behaviour, documents, learners' responses to interviews and interactions that took place during the study were recorded and analysed. The following are the results of the findings.

4.2 Findings from the pre-test in both primary schools

The following were the results of the pre-test administered to Grade 6 learners of Mlungisi and Xhalanga Primary Schools. The findings are presented per question for both schools. The questions range from Question A to Question D for both primary schools. Each question had a specific role to investigate as stated in chapter 3. The findings are presented using tables and graphical pictures as well as a narrative report.

The key for the numerical data presented in the chapter are as follows: Column 1 (**Question)** indicates the number of the question or sub-questions. Column 2 **(NW)** represents the number of learners who wrote the test. Columns 3 (**CR**) and 4 **(IR**) represent the number of **correct (CR)** and **incorrect responses (IR)** respectively**.** Column 5 **(% CR)** shows the percentage of correct responses.

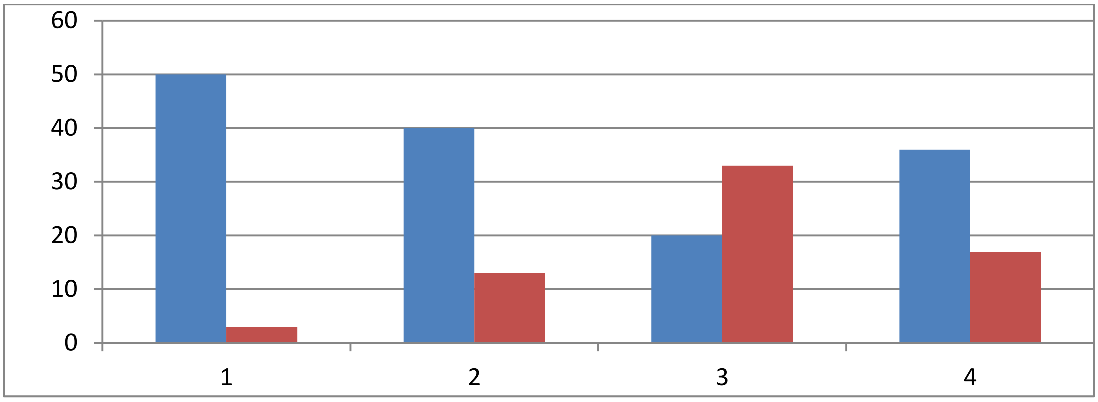
In the graphical representations the brown colour represents incorrect responses whilst the blue colour represents the correct responses.

4.2.1. FINDINGS FROM QUESTION A1 – A4

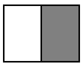
QUESTION	**N.W.**	**CR**	**IR**	**% CR**
A1	53	50	3	94%
A2	53	40	13	75%
A3	53	20	33	38%
A4	53	36	17	68%

Table 1: **Results of Xhalanga primary school in Question A**

The above are the results of Question A, where learners were required to answer as explained in the methodology section. The findings were that learners performed very well in sub – questions A1, A2 and A4 with 94% correct responses in A1, 75% in A2 and 68% in A4. However, although there were some good performances in these subs – questions, there were still some errors made by learners, for example: 6% of the learners in A1, 25% of learners in A2, 62% in A3 and 32% in A4 made some errors, thus the researcher could not get 100% correct responses in all the questions or sub – questions. This is the significant part of the study, to look at the type of errors made by learners (these are a pool of learners from which the interviews were conducted), the reasons for these errors and how the Cognitively Guided Instruction can assist in minimizing these errors. The graph below depicts the results of learners' responses in Xhalanga primary school.



Graph 1: **Results of Xhalanga Primary School in Question A**

The above graph shows that only 3 learners had errors in giving the correct numerical notation for diagram  in Question A1 for which the answer was$\frac{1}{2}$.

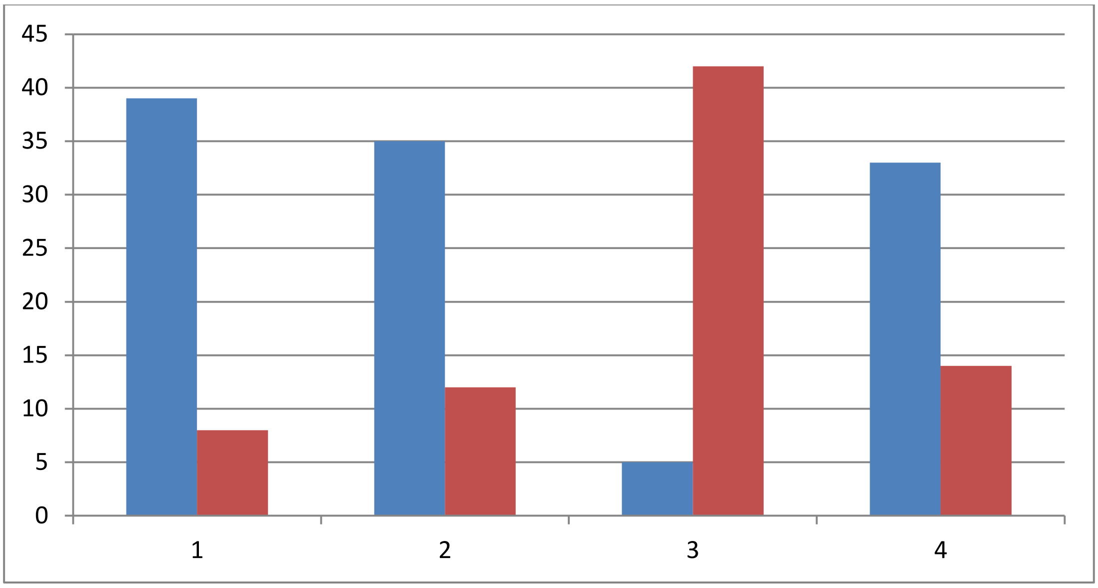
Question A2 required the learners to give the correct numerical notation for this diagram 

The answer was $\frac{2}{3}$ and 25% of the learners committed errors. The most significant of the errors were made when learners were requested to convert a diagram to a mixed fraction in Question A3. Only 38% of the learners gave correct responses to this question. For the same question in Mlungisi Primary School, the results of learner responses were as indicated in the table and the graph below:

	QUESTION A	**N. W.**	**CR**	**IR**	%CR
	A1	47	39	8	83
	A2	47	35	12	74
	A3	47	5	42	11
	A4	47	33	14	70

Table 2: **Results of Mlungisi primary school in Question A**

For Mlungisi Primary School the results were not much different from Xhalanga Primary School. In A1 most learners performed very well with 83% getting the correct answer; however, in A3 as in Xhalanga Primary school there were only 11% with correct responses and 89% of learners with errors. This is indicative that mixed fractions are a challenge for learners in both schools.



Graph 2: **Results of Mlungisi primary school in Question A**

In Question A the learners showed that the concept of diagrammatic representation was a challenge and therefore it is evident that one of the causes of errors and misconceptions in common fractions is that the fraction concept is not introduced sufficiently using diagrams and learners cannot clearly recognise the fractions.

4.2.2. FINDINGS FROM QUESTION B1 – B4

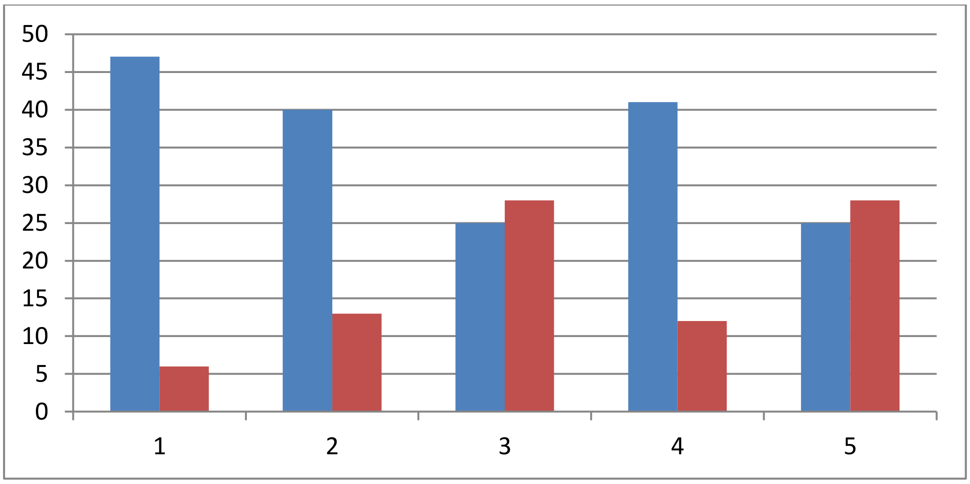
In Question B learners were expected to display skills, knowledge and understanding of converting given common fractions to diagrammatic representations. The following results depicted the learners' responses:

As in A1, in B1 there was an increased performance where the learners were given simple basic questions; however, when questions included a mixed number concept, the learners' errors and misconceptions start to show on a large scale. For example, in B1 learners were required to draw a diagrammatic representation of $\frac{2}{3}$ and 89% of the learners' responses were correct. But in B3 where the learners were required to draw $2\frac{5}{6}$ the number of errors increased, so that the total correct responses in B3 amounted only to 47% and 54% in B5 where the mixed fraction was $3\frac{3}{7}$. Similarly, in Mlungisi, although the correct responses percentage was not as high as Xhalanga

Primary School, it was reasonably fair. The pattern was the same as you observe from the responses from B3 and B5 which netted 30% and 31% correct responses respectively.

QUESTION B	NW	CR	IR	% CR
B1	53	47	6	89
B2	53	40	13	75
B3	53	25	28	47
B4	53	41	12	77
B5	53	25	28	54

Table 3: **Results of Xhalanga primary school in Question B.**



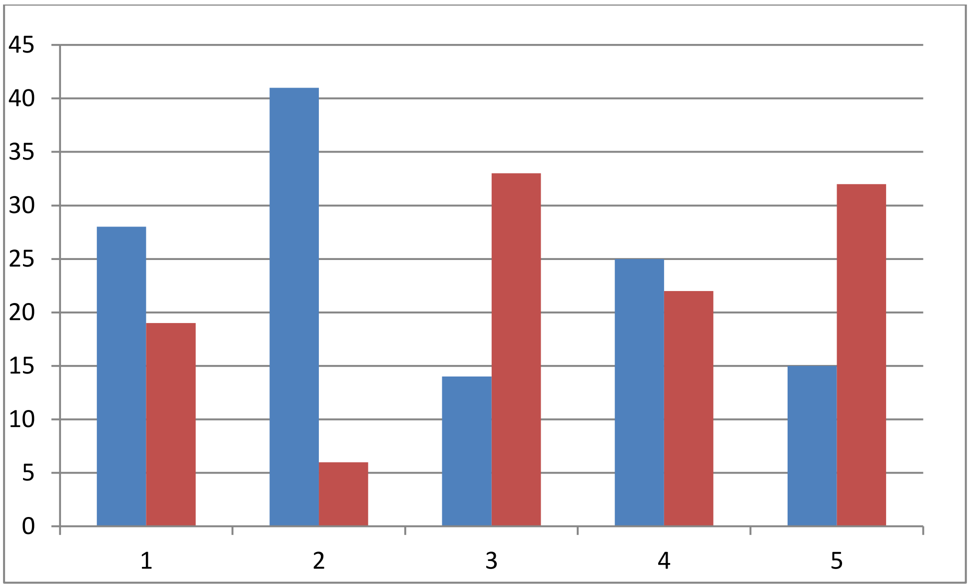
Graph 3: **Results of Xhalanga primary school in Question B**

In Mlungisi Primary School the responses are as displayed in the table below:

QUESTION B	N.W.	CR	IR	%CR
B1	47	28	19	60
B2	47	41	6	87
B3	47	14	33	30
B4	47	25	22	53
B5	47	15	32	31

Table 4: **Results of Mlungisi primary school in Question B**

As can be seen, there is not much difference between the schools in terms of errors and misconceptions displayed and the specific areas where these errors are not only evident but bigger when dealing with mixed fractions. The graphical representation of the data for Mlungisi Primary is shown below:



Graph 4: **Results of Mlungisi primary school in Question B**

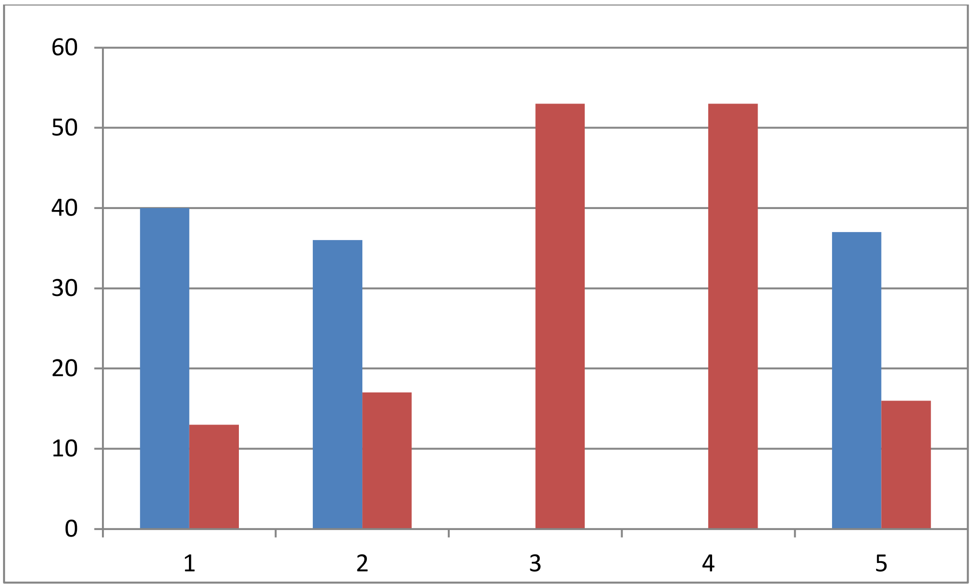
4.2.3 FINDINGS FROM QUESTION C1 – C4

In Question C, learners were requested to convert word problems to numerical notations, and simplify them. These sub questions included questions where learners had to deal with mixed fractions; this exercise tested the ability of the learners to name, and write the correct fractions and simplify them. The table below shows the learners' responses in this task:

QUESTION C	N.W.	CR	IR	% CR
C1	53	40	13	75
C2	53	36	17	68
C3	53	0	53	0
C4	53	0	53	0
C5	53	37	16	70

Table 5: **Results of Xhalanga primary school in Question C**

In the above table the learners' responses showed that they were able to give correct responses for simple word problems; however, in C3 and C4 the learners' responses showed 0% knowledge of word problems in mixed fraction form. This trend was consistent in both schools, where it was evident that the learners did not understand or did not have basic knowledge of naming and giving the correct responses for fractions in mixed form. The graph below shows learners' responses in Xhalanga primary school:

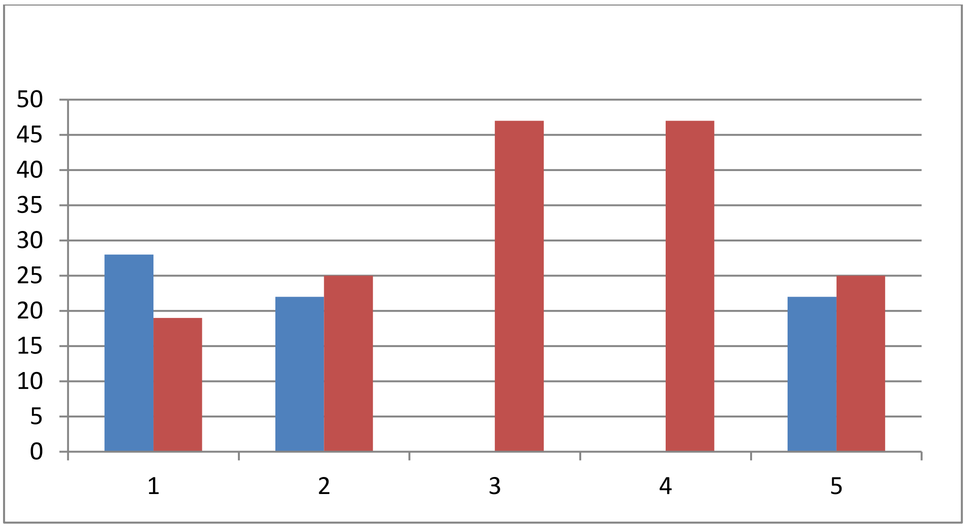


Graph 5: **Results of Xhalanga primary school in Question C**

In Mlungisi Primary School the table showed the following results which were similar to those given by learners in Xhalanga Primary School. In both schools learners recorded 0% correct responses for questions C3 and C4.

QUESTION C	N.W	CR	IR	% CR
C1	47	28	19	60
C2	47	22	25	47
C3	47	0	47	0
C4	47	0	47	0
C5	47	22	25	47

Table 6: **Results of Mlungisi Primary School in Question C**



Graph 6: **Results of Mlungisi primary school in Question C**

In C3 learners were required to give the correct numerical notation of three and a half litres of milk ($3\frac{1}{2}$) and five and a quarter ($5\frac{1}{4}$) litres of milk added together to give eight and three quarters of milk ($8\frac{3}{4}$), and in C4 they were instructed to convert two thirds ($\frac{2}{3}$) and four sixths ($\frac{4}{6}$) of a pizza and add.

As in Xhalanga primary school, learners struggled to convert word problems to numerical notations and simplify. The language might have also contributed to this challenge as these learners used their second additional language and are mainly from poor families where Xhosa is the main spoken language.

In Question C4 the learners were expected to simplify the given fractions. The questions were categorised as follows: addition and subtraction of fractions with common denominators, fractions with denominators as multiples and mixed fractions. The learners in both schools as seen from the table and graphs scored 0%.

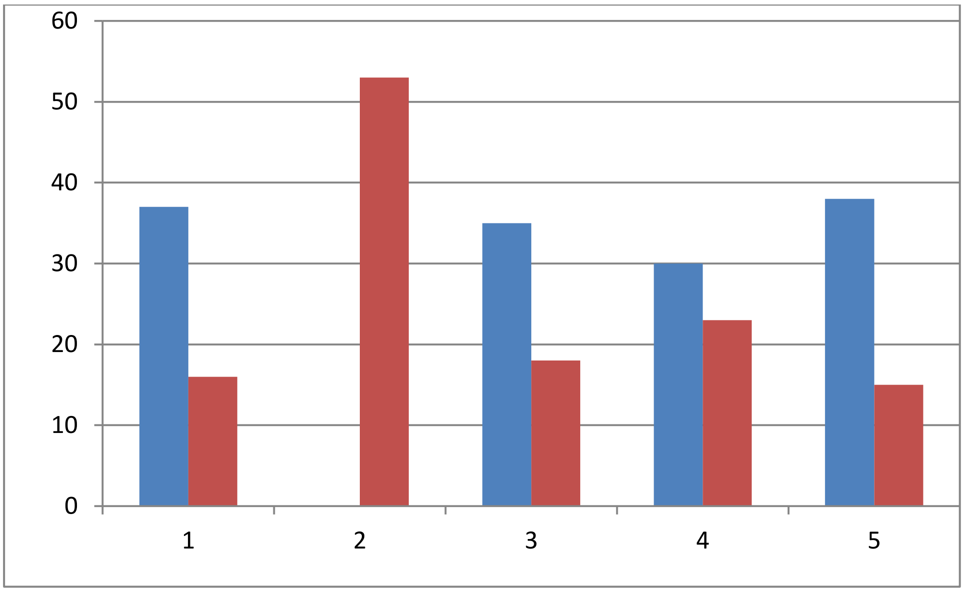
4.2.4 FINDINGS IN QUESTION D1 – D5

QUESTION D	N.W.	CR	IR	% CR
D1	53	37	16	70
D2	53	0	53	0
D3	53	35	18	66
D4	53	30	23	57
D5	53	38	15	72

Table 7: **Results of Xhalanga primary school in Question D**

In sub question D1 there were 70% correct responses where learners were given $\frac{7}{10}$ - $\frac{3}{5}$. The correct response was $\frac{1}{10}$ but some learners gave $\frac{4}{10}$, $\frac{4}{5}$ and or $\frac{10}{15}$. In sub question D2 learners obtained 0% correct responses. In this question learners were required to simplify $3\frac{3}{4}$ + $6\frac{1}{4}$ = 10. Written in words, this simply means three wholes and three quarters added to six wholes and a quarter equals to 10 wholes. However, because the learners did not have basic knowledge of the fraction concept they could not simplify this simple instruction. Although basic skills in identifying, naming and writing a fraction are requisite in the learning and understanding of the concept of 'fraction', the media which is used to translate this information is equally important. Hence, languages of instruction also come into play. As described in the contextual factors, these learners may also have some language barriers since English is their first additional language. More of these errors and misconceptions are reflected and discussed in the learner interviews.

The following graph shows the Xhalanga Primary School learners' responses in Question D:



Graph 7: **Results of Xhalanga primary school in Question D**

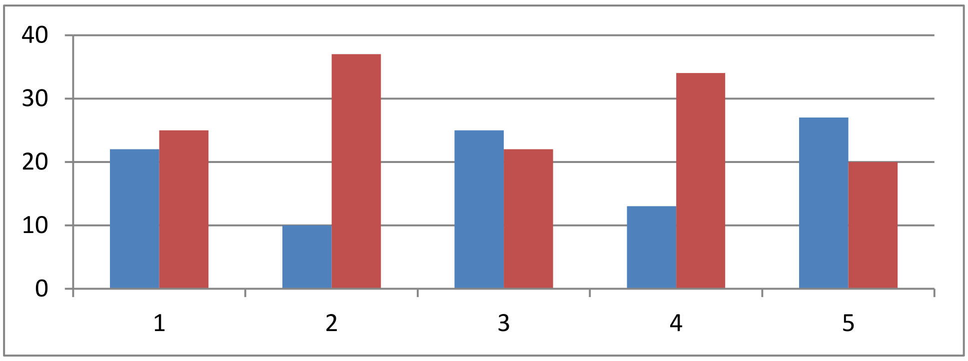
As shown in the graph above, in all sub questions in question D there are some incorrect responses that are as a result of some form of misconception. However, it is evident that in Question D2 all learners struggled to give a correct response.

For the same exercise (Question D) in Mlungisi Primary School the following table reflected the learners' responses:

QUESTION D	**N.W.**	**CR**	**IR**	**% CR**
D1	47	22	25	47
D2	47	10	37	21
D3	47	25	22	53
D4	47	13	34	28
D5	47	27	20	57

Table 8. **Results of Mlungisi primary school in Question D**

Although there was enough evidence to suggest that the Mlungisi Primary school learners were also struggling to provide correct responses, there were also improved performances in their responses than their counterparts in Xhalanga Primary School. For example in D2 where the learners in Xhalanga had recorded 0% correct responses, learners in Mlungisi Primary school recorded 21%, although there was not much improved performance.



Graph 8: **Results of Mlungisi primary school in Question D**

The graph above shows that all the learners were struggling in all the sub questions; however, the sub questions D1, D2 and D4 proved to be more challenging to learners as only less than 50% of learners were able to provide correct responses with 47%, 21% and 28% respectively. More discussions on the findings are done in the following chapter.

Subsequent to these findings, learners were selected as indicated in chapter 3 and they were interviewed so as to understand their thinking and reasoning which led to their incorrect responses. This is supported by Askew and Williams (2001) that we have to accept that learners will always make some generalizations that are not correct and many of these errors and misconceptions will remain hidden unless the teachers make specific efforts to uncover them. The interviews, therefore, became a tool to uncover these errors and misconceptions.

4.3. FINDINGS FROM INTERVIEWS

4.3.1. Learner interviews in Mlungisi primary school

There were ten (10) learners selected for the interviews in both primary schools, five (5) learners per school. However, there were only seven learners who participated in the interviews owing to the following reasons. Two parents did not grant permission for the interviews in time and the other one learner was absent from school. All the learners were simultaneously briefed about the procedure and format that the interview would take and those learners were free to answer or not to answer the questions (see Appendix C). The interviews were first done in School A. The first learner to be interviewed was 6J20. This was learner number 20 in Grade 6 of Ms. J's class. She was a 12 year old girl. The learner's response in Question A_1 was $\frac{1}{3}$ and incorrect as the expected answer was $\frac{1}{2}$.

QUESTION A

Q: How did you get this answer?

A: I drew my own lines on the diagram.

Q: Why did you do that?

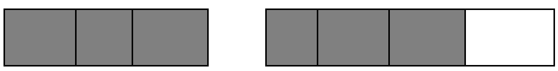
A: Did not understand the question correctly.

Q: What is the correct answer if you do not draw your own lines?

A: One half ($\frac{1}{2}$)

In question A_2 the learner got the answer correct ($\frac{2}{3}$) but could not give the reason for her response.

The answer to A_3 was incorrect as the expected answer was $1\frac{3}{4}$ but the learner wrote $\frac{6}{1}$:



Q: How did you get this answer?

A: I added all the shaded parts (6) out of 1.

Q: Why did you add the shaded parts?

A: No reason given.

The answer to QA_4 was incorrect. Also, instead of $\frac{3}{5}$ she wrote $\frac{3}{2}$

Q: How did you get this answer?

A: I wrote 3 shaded parts out of 2.

Q: What was the reason for you to give this answer?

A: No reason given

QUESTION B

In Question B learners were instructed to draw diagrams of given fractions.

B_1 was incorrect as the learner was instructed to draw a diagram for $\frac{2}{3}$ and drew:

Q: How did you reach this response?

A: I drew 3 shaded parts and left out 1 part.

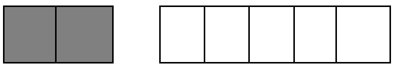
Q: Why did you do it like that?

A: Should have left 2.

There was no reason given for this response.

B_2 was correct but no reason given either.

B_3 was incorrect because the learner was expected to draw a diagram of $2\frac{5}{6}$ and she drew:



Q: Why did you draw this diagram?

A: I shaded these two to represent two wholes and left the 5.

Q: Where is the diagram that represents the 6 which is the denominator?

A: No reasons given.

QUESTION C

In this question the learner was expected to change the word problems into numerical notations and simplify: e.g. Five fifths of the class is boys and girls. If two fifths are boys, what fraction of the class are girls?

C_2 learner's response: $\frac{5}{5} + \frac{2}{5} = \frac{7}{5}$. This response was incorrect.

The response of learner C_3 in this sum was incorrect: $3\frac{1}{2} + 5\frac{1}{4} = 8\frac{2}{6}$. In this calculation the learner correctly added the whole numbers but also added the numerators and the denominators.

In C4 the learner's response was incorrect: $\frac{2}{3} + \frac{4}{6} = \frac{6}{9}$. She added the numerators and the denominators.

LEARNER 6J1

QUESTION A

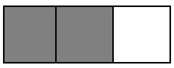
The learners were expected to write the correct numerical notation for the provided diagrams, e.g.

The correct answer was $\frac{1}{2}$, but the learner's response was $1\frac{1}{2}$. On probing:

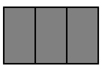
Q: How did you get this answer?

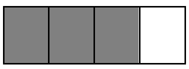
A: This answer means that there is one half.

There are many answers that he gave without valid reasons, e.g. for this diagram correct answer was $\frac{2}{3}$ but the learner's response was $3\frac{3}{2}$



For A_3 where the correct answer was supposed to be $1\frac{3}{4}$. On probing, the learner





could not supply a credible answer. For Question $A_{4,}$

The learner gave the answer as $\frac{3}{2}$ whilst the correct response should have been $\frac{3}{5}$ and on further questioning the learner said there were three (3) shaded out of two (2). When asked to count the number of parts, she found five. This helped him to realize his mistake and he gave the correct answer.

QUESTION B

Almost all the responses in this question were correct except for B_1 in which his response was correct as far as the number of parts was concerned but not equal, e.g.

This was his diagrammatic representation of$\frac{2}{3}$. The learner knows that, to have a third, the whole must be divided into three parts so that there is a bigger third, leaving out the most important aspect, the parts must be equal.

QUESTION C

In C1 the question was as follows: John has three quarters of a loaf of bread, Mary gives him one quarter more. How much bread does John have now? In this question, the learners were expected to change the word problems to numerical notations and simplify.

The learner's response was $3\frac{1}{4} + \frac{1}{4} = 3\frac{1}{2}$ which is similar to the one he gave in A1. According to him $3\frac{1}{4}$ means three quarters. The learner was correct in his calculations despite his error in interpreting the concept incorrectly.

QUESTION D

In question D1, the learner was instructed to simplify $\frac{7}{10}$ - $\frac{3}{5}$ = $\frac{4}{5}$. His response was incorrect; the learner subtracted the numerators and the denominators.

LEARNER 6J30

QUESTION A

The learner got A1 correct but for  A2

the learner's response was $\frac{2}{1}$ instead of $\frac{2}{3}$. On further probing, the learner having counted the number of parts into which the whole was divided got the correct response.

QUESTION B

In this question the learner got all the responses incorrect. On probing, the learner revealed that she did not read the question carefully. This was the case with all the other questions i.e. **QUESTIONS C & D.**

In this primary school all the learners had a uniform or consistent pattern of mistakes, errors or misconceptions.

4.3.2. Learner interviews in Xhalanga primary school

The same procedure of parental consent was followed after having selected five (5) learners for interviews. The interviews were done one on one with each learner after briefing them about the procedure to be followed, what was expected of them and their rights as volunteers for the study. To distinguish between the two types of learners, the learners in School B were just given numbers from 48 to 100.

LEARNER 89

Learner 89 was the first learner to be interviewed. He was a boy of 18 years of age. The selection was done purely on the learners' performance regardless of their age and gender.

QUESTION A1

The learner got this one correct as one half, but had a problem naming the fraction correctly as he named it as two quarters. On more questioning and interaction, I discovered that the boy had a hearing problem and that there were no means in place to assist him with this physical challenge.

In A2 the learner was expected to write the correct numerical notation of 

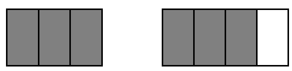
His answer was $\frac{1}{3}$; he could not give a reason for his response to question A. In B2 the answer was correct but again the naming was a problem; instead of two thirds ($\frac{2}{3}$) he called the fraction ($\frac{2}{1}$) and there was no reason given for this.

In C3, learners were expected to convert the word problems to numerical notations and simplify the fractions. He was able to write the correct notation, e.g. $\frac{3}{4} + \frac{1}{4}$ but the answer was incorrect as he added the numerators together and also did the same to the denominators. The learner was not even sure of the operational signs, e.g. In $\frac{5}{5} - \frac{2}{5}$ he got the answer as $\frac{7}{10}$. I could not get very far with the interview of learner 89 as he had this hearing impediment although he was really very committed and participating.

LEARNER 85

The second learner to be interviewed was a 12 years old girl. For A1 she was correct and was able to give a correct reason for her answer. However, what she wrote was not what she was saying orally.

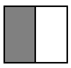
In A2 the learner responded incorrectly as she gave the answer as $\frac{2}{1}$ instead of $\frac{2}{3}$.

On inquiry, the learner said that the reason for her response was that there were two shaded parts and one unshaded. Further probing revealed that she made a mistake because she rushed through the work and she gave an incorrect answer. The other interesting response was in this diagram 

The learner gave $\frac{6}{1}$ as her response and which was incorrect. When asked how she reached that answer, she said that she added all the shaded parts to make six, which means that the learner knew that the shaded parts of a diagram represent the numerator whilst the unshaded parts represent the denominator, and hence her answer was $\frac{6}{1}$.

Another interesting response was in D_4, where she was instructed to simplify $3\frac{1}{2} + 5\frac{1}{4}$ and gave $\frac{10}{10}$. On questioning she revealed that she added 3 and 5 which were the whole numbers and she was correct but she continued to add the two numerators, i.e. 1 + 1 = 2 and added to 8 of the whole numbers to make 10. When asked what happened to the denominators, she did not respond. The next learner to be interviewed was learner 86.

LEARNER 86

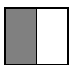
Learner 86 was a boy of 14 years of age. He was the most puzzling of them all. For example, in his first response, in which he was expected to give the correct numerical notation of the given  diagram….

He gave the answer as 1, $\frac{2}{3}$. When asked how he came to that response, he said, "I did not know what to write, this just came out of my mind." According to his explanation, his answer was not borne of any conceptual idea.

B_2 was correct and when asked how he got it right he said, "I just thought very hard and I thought it was the correct answer and wrote it down."

Where he was expected to change word problems to numerical notations, he got C_1 correct, i.e. $\frac{3}{4} + \frac{1}{4}$ but did not simplify as instructed. When asked how he had it correct and why did he not simplify it, he said, "I just thought with my head and wrote down." I could not get any further responses for other questions and therefore my interview stopped. The next learner was learner 95.

LEARNER 95

Learner 95 was a girl of 11, she got this one correct.  But the learner could not give a convincing reason until after an assisted effort. This could have been guess work or an intuition or she might well have copied the work.

In D1 the learner was expected to simplify the given fractions $\frac{7}{10}$ - $\frac{3}{5}$ = $\frac{16}{19}$ and the last one was $2\frac{5}{8}$ + $3\frac{3}{8}$ = $2\frac{8}{5}$ $3\frac{8}{3}$. There was no reason given for this response. All the other learners' responses were similar to those listed above.

4.4. TEACHER INTERVIEWS

4.4.1. Teacher interview in Mlungisi primary school

The first teacher to be interviewed was Mrs. Jobodwana from Mlungisi Primary School. She has some formal background of mathematics teaching and is well experienced in the teaching and learning of mathematics with well over 15 years' experience of teaching the same subject in various primary levels. Her learners consistently topped the District performance chart in ANA with well above 70% during the period under review. Her qualifications are relevant and she has attended various in-service trainings to develop her. The following questions were asked during the interview (note that the teachers' responses are recorded as they were spoken with no corrections made to their language):

Researcher: How do you think the learners performed in this test?

Teacher: I think in some parts of the test they performed well when the question was very simple or when the denominators were similar but had some challenges when the denominators were the multiples of each other. Clearly, there is a great challenge when learners have to simplify mixed fractions also.

Researcher: What do you think was the reason for this performance?

Teacher: One of the reason is that, this topic is not usually taught during the first quarter; the second being that, in Grade 5 the learners were taught by another teacher and I am not sure whether how much of the syllabus was covered because according to my knowledge, in Grade 5 learners should be introduced to the concept of equivalent fractions so that they can be able to simplify simple fractions and build on that in Grade 6. Another factor is that the learners do not seem to have understood when to add or subtract the numerators. In some instances they also add or subtract denominators. Another thing, they don't seem to understand the meaning of a given fraction. For example, in word problems they were unable to convert the word problem into a numerical notation. Learners here do not seem to have an in-depth background of the diagrammatic representation of fractions of which it should be the basic knowledge taught so that learners can be able to fully understand what a fraction is.

Researcher: How do you think these errors or mistakes can be improved?

Teacher: I think learners can be introduced to fractions using diagrams to teach them the fraction concept first. These diagrams should vary from halves, thirds, sixth up to double denominators. When learners are taught one type of fraction, they will not know the other types and will always think of the one fraction that they have been taught of. Learners must be taught to name, write and compare fractions so that they can be able to recognize a fraction and its size. For example, some learners don't even seem to realize which fraction is bigger than the other. They don't seem to understand the significance of a denominator nor numerator. After learners have learnt to name the fraction, they will be able to write it down in words and numerically and that will make them recognize and understand the value of each fraction. Hence it will be easier to simplify. For example in question C2 the question says: Five fifths of a class is boys and girls. If the boys make up two fifths of a class, what fraction of the class are girls? For a learner who understands the basic concept of fraction it would be easy to convert that to numerical notation; however, they might have a challenge in choosing the correct operation. Learners should be assisted to understand the difference between a whole number and a fraction so that when they are dealing with mixed fractions they know that they can only add whole numbers together first. The other concept that should be taught using material such as the fraction wall is equivalent fractions so that when learners are given denominators which are multiples of each other they are able to convert the denominators to be similar and add.

Researcher: What do you know about Cognitively Guided Instruction and how do you think it can help in solving this problem?

Teacher: What is that?

Researcher: A method or approach where learners organize and make meaning of information by themselves.

Teacher: I have not used it because there is so much work to do and therefore I prefer to teach the learners.

4.4.2. Teacher interview in Xhalanga primary school

Mrs. Xabendlini of Xhalanga Primary school is the sole teacher for Grade 6 learners. She is a humble lady who is well over 45years of age, and as much as she is an experienced teacher, she has been affected by redeployment from time to time and therefore would find herself being allocated various learning areas including mathematics, life orientation and so forth. However, in her earlier years of teaching she started as a mathematics teacher at a Junior Secondary School in the then Transkei Department of Education (TDE). She has attended various trainings and workshops and she is a hardworking, dedicated and passionate lady but a little shy. The interview proceeded as follows:

Researcher: How do you think learners performed in this test?

Teacher: They performed a little bit average.

Researcher: What do you think caused this performance?

Teacher: I think first the learners did not understand what the question required, not because the question was not clear but maybe they had not done these forms of tasks in their previous grade although it is requisite to do so. The learners have a tendency of being careless because as you probed them during the interviews; they supplied correct responses which means that they don't do enough to think mathematically on their own. These learners seem not to understand the concept of fraction as equal parts of a whole. Thus you would find that some of the diagrams were divided skewly. Furthermore, learners didn't seem to understand the relative sizes and the meaning thereof various common fractions they had to simplify. Learners do not know the function or importance of either denominator or numerator and they don't seem to understand when or which to add or subtract between the denominators and numerators, they are struggling with mixed fraction as they recorded 0% above. They don't seem to have a conceived approach; hence they vary their approaches as if they are doing guess work. Another thing is that the learners might have what we call barriers including one of the learners who could not hear clearly, the language barrier since they are second language learners.

Researcher: How do you think this performance can be improved?

Teacher: The learners have been in constant dealings with the concept of fractions before formal interaction with them. This early interaction with informal fractions bred the formation of varied misconception on common fraction. One of the most prevalent of these misconceptions is that learners understand that a fraction is formed when a whole is divided into more parts, with disregard of the concept of equal sharing, thus learners will always compete for a bigger half.

This presents the teacher with a challenge to prepare a lesson plan which would address these misconceptions. One of the important elements of the lesson will be to prepare a lesson using simple manipulative materials such as fraction wall charts, fraction strips, and other resource material that will enhance the lesson. Also the introduction of the concept of equivalent fractions must be used to assist in the addition and subtraction of common fractions. The common denominator concept can only be introduced when the learners have understood the equivalence fraction concept. Diagrammatic representations also play an important role in teaching the fraction concept through dividing the whole into equal parts, thus inculcating the sharing concept in learners.

She believed that the learners lacked conceptual understanding of fractions. i.e. they did not know the meaning of the fraction, different types of fractions, how to represent them, how to name them, and how and which fractions to add. However, there were some positives in that in mixed fractions learners were able to identify whole numbers and added them correctly.

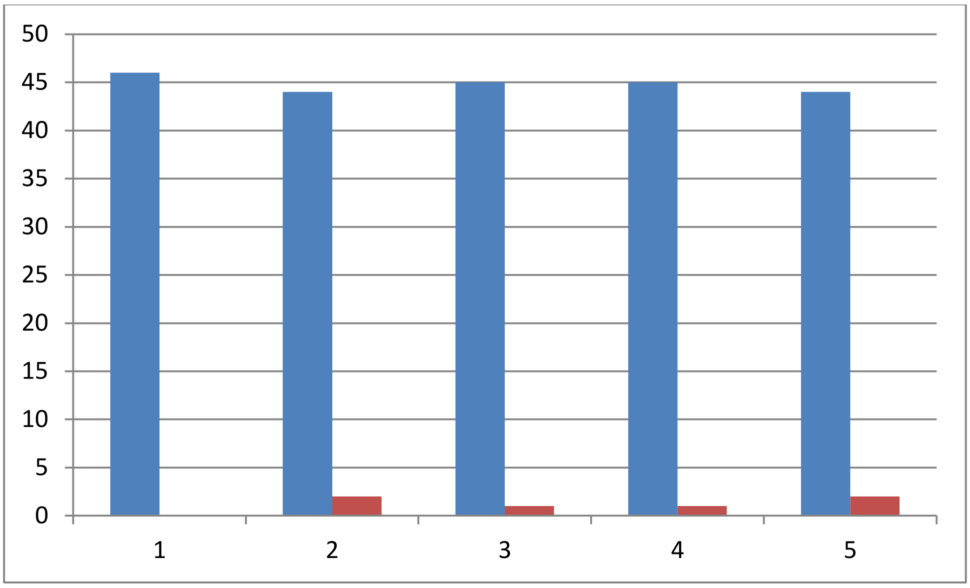
4.5 Post-test results findings

The post – test results after the CGI was used showed some slight improvement from the pre-test, for example in Question A the lowest percentage was 96% for both schools and the highest percentage in Mlungisi and Xhalanga primary was 100% and 98% respectively. However, these results do not imply that there were no errors or misconceptions but these were minimized. In the pre – test for a similar form of questions, the highest percentage was 83% and the lowest percentage was 11% for A1 and A3 respectively. There was great improvement in A3: from 11% to 98% correct responses.

4.5.1 FINDINGS FROM A1 – A4

QUESTION	N/W	CR	IR	%CR
A_1	46	46	0	100
A_2	46	46	0	100
A_3	46	44	2	96
A_4	46	45	1	97
A_5	46	44	2	96

Table 9: **Results of Mlungisi primary school after CGI in Question A**



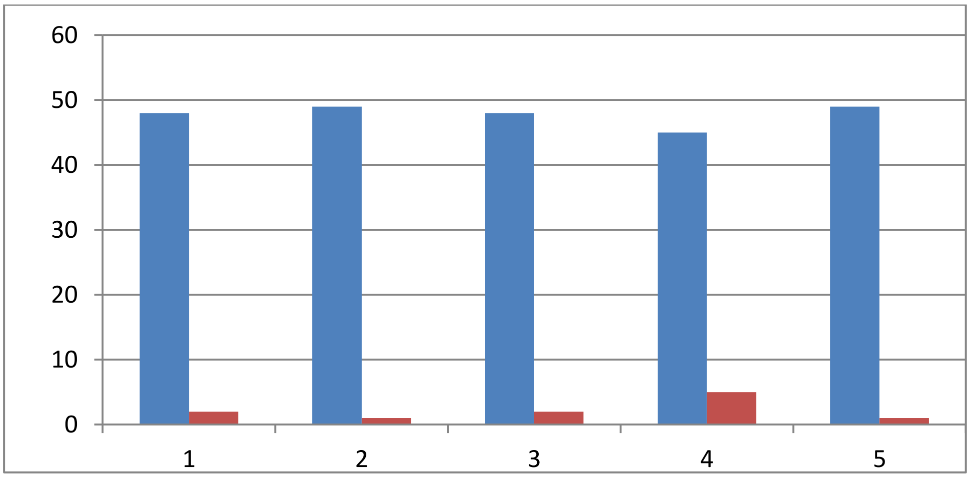
Graph 9: **Results of Mlungisi Primary School after the CGI in Question A**

The above graph shows learners' responses in Mlungisi Primary School to Question A after the Cognitively Guided Instruction was used. Here learners were expected give the correct numerical notation for given shaded diagrams.

QUESTION	N W	CR	IR	% CR
A1	50	48	2	96
A2	50	49	1	98
A3	50	48	2	96
A4	50	45	5	90
A5	50	49	1	98

Table 10: **Results of Xhalanga Primary School after the CGI in Question A**

In Xhalanga Primary School the learners' errors improved and hence the percentage of correct responses increased noticeably. For example, for a similar set of questions in the pre – test learners recorded the following noticeable percentage improvements for questions A1 – A4: 94%, 75%, 38% and 68% respectively. There was a remarkable change after the learners were taught with percentages as follows for A1-A5: 96%, 98%, 96%, 90% and 98%, although in A1 in the pre-test 94% of the learners responded correctly but there was a remarkable drop in incorrect responses to 38%. However, after the CGI was introduced in the post – test the learners' recorded 96% and 90% for the sub question in which they had initially recorded 38%.



Graph 10: **Results of Xhalanga primary school after the CGI in Question A**

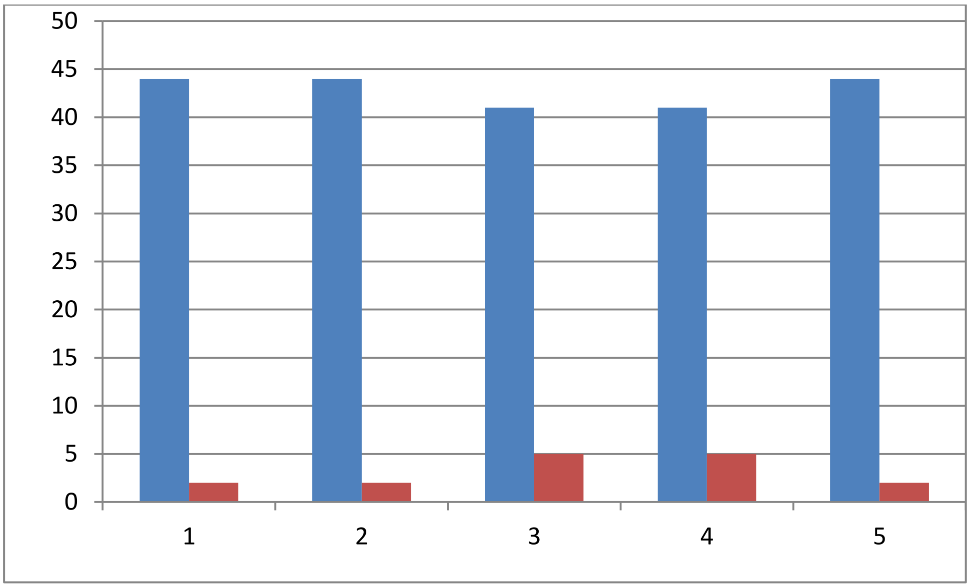
The above graph shows improvement in Xhalanga Primary School learners respons Question A after the CGI was introduced.

4.5.2 FINDINGS FROM B1 – B4

QUESTION	NW	CR	IR	% CR
B1	46	44	2	96
B2	46	44	2	96
B3	46	41	5	89
B4	46	41	5	89
B5	46	44	2	96

Table 11: **Results of Mlungisi primary school after the CGI in Question B**

In Question B the learners were expected to draw the correct diagrammatic notation of a given fraction, and here the number of correct responses increased remarkably. For example, in this question learners' correct responses from Mlungisi Primary School recorded the following percentages in the pre-test: 60%, 87%, 30%, 53% and 31% (B1-B 5 respectively) whilst for the similar type of questions in the post-test the percentages recorded were 96%, 96%, 89%, 89% and 90%. This was a notable improvement, more especially for questions B3 and B5 where learners' responses in the pre-test were at 30% and 31% respectively with an improved response percentage of 89% and 96% respectively.



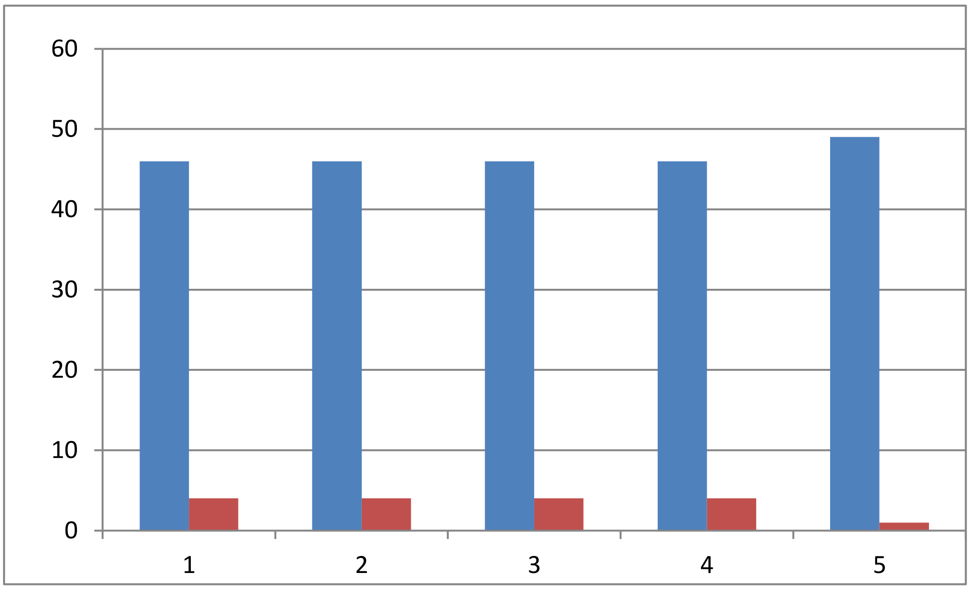
Graph 11: **Results of Mlungisi primary school after the CGI in Question B**

The graph above shows improved percentages in learners' responses in Mlungisi Primary School for Question B

QUESTION	NW	CR	IR	% CR
B1	50	46	4	92
B2	50	46	4	92
B3	50	46	4	92
B4	50	46	4	92
B5	50	49	1	98

Table 12: R**esults of Xhalanga primary school after CGI in Question B**

In Xhalanga Primary School learners also showed a notable improvement in their correct responses in Question B following the interviews and engaging in the CGI learning and teaching approach. This is shown in the above table where the learners recorded 92%, 92%, 92%, 92% and 98% from B1 to B5 respectively in the post -test, whilst the results in the pre- test were as follows: 89%, 75%, 47%, 77% and 32%. There was a great improvement in B3 and B5 where the learners had recorded 47% and 32% correct responses in the pre-test with an improvement of 92% and 98% of similar questions in the post-test respectively.



Graph 12: **Results of Xhalanga primary school after CGI in Question B**

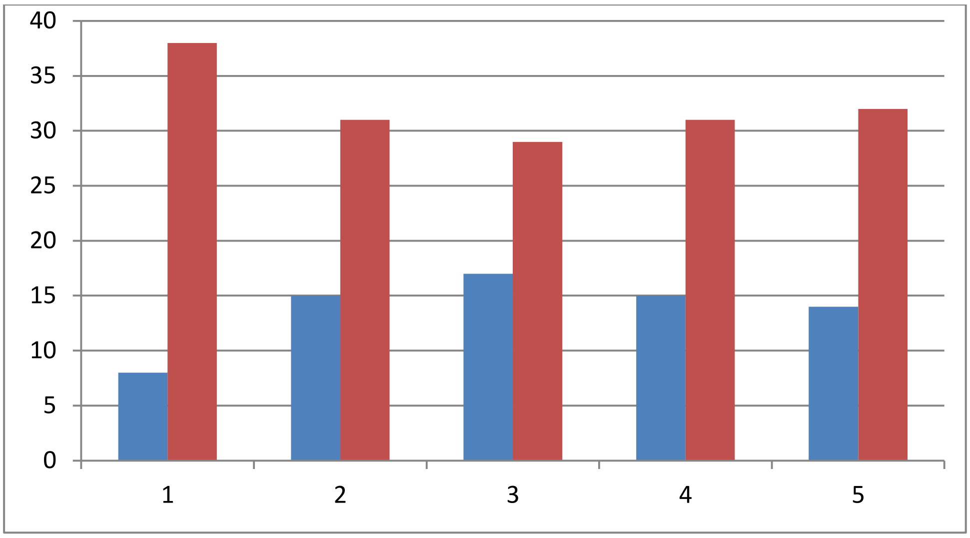
Above is a graph showing improved learners' responses in Xhalanga Primary School for Question B after the CGI model of instruction was introduced.

4.5.3 FINDINGS FROM C1 – C4

QUESTION	NW	CR	IR	% CR
C1	46	8	38	17
C2	46	15	31	32
C3	46	17	29	37
C4	46	15	31	33
C5	46	14	32	30

Table13 Results of Mlungisi primary school after the CGI in Question C

Question C, which had proved to be a major challenge to the learners in the pre-test, did not produce very different results either in the post-test. In the pre-test at Mlungisi Primary School the highest percentage was 60% whilst the lowest percentage was 0% in C3 and C4. After the CGI learners did not show much improvement but there was a slight improvement in C3 and C4 with learners' correct responses 37% and 33% respectively. However, the results of this test might not be attributed to the failure of the CGI but rather too many disturbances which took place during the study. For example, the word problems were scheduled to be taught on Wednesday which is a sport day according to the school's timetable. Therefore, many learners were either absent or the disruptions that took place disturbed their focus and attention. Also this result might be attributed to the fact that there was not enough time to give individual attention given the sizes of the classes.



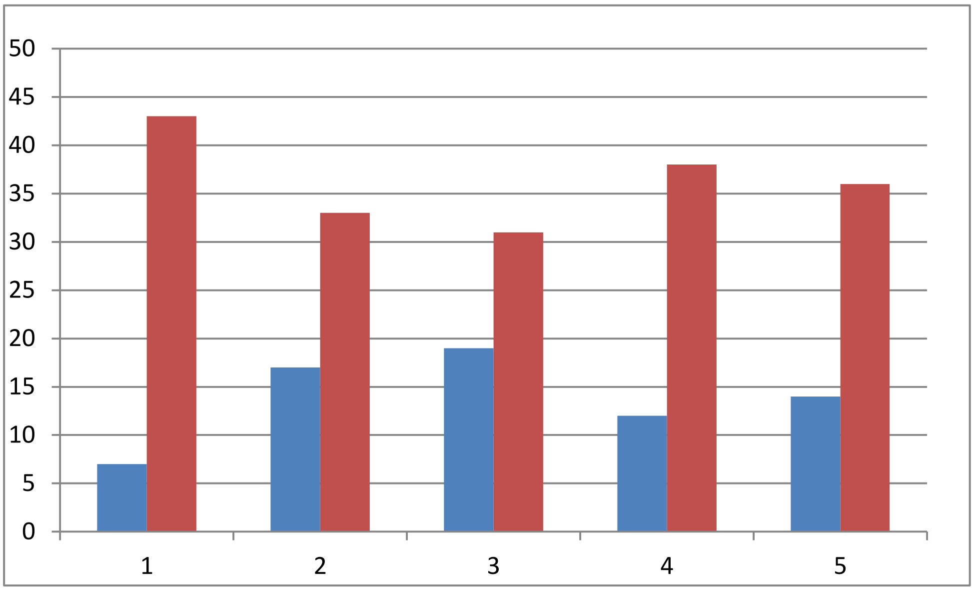
Graph 13: R**esults of Mlungisi primary school after the CGI in Question C**

The above graph shows the responses of Mlungisi Primary School learners to Question C.

QUESTION	**NW**	**CR**	**IR**	**% CR**
C1	50	7	43	14
C2	50	17	33	34
C3	50	19	31	38
C4	50	12	38	24
C5	50	14	36	28

Table 14: R**esults of Xhalanga primary school after the CGI in Question C**

In Xhalanga Primary School the results were also not much different in the post-test as they were in the pre-test. In the pre-test the highest percentage of the learners' responses was 75% whilst the lowest percentage was 0% for C3 and C4 as in Mlungisi Primary School. The 0% was obtained where learners were to deal with word problems which required them to convert them to numerical notations and simplify. Although there was a change in C3 and C4, with 38% and 24% increase in correct responses, the highest percentage in responses fell from 75% to 38%. The same as in Mlungisi Primary School, learners' response percentages fell notably. For example, in the pre-test, the learners' percentage in C1 – C4 was 75%, 68%, 0%, 0% and 70% respectively, whilst in the post-test for the similar questions the correct response percentages were 14%, 34%, 38%, 24% and 28%. This result might be attributed to the reasons given for the learners' responses in Mlungisi Primary School.



Graph 14: **Results of Xhalanga primary school after the CGI in Question C**

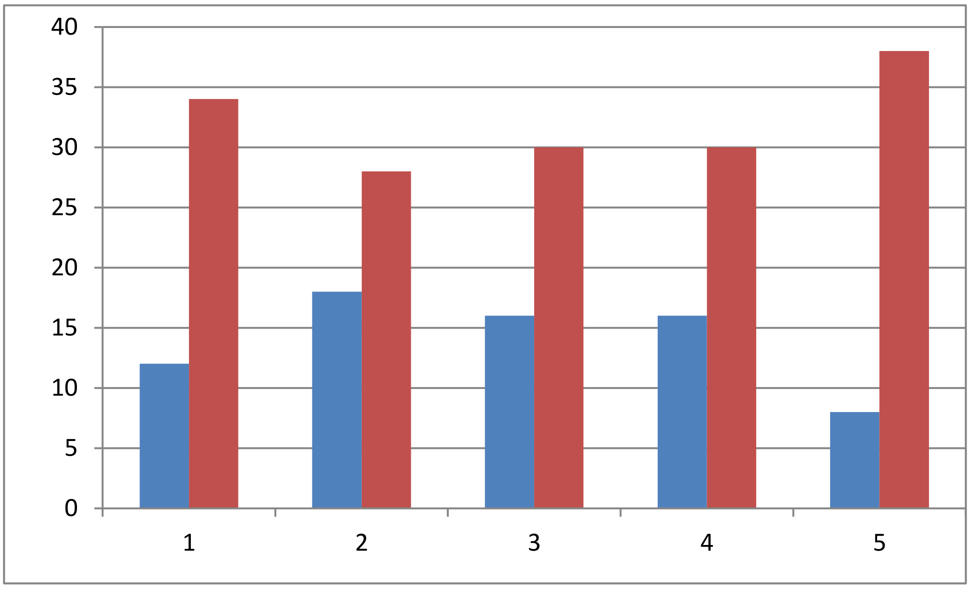
Graph 14 shows the learners' responses for the Xhalanga Primary School in Question C. The red bars which represent the incorrect responses are getting longer whilst the blue bars which represent correct responses are gradually shrinking. This is a cause for concern for any educationist.

4.5.4 FINDINGS FROM D1 – D4

QUESTION	NW	CR	IR	% CR
D1	46	12	34	26
D2	46	18	28	39
D3	46	16	30	35
D4	46	16	30	35
D5	46	8	38	17

Table 15: **Results of Mlungisi primary school after the CGI in Question D**

In Question D the learners were required to simplify common fractions with the same denominator and those fractions with denominators which are multiples of each other. In the pre-test the highest percentage of correct responses was 53% and the lowest was 21%. In the post-test the highest percentage of correct responses decreased to 39% with the lowest percentage recorded at 17%. In the pre-test it was evident that learners had challenges with the denominators of fractions which were multiples of each other. Therefore, the researcher introduced the equivalency concept to simplify fractions. This equivalency concept was introduced through the use of the fraction wall chart and some fraction pieces. From the class work and home work learners showed signs of understanding the concept. However, the result showed an opposite reality.



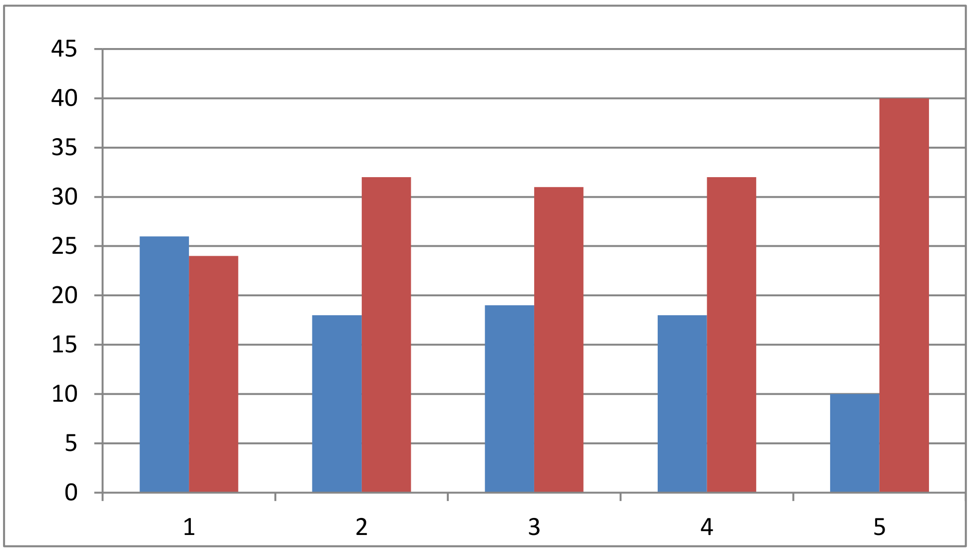
Graph 15: **Results of Mlungisi primary school after the CGI in Question D**

The above graph shows the drop in learners' responses from Mlungisi Primary School after the CGI was used to improve learners' responses in simplifying given common fractions. The greatest challenge shown here was that learners seemed to have difficulty in finding the common denominator using equivalency.

QUESTION	NW	CR	IR	% CR
D1	50	26	24	52
D2	50	18	32	36
D3	50	19	31	38
D4	50	18	32	36
D5	50	10	40	20

Table 16: **Results of Xhalanga primary school after the CGI in Question D**

For the pre-test in the Xhalanga Primary School learners were able to obtain 72% highest number of correct responses and the lowest response was 0% in sub-question D2. In the post-test, the 0% improved to 36% of correct responses. Although there was this increase in correct responses in D2, the concern remains that the number of correct responses dropped from 72% in the pre-test to 52% in the post-test. The learners' seemed to make more errors due to various misconceptions that they picked up. The misconception of adding numerators and denominators became the major challenge, as was the fact that learners seem to disregard the meaning of the given fraction and just simplified anyway.



Graph 16: **Results of Xhalanga primary school after the CGI in Question D**

The above bar graph shows the learners' responses in Question D. It is only in sub-question D1 where there were more correct responses than incorrect. Part of the reason may be that the learners were requested to simplify fractions with the same denominator.

4.6. Findings from observation

Cohen (2011) views the use of immediate awareness or direct cognition as a principle mode of research that has the potential to yield more valid or authentic data than would otherwise be the case with mediated or inferential methods. As stated by Robinson (2002), observation can be of facts, e.g. the number of books in the classroom, the number of the students in the classroom, but it can also focus on events as they happen in the classroom.

For this study, the researcher focused on events that had to do with the study. These included: the classroom lay-out including learner arrangement, classroom discipline (intra and extra), the learners interaction with the task, time spent on the task, learner responses to questions (for both interviews and the test), and document analysis.

Learners were upbeat about the use of visuals and the fraction pieces. The researcher also used paper folding to develop the fraction sense in terms of magnitude. The learners were able to point, name and write the various forms of fractions on the fraction wall. Learners were able to draw diagrams of different shapes to represent and to interpret various fractions. Some learners though, including the 11 year old girl who was part of the learners who were interviewed, were always the last ones to submit their workbook for instruction. Moving around the desks observing the learners as they worked, some were working in pairs in their desks; some learners including the 11 year old were working individually with their work buried underneath their arms and foreheads. Those learners took their time to submit their work for marking up until the bell rung for the period change. Some learners were actively engaging with the task, each other and curiously asking questions of researcher. Although the teachers were invited to be part of this study, they chose to stay away and only participate as resource persons.

4.6.1. The classroom lay-out

In Xhalanga primary school, there were 56 learners in the selected classroom, which is a very large class by any standards. It is unacceptably large according to the Educational framework which is 35-40 learners per class per teacher (DoE, 1996). However, on the day of the pre-test there were 53 learners who wrote the test. Because of the numbers stated above and the available furniture, learners could not or were not arranged in the proposed social learning groups. Full light was only coming from one side of the classroom with the other side having high windows to prevent external learner distraction from work. The classroom is divided from the adjacent classroom by a rolling iron sheet door which could not contain noise and disturbances from the adjacent classroom. I am of the opinion that the classroom setting and arrangement are not conducive to effective learning.

However, in Mlungisi primary school the lay-out was a little different, with a total number of 47 learners and with full light coming from two sides of the classroom and two stable walls which separate the classrooms, hence containing the disturbances from the adjacent classrooms. The desks are arranged such that learners can work in groups and form some social working groups as proposed in learner centred approaches.

4.6.2. Classroom discipline

Intra-classroom discipline was at its high with learners upbeat about the test and study content, methods or approach to teaching and learning. They were always willing to participate in all activities. Extra classroom activities like feeding (nutrition) which happened between 9:30am and about 10:30am distracted the focus and caused learners to divert their attention to feeding and effective learning could not take place. Other factors like extra-mural activities, including choral music and sport, created a fair share of disturbance during optimal learning time. Absenteeism in these big classes is also a norm and therefore some quiet learners can be absent for days without being noticed.

4.6.3. Learners' interaction with the task

The learners were willing to work to the best of their abilities, including interactive dialogue. However, naming or drawing diagrams with mixed fractions, the word problems and the simplification which involved mixed fractions with the same or different denominators in both schools proved to be difficult for all the learners in the pre-test. Some of the learners could not give reasons for their responses which became almost difficult for the researcher to identify whether the response was a misconception or an error. Some learners took too much time on questions that needed just mere counting of the number of shaded areas against the unshaded ones. Language also seemed to be a barrier because when probing questions were asked during the interviews in their own language you would find out that the learners responded correctly which brings us to the question: could the language also lead to errors and misconceptions?

4.7. Findings from document analysis

The researcher intended to analyse the learners' books showing previous work when they were in Grade 5, but even the learners selected for the interviews failed to bring their previous years' work and by the ethical code the researcher could not go beyond asking. Available documentary material included the previous year's final examinations (Grade 5) papers, previous years' Grade 6 final examinations and Grade 6 ANA exam papers, mathematics mark sheets which were not specific to common or mixed fractions, and their class tasks including homework, classwork, the pre-test and the post-test). However, from question or answering papers, samples of common fraction questions were available for inspection. Some workbooks had signatures appended on them with not much feedback to the learner.

During my interaction with the work of some of the learners' who made about up 97% of both classes, they showed understanding of the work, although, according to Ashlock (2002), they were being rushed through the work. When misconceptions were identified by the researcher from one or more learners, that misconception was dealt with and discussed with the whole class using resource material or if the case was unique, the researcher dealt with it as such.

In these documents the researcher discovered that most learners did not regularly submit their task books for inspection or corrections by the teacher, hence most of the work on fractions (most importantly mixed fractions) was not marked, and one of the reasons which can be attributed to this unmarked work was the sizes of the class, because it proved to be difficult for the researcher to interact fully with each learners' errors or misconceptions individually; rather, the researcher tried to address these mistakes at once with the whole class.

Other learners would just stall and waste time until the period was over and therefore would not be obliged to submit their work that day, and absenteeism was another bad tendency which also contributed to this unmarked work. The teacher contributed as well by indecisive marking which does not give positive feedback to the learners. Because of this and other tendencies, learners could not get corrective feedback and therefore misconceptions or errors made remained uncorrected and they thus become part of the learners' knowledge and understanding. Very few of those learners whose work was marked did not commit any errors or have any misconceptions in the said concepts in these documents.

4.8. Conclusion

In this section, the researcher presented the findings from the pre-test and interviews of both learners and teachers, observed the learner behaviour when interacting with the task, analysed the learners' documents and gave a brief description of the learning environment and the extra class environment, compiled lists of tables and drew graphs showing the learners' responses for both the pre-and post-test written by learners. Although not according to expectations, the learners performed better in the post-test than in the pre-test. The majority of learners had challenges in doing word problems and simplification, more especially when dealing with mixed fractions of any kind.

CHAPTER 5

DATA ANALYSIS, RECOMMENDATIONS AND CONCLUSION

5.1. Introduction

This chapter presents data analysis followed by recommendations and conclusion. During analysis, the researcher tested whether the data collected were able to answer the research questions posed. As stated in the methodology section, the researcher used the four categories of errors and misconceptions noted in Hodes and Nolting's (1998) theory to interpret the forms of errors and misconceptions made by learners in the two schools. The researcher also compared the results of studies undertaken by other researchers with this study. The researcher evaluated whether the CGI did improve the learning of common fractions in the two primary schools. Being an interpretive study the data from interviews, observations and document analysis were also used in interpreting the findings.

5.2. DATA ANALYSIS

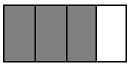
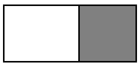
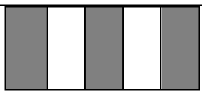
In analysing the findings, the researcher used triangulation as stated in chapter 3. The main focus of the analysis was based on the Hodes and Nolting theory (1998), the interviews of learners and teachers, the findings from the pre-test and post-test, document analysis and direct observation. In analysing the data the researcher also focused on the answers to the three research questions which were:

(1) What are the common errors and misconceptions made by Grade 6 learners in common fractions?

(2) What are the reasons for the common errors and misconceptions made by Grade 6 learners?

(3) How can Cognitively Guided Instruction be used to improve learning and understanding of common fractions?

The common errors and misconceptions made by Grade 6 learners in both schools: The errors by the learners are categorised as shown in Table 17 below following the suggestions by Hodes and Nolting

No.	QUESTION	LEARNERS' INCORRECT RESPONSES	TYPE OF ERROR OR MISTAKE
1.	 Give the correct numeric representation of this diagram.	The learner's response was $\frac{1}{3}$	Careless error
2.	  Give the correct numeric representation of this diagram	The learner's response was $\frac{6}{1}$	Conceptual error
3.	 Give the correct numeric representation for the above diagram. The same learner for another question 	The learner's response was $3\frac{3}{2}$ instead of $\frac{2}{3}$ The second response was $1\frac{1}{2}$ instead of $\frac{1}{2}$.	Conceptual error
4.	 Give the correct numeric representation of the diagram above	Learner's response was $\frac{3}{2}$	Conceptual error/Careless mistake.

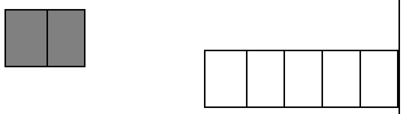
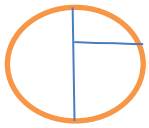
5.	Draw a shaded diagram for the fraction $\frac{2}{3}$	Learner's response was 	Conceptual error
6.	Draw a diagram representing the following fraction $2\frac{5}{6}$.	Learner's response was 	Conceptual error
7.	Draw a diagram representing $\frac{2}{3}$	Learner's response was 	Conceptual error
8.	Five fifths of a class are boys and girls. If two fifths are boys, what fraction of the class are girls?	Learners' responses (1) $\frac{5}{5} + \frac{2}{5} = \frac{7}{5}$	Application error
9.	Three and a half added to five quarter litres of milk.	$3\frac{1}{2} + 5\frac{1}{4} = 8\frac{2}{6}$	Procedural error
10.	Simplify in $\frac{7}{10} - \frac{3}{5}$	Learners' response was $\frac{7}{10} - \frac{3}{5} = \frac{4}{5}$	Procedural error

Table 17: **Category of learners' errors and misconceptions**

5.2.1 Discussions and recommendations

Hodes and Nolting (1998) suggested four types of common errors made by primary school learners when learning fractions as discussed in chapter 2.

5.2.2 Careless errors or mistakes

In Table 17 **error number 1** is a **careless error or mistake**. The learner who was interviewed gave $\frac{1}{3}$ as her response instead of $\frac{1}{2}$. When probed about her response, the learner said that she drew her own line to make three partitions. Given her response after the construction of the diagram, the response was correct. However, the learner erred because they were not instructed to reconstruct the diagram but to give the correct numeric notation of the already shaded diagram.

Another learner gave $1\frac{1}{2}$ as his response meaning one half. Orally the response is correct and you might not notice that the learner was unable to write the response. During the pre-test there were 11 learners who got incorrect responses in Question A_1 of which eight displayed careless errors or mistakes. During interview Mrs Jobodwana (the teacher from Mlungisi) said, … “in some parts of the test they performed well when the question was very simple or when the denominator was similar but had some challenge when the denominators were multiples of each other…”

5.2.3 Reasons for these careless mistakes or errors?

When learner 6J20 was probed about her response, the learner gave a correct response. When the researcher followed up on why she had given the incorrect response, she said that she drew her own lines because she did not understand the question correctly. Which brings us to the suggestion that, some of the errors that learners make are caused by them not understanding the question correctly. The learner here might have been careless by not carefully reading the question. Hodes and Nolting (1998) defined careless errors as mistakes learners make during the calculation process and can be easily detected and corrected when learners review their work. I concur with Hodes and Nolting (1998) in this regard because when the learner was interviewed she easily detected her error and gave a correct response. This type of careless errors is also confirmed by Dawkins (2006) that learners who do not carefully read the instructions and problem statements before starting to work on a problem do make mistakes.

Cockburn (1999) suggested the following model to explain some of the commonest sources of mathematical errors, the expertise of the learner with the basic rules for completing a given task. According to Cockburn, misconceptions and careless errors may occur when a learner lacks ability to understand what is required from the task. Cockburn further suggested that the mood of the learner with which the task is being tackled can affect the learner's performance in that, if the learner is not in the right mood for working or rushes through a task, careless mistakes may be made.

Idris and Narayanan (2011) also suggested that teachers tend to use the same examples of fractions, e.g. halves and quarters, without introducing learners to other fractions, hence the learners will think that the teachers have erred when introducing a new fraction, e.g. a third in a task. This can be possible as the learner decided to draw another partition to give a third as a correct answer. They also suggest that learners' corrections are sometimes ignored and not marked and analysed during feedback and this causes an incorrect misconception to persist. During the document analysis into the learners' exam answer sheets, the researcher found that similar types of careless errors were prevalent showing that the teachers might not have correct the misconceptions learners displayed in their earlier learning of the concept.

Examining the learners' workbooks, I found that it is normal just to append a signature to the learners' work with not much of a feedback to the learners. The signature might suggest that the work has been seen and that it is correct, or even the learners might copy the teacher's signature and append it as if they have done the work. Above all it seems as if that no individual attention is given to these learners as the class was full and some learners might have stalled and did not submit their work for inspection and therefore effectiveness cannot be guaranteed. A look at the learners' tasks confirmed the argument by Idris et al. that learners' corrections are sometimes ignored and not marked and analysed. Just appending a signature to the learners' work is neither marking nor feedback. These careless errors, therefore, persist uncorrected and they become misconceptions.

Concurring with the above suggestions, Mrs. Xabendlini had this to say about errors committed by learners in Xhalanga Primary School: "The learners have a tendency of being careless because as you probed them during the interviews, they supplied correct responses which means that they don't do enough to think mathematically on their own." Mrs. Xabendlini went further to say that learners might have barriers like language and physical impediments.

In schools, Head of Departments have been employed to supervise the teachers' work. In this case there seems to be no or minimal supervision as some work goes without being marked. This raises the question as to how the learners in lower classes are progressing in large numbers to the higher classes. Can we therefore trust the SBAs (School Based Assessments)?

5.2.4 Recommendations

When giving a task to learners, teachers should always make sure that the task is within the learners' level of development. The questions should be clear and unambiguous. The time allocated to the task should be sufficient to afford them the opportunity to read the instructions carefully (learners should be trained in reading instructions carefully with understanding). A variety of fractions may be taught using concrete material; no new fractions may be introduced during a test without learners interacting with them during class and homework exercises. Corrections can be done in class and teachers should make sure that the errors and misconceptions displayed by learners are thoroughly tackled and monitored before introducing a new concept. Feedback should not only be given in written classworks but also in corrections.

In Question B learner 6J30 got all the responses incorrect and when asked why, she revealed that she did not read the questions carefully. Here the learner displays not only carelessness but a lack of interest and commitment which might suggest that she might have been trained or supported adequately in reading with understanding. Teachers should always make sure that they assist learners to understand the questions correctly. This they can do by reading to them the questions during classwork or before writing a task. The language of testing must be simple as the language of teaching and learning is not their home language.

The Department of Basic Education has embarked on a campaign to have mathematics taught in the mother tongue of all languages including isiXhosa. However, I believe that the teaching in their home language will not assist in the improvement of performance in Mathematics learning unless the correct methodology and mathematical language are used.

Heads of departments (HODs) and principals should afford much needed support to the teachers by regularly monitoring and evaluating them including visiting their classrooms. They should also inspect the learners' books for feedback. However, in some schools there are no knowledgeable HODs and therefore those mathematics teachers are left to themselves to figure out strategies and lesson planning. A suggestive action is that, when the school governing bodies are recommending educators to the Department to ensure that at least one HOD or the principal has specialised in mathematics. Otherwise our Department of Education and powers that be only think about importance of mathematics when it is results time.

5.2.5 Conceptual errors made by learners

Error number 2 in Table 17 where the learner 6J1 gave $\frac{6}{1}$ instead of $1\frac{3}{4}$ can be diagnosed as a **conceptual error**. Idris et al. (2011) defined conceptual knowledge of fractions as a knowledge of what fractions mean, for example their magnitude and relations to physical quantities, an understanding of why arithmetic procedures with fractions are mathematically justified and why they yield the answers they do. To confirm this suggested definition by Idris et al., the researcher interviewed the learner so as to establish why they gave such a response; the learner said that she added all the shaded parts to make six, meaning that the learner knows that the shaded parts of a diagram represent the numerator whilst the unshaded parts represent the denominator which is a misconception. Had this misconception been corrected earlier, the learner would have known better.

Most of the errors committed by the learners were arguably conceptual errors, confirmed by the fact that learners could not understand the principles applying to those specific situations. For example, learner 86 said when interviewed, "I did not know what to write, this just came out of my mind," This explanation reveals that the learner did not understand the concept but had to answer anyway. The learner further explained to say, "I just thought hard and I thought it was a correct answer and I wrote down." Clearly the thinking given by learner 86 had nothing to do with conceptual thinking but was just guesswork because he did not know what to write.

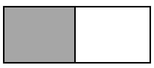
The error here is that the learner saw six parts of the diagram shaded and one unshaded. The fact that the learner could not notice that there were two diagrams and that the first diagram was all shaded suggests that the learner was not conceptually aware of what they were doing or required to do, i.e. three out of three parts of the diagram were shaded meaning that the whole of the first diagram was shaded; which means that it represents a whole (1) and that the second diagram which had only three parts out of four parts shaded represents ($\frac{3}{4}$). These are the basic fundamental facts about fractions. This might suggest that the learner did not understand and does not know about these facts, and that the learner would never be able to understand more complex concepts on fractions.

Mrs. Jobodwana cited a case of syllabus coverage as might be the cause of these errors. When probed learner 86 revealed that he did not know what to write, which might suggest that they did not get an in-depth learning of this concept. Even when he was thinking hard, there was nothing coming as there was nothing registered on these concepts in his mind.

Error Number 3 in Table 17 is also another **conceptual error.** In this question the learner was expected to give a numeric representation of these diagrams (a) and (b)

(a)

(b)



And whose correct responses were $\frac{2}{3}$ and $\frac{1}{2}$. The learners' responses were as follows: $3\frac{3}{2}$ and $1\frac{1}{2}$ respectively. It was interesting to understand why they responded this way. The learners were able to provide reasons for these responses although the reasons were invalid or conceptually incorrect. This also gave the researcher an indication as to what type of errors were they and why they were committed.

The error for the above response is that, in diagram (a), the whole was partitioned into three parts and of those three parts, two parts were shaded. The question requires the learner to give the numerical notation of the shaded parts, which is two shaded parts out of three parts written as $\frac{2}{3}$; the learner's response was $3\frac{3}{2}$.

For the second diagram, the whole is partitioned into two parts and of those two parts, one part was shaded. The same question as above was asked; the numeric notation of the shaded part was one shaded part out of two shaded parts which was written as $\frac{1}{2}$, and the learner gave $1\frac{1}{2}$ as the response.

When the learner was interviewed regarding these responses, he said for $1\frac{1}{2}$ this means one (1) half ($\frac{1}{2}$). For $3\frac{3}{2}$, the learner's answer was not consistent with the response given above for ($1\frac{1}{2}$) because he said three (whole number) represented the number of parts into which the whole has been partitioned and the other three (numerator) represented the numerator parts which were three and two represented the denominator. As to how these numbers represented as described could not be answered, which showed that the learner did some generalisation when responding to this question.

Error number 4 in the Table is a **conceptual error.** The question required the learner to give the fraction of the shaded part which was three out of five parts ($\frac{3}{5}$); the learner's response was ($\frac{3}{2}$). The error here is that the learner gave the number into which the whole was divided as 2 and which was incorrect.

Error number 5 was also a conceptual error. This is because the learner was instructed to draw a diagram representing the following fraction $\frac{2}{3}$ which was expected to be a diagram partitioned into three parts and two shaded parts instead of the diagram which is divided into four parts with three shade parts. There was no explanation given for this response by the learner.

Error number 6. In this question, the learner was expected to draw a diagrammatic representation of the mixed fraction $2\frac{5}{6}$ and the learner's response was two diagrams with one whole diagram divided into two parts which were shaded and the other diagram was divided into five parts which were not shaded. The denominator six was not represented in the diagram.

In error number 7, the learner was required to draw a diagram representing $\frac{2}{3}$. The learner's response was a circle which was not drawn correctly but dissected into three unequal parts as shown in Table 5.1. The error here is that the circle itself is not drawn according to scale. The second error is that the partitions are not equal and, thirdly, the partitioned parts are not shaded to represent the given fraction.

5.2.6 Reasons for the conceptual errors

In error number 3, the learner is conceptually incorrect because he shows lack of understanding of the basic rules for completing a given task. This reason relates to Cockburn's (1999) suggestion that some of the commonest sources of mathematical errors are caused by the expertise of the learners with regard to the basic rules for completing a given task.

Hodes and Nolting (1998) defined conceptual errors as mistakes made by learners when they do not understand the properties or principles covered in the textbook and/or lecture. The researcher is of the opinion that the error above is of the conceptual nature which is borne out of incorrect naming of the fraction. For example, although the researcher cannot give a valid suggestion for the learner's error for the response ($3\frac{3}{2}$), for the response of $1\frac{1}{2}$ and also from the learner's response during interviews, the learner might have had a naming misconception which resulted in the commission of this error.

When the learner responded orally he gave a correct response when naming the response as one half. There was a contradiction between the given written response and the orally given one. Here the learner used a whole number instead of the numerator to name the fraction. The reason could be that the naming skill of the fraction concept during the teaching or lesson was not done correctly by the teacher as suggested by Kerslake (2000) and Denise (2007). Kerslake (2000) stresses mainly the teaching of the skills of naming and writing the fraction. Denise agreed with Kerslake in that elementary learners need to know how to read fractions, how to write fractions and how to work with fraction families.

From the discussion above it is evident that the learner was taught about fractions and it might be that not much emphasis was made on how they should name and write fractions. There could be various reasons that cause **error number 4.** One of the reasons was suggested by Ashlock (2002) as that such errors are borne out of misconceptions learners have in learning common fractions such as learners equating the number of shaded parts in a diagram to the numerator and the number of unshaded parts to the denominator or vice versa. For example, the learner 6J30 in Mlungisi Primary School counted the number of shaded parts and equated them to the numerator to get $\frac{2}{1}$ which was incorrect. Also learner 85 in Xhalanga Primary School had a similar answer when given a mixed fraction diagram to write in a numerical form. Instead of giving the answer $1\frac{5}{6}$, he gave the answer $\frac{6}{1}$. Generally in both the schools there were common errors and misconceptions displayed for this type of error.

In an experiment carried out by Hansen (2006), she found that the learner does not see the five parts as all part of a whole. According to her, understanding the whole and that fractions are a part of a whole is very important but the learner could have focused on individual parts of the diagram. Hansen also suggested that the learner might have been giving a ratio of the fraction 3:2; that is three shaded parts are to two unshaded parts which was incorrect and this is confirmed by Yusof and Malone (2009) in chapter 2. Teachers tend to overuse the direct instruction method which focuses on content rather than deeper understanding of the concept.

I concur with the suggestion made by Hansen (2006) and Yusof et al. (2009) because in my study, when interviewed the teacher in Mlungisi Primary School about the use of CGI, she said, “I have not used it because there is so much work to do and therefore I prefer to teach the learners”. In a nutshell, this suggests that the teacher here was saying the most important thing to her was to go to class every day and have something for the learners with not much planning for effective learning and understanding. The profile of the said teacher is good in terms of experience and support. However, when it comes to delivering quality lessons, this profile becomes inadequate.

In error number 5 the learner was expected to draw a diagram represented by $\frac{2}{3}$, but instead drew a diagram showing $\frac{1}{4}$. There may be varied reasons for such an error, one of them can be that the learners are used to quarters only, or that they always got a correct mark when they used a quarter, or maybe they understood quarters better than any other fraction because their teacher loved them too. Newstead and Murray (2000) agreed with this assumption by concluding that one of the reasons for learners poor understanding of fractions is the way and sequence in which the content is presented to learners, in particular exposure to a limited variety of fractions (only halves and quarters) and the use of pre-partitioned manipulatives.

In error number 6, the learners simply did not know what to do. Probably the reason for this error is that learners never practiced doing diagrammatic representations of mixed fractions in class. The learners could have done diagrammatic representations only on common fractions and not intensive training on mixed fractions; hence they gave only the diagrammatic representations of two components of the fraction, i.e. two (2) and five (5) which are whole numbers of the fraction.

When interviewed, the learner could not give a reason for his response which confirmed that, he did not only misunderstand the question but also he did not understand the principles and properties of the task as suggested by Hodes and Nolting (1998).

In error number 7, the learner thinks logically but (1) did not draw the circle correctly to shape, (2) the learner did partition the circle correctly but the parts were not equal, and (3) the learner did not shade the diagram to show the given fraction. The possible reasons for this are that when the learner was first introduced to the fractions, the teacher drew circles on the chalkboard using a free hand method. This method of drawing results in a circle that is not perfect but sometimes crooked. The learner then is given the impression that the shape and perfectness of the circle is of not much importance and the focus is mainly given to the partitioning.

The second possible reason is that as the circle was not drawn to scale the parts could not be drawn to scale either. This confused the learner that they forgot or could not be sure which parts to shade. Here the teacher might have failed to teach the learner the correct drawing procedures for circles, or even might have failed to monitor the learner's drawing in class, as a result of which this persisted and was taken as an accepted norm. This confirms what Newstead et al. (2000) suggested as some of the reasons for learners' errors and difficulties which they termed a classroom environment, in which, through lack of opportunity, incorrect intuition and informal (everyday) conceptions of fractions are not monitored or resolved. Subsequent to the confusion that happened, the learner either forgot to draw or could not decide which parts to shade.

5.2.7 Recommendations

Behr et al. (1983) suggested that the teaching of fractions for understanding is one of the main goals or objectives of mathematics education. It is imperative therefore, that teachers employ effective approaches to assist learners in understanding the fractional concept by first using concrete materials where learners will be able to build their knowledge base. Swan (2001) argued that frequently a misconception is not wrong thinking but a concept in embryo or a local generalization that the learner makes.

Siegler and Fazio (2011) recommended that learners be introduced to fractions at an early age by building on their informal understanding of sharing and proportionality. According to them, this early knowledge can be used to introduce the concept of fractions connecting students' intuitive knowledge to formal fraction concepts with activities which involve simple equal sharing and proportional reasoning. They further suggested that students need to know that fractions are numbers with magnitudes.

Siegler et al. (2011) further recommended that visual representations of fractions help develop conceptual understanding of computational procedures. According to them, one of the reasons that cause learners to have misconceptions is that learners are often taught computational procedures without adequate explanation of why the procedures work. They further suggested that learners who understand why a denominator is necessary when adding fractions are more likely to remember the correct procedure than learners who do not understand why common denominators are required. They cited an example where visual representations can be used to help illustrate the need for common denominators when adding and subtracting fractions. They used examples where a teacher can demonstrate addition by using fractions of an object (e.g. $\frac{1}{3}$ of a rectangle and $\frac{1}{2}$ of a rectangle). By placing the third of a rectangle together with a half of the rectangle together inside a third rectangle, the teacher can show the approximate sum. The teacher can therefore show that $\frac{1}{3}$ of the rectangle equals $\frac{2}{6}$ and that $\frac{1}{2}$ of the rectangle equals $\frac{3}{6}$ and when these combine exactly equals$\frac{5}{6}$. This type of concrete demonstration can help develop the learners' conceptual understanding of fractions.

Another misconception here is that learners tend to treat the numerator and denominator as separate whole numbers. It is therefore important for teachers to encourage learners to use correct mathematical language when naming a fraction e.g. $\frac{2}{5}$ (two fifths) plus $\frac{1}{5}$ (one fifth) equals $\frac{3}{5}$. An early introduction to equivalent fractions can also develop the common denominator principle and therefore addition and subtraction of fractions can be made easy. Naming and writing of fraction exercises are also recommended at this stage, because it is another way of making learners understand the meaning and magnitude of the fraction.

Mrs. Jobodwana suggested that learners can be introduced to fractions by using diagrams to teach them the fraction concept first. According to her, these diagrams should vary from halves, thirds, sixths up to double denominators. She further argues that when learners are taught one type of fraction they will not know the other types and will always think of the ones they have been taught. She recommends that leaners should be taught to name, write and compare fractions so that they can recognise a fraction and its size. For example, learner 6J1 could not correctly name the fraction $\frac{1}{2}$ and instead named it as $1\frac{1}{2}$ which showed that she did not know either the meanings of the fractions given or their relative sizes.

The researcher agrees fully with the above recommendations that teachers need to assist learners to develop the conceptual knowledge and understanding of the fraction concept and computational procedures, including the use of correct denominators using visual manipulatives like diagrams, fraction wall charts and fraction pieces. Once learners are able to visualize, name and write fractions, fewer errors can be made by learners. The teachers should be able to develop their own manipulatives.

5.2.8 Application errors

Error number 8 is an application error. Hodes and Nolting defined the application errors as those errors in which learners know the concept but cannot apply it to specific situations or questions. For example, in the above question the learner is required to convert the word problem and simplify. The learner is required to simplify a given problem. Here the learner was logical and was able to convert the word problem correctly. However, the learner could not apply the procedure correctly. For example, the learner was able to convert to numeric notation but used an incorrect operation, i.e. instead of subtracting the learner added the fractions.

6J20 made an application error in Question C by adding the numerators and denominators as the problem required the learner to subtract to get the correct response. The learner knows the concept of addition and/or subtraction of fractions but on application she used the incorrect symbol; instead of subtracting she added the fraction.

5.2.9 Reasons for the application error.

The reason for this error might have been language comprehension. Most learners in both schools come from impoverished backgrounds and the language of teaching and learning is English which is their First Additional Language (FAL) is. A study carried out by Yusof and Malone (2009) in Brunei Malaysia found that language was one of the challenges that caused learners to commit some computational errors. The researcher noted that, although the learner was able to convert the word problem, they were not able to apply the correct procedure.

Mrs. Jobodwana attributed the error to the fact that the learners do not seem to understand when to add or subtract the numerators and in some instances they also add or subtract the denominators. According to Mrs. Xabendlini, learners do not know the function or importance of either the denominator or the numerator and they don't seem to understand when or which one to add or subtract between the denominators and the numerators. For example, in Question D_1 learner 95 was given a fraction to simplify. The answer she got was incorrect and she could not give an explanation for her response but she applied a principle and could not give a reason why she chose to calculate like that.

Mrs. Xabendlini suggested that the learners have been in constant dealings with the concept of fractions before formal interaction. According to her this early interaction with informal fractions bred the formation of varied misconceptions on common fraction.

5.2.10 Recommendations

In part, the application error occurs sometimes because the learner had language challenges which bring us to the suggestion that the teachers:

(1) Always use correct mathematical language when teaching, receiving or asking questions. Learners need to be encouraged to practice using correct mathematical

language whenever they are in a mathematics classroom. This helps learners to be acquainted with the vocabulary used in common fractions.

(2) Learners should be given chance to visualize and handle the concrete materials used so that they can build a knowledge base for future computational development.

(3) The above suggestions include naming and writing numerical representations of fractions, doing and practising word problems, and giving regular feedback to learners.

When learners are taught fraction conceptual knowledge at an earlier age or grade, they are able to correctly choose and use procedures that are relevant, to solve their mathematical problems (Ashlock, 2002).

Mrs. Jobodwana recommended that learners should be introduced to the concept of equivalent fractions so that they can simplify simple fractions and build on that in Grade 6. According to her, the other concept that should be taught using material such as the fraction wall is equivalent fractions so that when learners are given denominators which are multiples of each other they are able to convert denominators to be similar and add.

Mrs. Xabendlini in Xhalanga Primary School also advocates for the introduction of equivalent fractions to assist in the addition and subtraction of common fraction. According to her, the common denominator concept can only be introduced when the learners have understood the equivalent fraction concept.

5.2.11 Procedural errors

Errors 9 and 10 are procedural errors. Hodes and Nolting (1998) defined procedural error as a mistake made by the learners when they skip steps or directions, or misunderstand directions but answer the question anyway. **In error number 9**, it will be noted that the learner was given the task to complete $3\frac{1}{2} + 5\frac{1}{4}$ and the given response was $8\frac{2}{6}$, and in **error number 10** the learner was required to simplify $\frac{7}{10} - \frac{3}{5}$ and the learner's response was $\frac{4}{5}$. The main error here as defined by Hart (1981, p.39) was "adds tops and bottom rule". According to Hart, this form of error was more prevalent where the two denominators were different. Hart further argued that it was interesting to note that this particular error occurred more when the question was posed in computation form than in word problem form. Another computational error here is procedural in that the learner just added tops and bottoms without finding a common denominator.

According to Hodes and Nolting, the procedural errors are the fourth type of errors which occur when learners skip directions or steps, or misunderstand directions, but answer the question or the problem anyway. Learner 85 committed a procedural error in Question D4 where she correctly added the whole numbers three and five but she continued to add the numerators.

5.2.12 Reasons for procedural errors

There are multiple reasons for these errors to be made by the learner: (1) the learner could not find a common denominator, or (2) the learner does not understand the methods for finding a common denominator. In a similar study by Lukhele et al. (2002), they found that this type of error occurred mostly because many learners approached these problems from a whole number point of view, implementing the whole number procedures without regarding fractions as fractions.

Here the learner (85) might have not understood the concept of equivalent fractions, but might have known from previous knowledge that in the addition of mixed fractions the whole numbers are added, but she went on to add the numerators but she could not account for the denominator. When asked how she got the response, she revealed that she added three and five which were the whole numbers and she was correct but she continued to add the numerators, i.e. 1 + 1 = 2 and added to eight of the whole number sum to make ten. When further quizzed, she did not respond.

5.2.13 Recommendations

Idris and Narayanan (2011) suggested that teaching of fractions must be done meaningfully so that formal mathematics is seen as a subject for whole class discussion in primary school. They further advised that conceptual oriented instruction enables students to achieve a level of computational competence they would not have achieved had they been in a procedurally oriented classroom. Conceptually oriented instruction enhances learners' ability to understand, and through these understandings computational competence is achieved. Learners should have in-depth background knowledge of the diagrammatic representation of fractions which should be the basic knowledge taught so that learners can understand what a fraction is, advised Mrs. Jobodwana.

5.3. How can Cognitively Guided Instruction (CGI) be used to improve learning and understanding of common fractions?

This is the third research question where the researcher used Cognitively Guided Instruction to improve and minimize the commission of errors in the learning and teaching of common fraction in both primary schools. After the analysis of the results of the pre-test, the researcher prepared a series of lessons which tried to accommodate all the incorrect responses learners had done.

The length of the lessons stretched from Monday to Thursday with Friday set aside for the post-test and the scope of the lesson consisted of diagrammatic representation of fractions, naming and writing common fractions, equivalence, finding the common denominator, addition and subtraction of common fractions with the same or denominators which are multiples of each other, addition and subtraction of mixed fractions with the same or denominators which are multiples of each other. See Appendix C for the lesson plan.

Evidently, there was a lot of work to be covered within a space of four days (i.e. four hours in each school). The reason for this large amount of work was that most of the misconceptions and errors found in the pre-test was work learners should have covered in previous grades. Had the learners learnt with understanding and developed conceptual and computational procedures in the earlier stages, the work would have only covered those aspects that were not yet taught.

The use of concrete or visual material was limited to fraction pieces and a fraction wall chart. This was due to time constraints which could not be stretched beyond the teaching time allocated for the learning area according to Curriculum and Assessment Policy Statement (CAPS), the other programmes of the respective schools, and according to the researcher’s own daily work programme which also contributed to the time constraint.

5.4. Findings from the post-test

After the prepared lesson was taught, a post-test was administered at both schools although not on the same day because of time constraints. The number of learners dropped from both schools. The summarized findings of the post-test were as follows:

In both schools for Question A where learners were expected to give a correct numeric notation for a given diagram, the correct responses for Mlungisi Primary School ranged from 96% to 100% and for Xhalanga Primary School the correct responses range between 90% and 98%; an improvement from before the CGI, where the learners' correct responses were between 11% to 83% and 38% to 94% respectively after the pre-test.

For Question B where the learners were requested to draw a diagrammatic representation of a given fraction, in Mlungisi Primary School the correct responses ranged from 89% to 96% whilst in Xhalanga Primary School the correct responses ranged from 92% to 98%, an improvement from the earlier pre-test which saw correct responses for the similar questions ranging from 30% to 87% and 47% to 89% respectively.

In Question C, the results of correct responses ranged from 17% to 37% in Mlungisi Primary School and the results ranged between 14% and 38% in Xhalanga Primary School, which is a significant decrease in terms of quantity in correct responses from 0% to 60% and 0% to 75% respectively. However, although there was a significant decrease in terms of quantity of correct responses, there was a slight improvement in C3 and C4 where learners displayed 0% correct responses in both schools. For Xhalanga Primary School (C3) the number of correct answers improved from 0% to 38% and for C4 the number of correct responses increased from 0% to 24%. In Mlungisi Primary School the results improved from 0% to 37% for C3 and 0% to 33% in C4.

In Question D, the results followed this pattern; the learners' correct responses in Mlungisi Primary School were 17% to 39% in the post-test whilst they were between 21% and 57% in the pre-test. For Xhalanga Primary School the correct responses ranged from 20% to 52% during the post-test and 0% and 72% in the pre-test. The results showed a slight improvement from 0% in D2 to 36% in the pre-test and post-test respectively.

5.5 Possible reasons for the decrease in correct responses

Reasons for the drop may be contextual in that the time for the amount of work covered during one week was very short. According to the Curriculum and Assessment Policy Statement for the Intermediate Grades (Grades 4 – 6, 2012) , learners in Grades 6 have to learn only addition and subtraction of common and mixed fractions with denominators which are multiples of each other and of which the researcher had to prepare a lesson which incorporates work from Grades R-6.

The other reason could have been that the lessons prepared could not be completed during the planned period because of some unforeseen circumstances including choral competitions and sporting activities. These disruptions caused lessons to be postponed for another time schedule, breaking the learning momentum before testing. For example, in Mlungisi Primary School the lesson was planned for the second week of June 2015 but could not be done because of mid-year exams which were unscheduled. They were then rescheduled for the third week of the same month. After the examinations, the number of learners attending school decreased dramatically each day. The lessons had to be postponed for the re-opening of the schools for the third quarter because the time allocated was not enough as the school had other programmes to attend to.

Although in Xhalanga Primary School the lessons commenced as scheduled, on Wednesday there was a choral music competition which led to another rescheduling of lessons. The most challenging aspect of this study was that the post-test was written on Tuesday after a long weekend.

5.6 Can Cognitively Guided Instruction improve the learning of common fractions?

Carpenter et al (1999) described CGI as an inquiry-based approach to teach mathematics which was developed at Wisconsin Centre for Education Research. They stated that this approach provides teachers with knowledge about the development stages of children’s mathematical reasoning. From the pre-test, interviews, observations and document analysis that were done during the data collection stages, the researcher was able to analyse the learners' thinking and also during the lesson where learners' were either answering oral or written questions. This enabled the researcher to plan activities whereby learners were able to develop reasoning and conceptual development. The use of visual and concrete material such as a fraction wall chart, paper folds and fraction pieces helped the learners to increase their fraction sense development as seen from the results of Questions A and B. However, for Questions C and D the results were not that impressive as shown in the table below in 5.7.

5.7 Results of (CR) in Questions C and D in the pre- and post-tests

QUESTION C	MLUNGISI PRIMARY SCHOOL		XHALANGA PRIMARY SCHOOL	
	PRE – TEST	POST – TEST	PRE - TEST	POST – TEST
C_1	28	8	40	7
C_2	22	15	36	17
C_3	0	17	0	19
C_4	0	15	0	12
C_5	22	14	37	14

Table 18: Results of Questions C after the CGI in both schools

QUESTION D	MLUNGISI PRIMARY SCHOOL		XHALANGA PRIMARY SCHOOL	
	PRE -TEST	POST – TEST	PRE – TEST	POST - TEST
D_1	22	26	37	12
D_2	10	18	0	18
D_3	25	19	35	16
D_4	13	18	30	16
D_5	27	10	38	8

Table 19 Results of Question D after the CGI in both schools

Although the results of the post-test showed a considerable decrease in Questions C and D, for Questions A and B there was a significant increase in correct responses which saw a range of between 92% to 100% for Question A and between 86% to 96% for Question B. This range of results was an increase of correct responses for both schools. In the pre-test Questions C3 and C4 netted 0% correct responses, whilst in the post-test for a similar set of questions the learners showed an improved level of correct responses from 24% to 39% for both schools.

The above results show that the CGI had an effect in the improvement of the errors; however, it should be noted that the decrease in correct responses cannot be attributed to the failure of the CGI to reduce the number of correct responses but can be seen as a direct effect of the work load that had to be done within the short space of time and also the timing of the test.

5.8 Recommendations

From Watson's (2012) definition of CGI as an approach to teaching mathematics using learners' own mathematics thinking as the basis for instruction, and for CGI to be successful, teachers should have a thorough understanding of the grade level curriculum and the distinctions between the different problem types within that curriculum, the researcher recommends that a form of baseline assessment should be employed to gauge the level of the learners' understanding of the concept and further determine the types of misconceptions and errors learners commit.

Learners are to be encouraged to make their own physical representations by affording them the opportunity to use models for mathematical reasoning and also to help them figure out an answer or as a way to think about mathematical relationships (Gravemeijer, 2001). This argument is true as this was evident during the lessons where learners were able to find correct answers using paper folding, the fraction wall chart and the fraction pieces. Learners should be given enough time as the physical modelling takes a lot of time for learners to use as an effective learning approach.

5.9 Limitations of the study

The data in this study was collected during the second quarter of the school calendar but the fractions were not taught in the first quarter. Learners in this class were taught fractions in Grade 5 by another teacher. I could not get information from the previous teacher as to what they had covered, thus I could not be certain as to whether the learners were taught sufficiently.

The time constraint which made me to rush from school to school, sometimes without adequately summarising my lesson was another challenge. The huge amount of work which had to be covered within a limited time and some learners absenting themselves from interviews and from the prepared lesson was also problematic. One of the challenges was the extra class activities which provided a fair share of disruptions of lessons.

The timing of the study was not perfect as the fractions were taught only during the second quarter in the school calendar and therefore I might have missed some true reflections of the learning process.

The inability to take learners' books home did not enable me to analyse more closely some other errors and misconception of the learners. Financial constraints limited the scope of the study to Queenstown. The drawing of the diagrams for the illustrations and the tables were found hectic and time consuming.

5.10 Possible areas for further research

During the study some interesting patterns emerged which I think can be explored further. This study was limited only to the Queenstown Education District. It is fair enough to state that the scope of the study can be expanded to include more than one district. There also seemed to be some connection between the learner age and the learning of the concept of fractions.

Teachers' actual planning and presentation of the lessons was not under scrutiny during this study as they might be the cause of these errors. The study was also limited to Grade 6 learners which mean that the study could be extended to other grades and phases.

5.11Conclusion

From the discussions above it is evident that misconceptions about fractions cause learners to commit various types of errors when dealing with fractions. The common errors found to be most prevalent in the learners' daily computations with fractions are careless errors, conceptual errors, application errors and procedural errors. Various studies have found that misconception leads to errors. However, misconceptions can be used to analyse and understand learners' thinking and understanding of fractions. Teachers can use those errors to develop the learners' conceptual knowledge about fractions.

Having noted these errors and recommendations, the researcher prepared a lesson using Cognitively Guided Instruction to test whether this method was able to minimize these errors. The results of the study showed that Cognitively Guided Instruction used correctly can assist the teachers and learners to reduce the misconceptions and errors learners make during the learning of fractions and could be applicable to other topics in mathematics as well as other learning areas.

Many authors referred in this document emphasized the importance of using a variety of physical models to develop the fraction concept. However, with the financial bankruptcy of our ordinary public schools which are no fee schools, acquiring these materials are a challenge. Hence the use of cheaper materials such as the fraction wall charts, paper folding and diagrammatic representations can be alternative resource materials. Learners can also prepare these and use them in their spare time.

Mathematics and mostly fractions need to be taught using physical models as we may have noted the enthusiasm shown by learners when these physical models were introduced. Simply focussing on syllabus coverage and content might render our teaching irrelevant and insignificant from the learners' point of view.

Teachers are encouraged to give their utmost when preparing, planning and presenting their lessons so as to improve learner attainment and minimise errors and misconceptions.

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APPENDICES

APPENDIX A_1

TO : THE DISTRICT DIRECTOR (QUEENSTOWN DISTRICT)
FROM : B.P. LIBAZI – A STUDENT RESEARCHER
SUBJECT : REQUEST TO UNDERTAKE A RESEARCH IN YOUR DISTRICT
DATE : 27 OCTOBER 2014

I, Babini Precious Libazi, an employee of the Department in your District, would like to request your permission to undertake a research study in two primary schools in your District. The intended study is to be carried out at Mpendulo Primary School which is situated at Mlungisi Township and Unathi Mkefa Primary School which is situated at Ezibeleni Township. I am planning to undertake the study in the academic year February and April 2015 and the maximum period of the study will be two weeks.

The purpose of the study is to investigate errors and misconceptions of Grade 6 learners in common fractions, the reasons for these errors and ways to eliminate them. The significance of the study is that it will inform the best practices to teaching and learning of this concept in Grade 6 and improve the performance of learners in learning common fractions.

I hope that my request will be well received.

Thanking you in advance
Yours in Education

…………………………………………..
B.P. LIBAZI
073 828 4971

APPENDIX A_2

TO : B.P. LIBAZI – A STUDENT RESEARCHER

FROM : THE DISTRICT DIRECTOR (QUEENSTOWN DISTRICT)

SUBJECT : PERMISSION TO UNDERTAKE A RESEARCH IN THE DISTRICT

DATE : 27 OCTOBER 2014

Your application to undertake a research study in two primary schools in the District has been received by my office and I would like to inform you that the permission to undertake this study has been granted as requested.

I am looking forward to your results and recommendations.

Yours in Education

…………………………………………..

H.N. GODLO

DISTRICT DIRECTOR

APPENDIX A_3

TO : THE PRINCIPAL & SGB

FROM : DISTRICT DIRECTOR

SUBJECT : RESEARCH STUDY AT YOUR SCHOOL – BY B.P.LIBAZI

DATE : 27 OCTOBER 2014

Due to general poor performance of learners in Mathematics in our district and in South Africa in general, both the Provincial and National Departments of Education including ourselves as Queenstown Education District have taken a strategic view to commission a study to investigate in-depth the causes of this chronic poor performance in this subject.

Your school therefore has been purposively selected to be a participant in this study. We therefore humbly hope that you will accept this invitation with open arms as it will not only assist in the investigation but will assist you to understand your own learners' challenges in this subject and assist in their effective teaching and learning.

Detailed information of the study will be relayed to you in due course before the commencement of the investigation; however, we want to put it up front that details of the school, the participants (teachers and learners identity) will remain confidential and those issues participants feel confidential about will be kept as such.

The study is earmarked to take about two weeks at the most during teaching hours. The researcher, who is Mr. B.P. Libazi, will do the study which will include, amongst others, performance test, interviews (teachers and learners), observations and may take photographs of the learners' work as evidence. As the department, we would like to urge you to assist him in any way possible so that this study can achieve its desired objectives.

Hoping that, this plea will be kindly and objectively received.

Thanking you in advance

Yours in Education

H. N. GODLO ………………………………………..DATE :

APPENDIX A_4

TO : THE MATHEMATICS TEACHER

FROM : Mr. B. P.LIBAZI (STUDENT AT FORT HARE UNIVERSITY)

SUBJECT : RESEARCH STUDY IN YOUR CLASS – BY B.P.LIBAZI

DATE : 27 OCTOBER 2014

I am doing a research study at the above named institution (Fort Hare) in Mathematics Education. I therefore like to use yourself and your learners in Grade 6 as subjects of my study in the learning and teaching of common fractions in the academic year February 2015.

The study will take a period of two (2) weeks maximum. No photographic pictures or audio visual material will be done during the course of the study; only photos of learners work will be taken. Confidentiality of your persons will be upheld during the study period. If you feel uncomfortable during the period of study you are free to withdraw as this is a voluntary exercise.

I would beg you therefore to respond in writing if you accept this invitation as it will be to your class and personal benefit.

Looking forward to working with you in this endeavour.

Yours in education.

……………………………………

B.P. LIBAZI

073 828 4971

APPENDIX A_5

No. 334 Zone 1
Ezibeleni
Queenstown
13 March 2015

Dear Parent

REF: FORMAL REQUEST TO INTERVIEW YOUR CHILD

I **Babini Precious Libazi**, who is undertaking a research investigation on the errors and misconceptions of grade 6 learners in common fractions in the school in which your child has been enrolled as a learner would like to request permission to interview your child whose name is………………………………………………………………………………………. I have discovered that there are interesting errors and misconceptions she/he committed during a Pre – Test exercise they did.

I want to categorically and clearly state the following conditions of your child's participation in this study:

1. Is purely voluntary and may not turn to monetary or material benefit except that it might have an educational benefit to them.
2. The identity of the location and participants including your child will be treated with confidence.
3. May be withdrawn anytime they feel uncomfortable.
4. The interview is one on one and private and the responses will be purely used for the purpose of this study as stipulated.

This study is in the public interest and therefore your child's participation will be an honour and great value to the study and learning of common fractions.

Hoping that my request will receive your kind attention

Humbly Yours

Babini Precious Libazi

073 828 4971

babs.precious@gmail.com

Please reply below:

I ……………………………………………………………………………..who is the parent of the learner mentioned above would like to reject /permit my child's permission in this study.

…………………………………………….

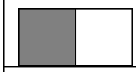
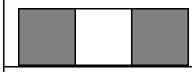
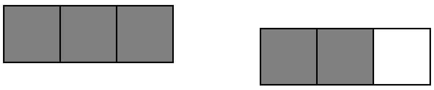
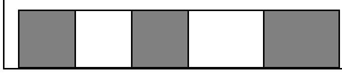
Cell:

APPENDIX B

PRE – TEST QUESTIONS FOR GRADE SIX IN TWO PRIMARY SCHOOLS IN QUEENSTOWN: DATE (16 & 17 FEBRUARY 2015)

QUESTION A

WRITE THE NUMERIC NOTATIONS FOR THE FOLLOWING DIAGRAMS:

DIAGRAMMATIC REPRESENTATIONS	NUMERIC NOTATIONS
1. 	
2. 	
3. 	
4. 	

QUESTION B

DRAW THE CORRECT DIAGRAM TO REPRESENT THE FOLLOWING FRACTIONS:

SYMBOLIC NOTATIONS	DIAGRAMMATIC REPRESENTATIONS
1. $\frac{2}{3}$	
2. $\frac{4}{5}$	
3. $2\frac{5}{6}$	
4. $1\frac{2}{4}$	
5. $3\frac{3}{7}$	

QUESTION C

CONVERT THE WORD PROBLEMS AND SIMPLIFY

1. The sum of three quarters and one quarter.	
2. From eight tenths of a class of girls; three eights of girls are dark in complex. What fraction of the girls is light in complexion?	
3. Three and a half loaves of bread brought by ANAs uncle and five and a quarter loaves brought by her mom equals to …….	
4. Four fifths of a glass of milk added to three tenths of glass milk equals to ….	
5. When Zola spilt two and a third from 5 litres of milk. The leftover was ……	

QUESTION D

SIMPLIFY THE FOLLOWING FRACTIONS:

1. $\frac{7}{10} - \frac{3}{5}$	
2. $2\frac{5}{8} + 3\frac{3}{8}$	
3. $4\frac{5}{7} - 3\frac{4}{7}$	
4. $\mathbf{7\frac{5}{12} - 3\frac{4}{12}}$	
5. $\mathbf{6\frac{2}{4} + 3\frac{1}{2}}$	

APPENDIX C - INTERVIEW QUESTIONS GUIDE

Thesis Interview Guide for Learners

The questions for this study are semi – structured, however there are some basic questions drawn from the learners performance and some probing questions will depend on learners responses:

1. How did you get this answer?
2. Why did you do that?
3. What is the correct answer if you do no draw your own lines?
4. Where is the diagram that represents the 6 which is the denominator?
5. Why did you add the shaded parts?
6. What was the reason for you to give this answer?
7. How did you reach this response?
8. Why did you do it like that?

Thesis Interview Guide for Teachers

1. How do you think the learners performed in this test?
2. What do you think caused this performance?
3. How do you think this performance can be improved?
4. What type of errors do you think learners do here?
5. What do you think causes these errors?
6. How do you think these errors can be improved?
7. What do you know about Cognitively Guided Instruction?

How do you think it can assist in eliminating these errors ?

APPENDIX D – THE COGNITIVELY GUIDED INSTRUCTION (CGI) LESSON PLAN

<u>LESSON PLAN – INTERMEDIATE PHASE</u> **<u>RESEARCHER/TEACHER</u>: B.P. LIBAZI**

<u>DATE:</u> FEBRUARY 2015 **<u>LEARNING AREA</u>: MATHEMATICS** **<u>GRADE</u>: 6**

<u>DURATION</u>: <u>CONTENT IN CONTEXT</u>: ADDITION AND SUBTRACTION OF COMMON AND MIXED FRACTIONS

<u>VOCABULARY</u>: COMMON FRACTIONS, DENOMINATOR, MULTIPLES, COMMON RATIO, MIXED NUMBERS, EQUIVALENT FRACTIONS

SELECTED CONTENT AREAS AND TOPICS	TEACHING &LEARNING ACTIVITIES	DETAILS OF ASSESSMENT : FORMS , METHODS, AND TOOLS	PROVISIONS FOR LEARNERS WITH BARRIERS
Content Area (1): Number, Operations and Relationships – Common Fractions. Revision/ Introduction : The teacher revises the work done in Grade 5 by giving learners a task on common fraction using concrete materials/ manipulatives / physical models : 1. Addition of common fractions with the same denominators. 2. Subtraction of common fractions with the same denominators. 3. Addition of mixed numbers with the same denominators. 4. Subtraction of mixed numbers with the same denominators. <u>CONTENT SPECIFICATION</u> : • The range of numbers developed by the end of the intermediate phase is extended to at least 9 – digit whole numbers,	<u>ACTIVITY</u> 1<u>: MENTAL</u> <u>Ice breaker</u> 1.1. Learners are each given a piece of paper to fold so as to create personal experiences. As they fold; learners are requested to name, write and diagrammatically represent the fractions formed. 1.2. The learners cut out the pieces of formed fractions and label/ name them. 1.3. Learners are required to arrange the fractions according to their sizes. 1.4. The learners add / subtract the fraction pieces together. E.g.	<u>Assessment Forms</u> : Group work , observations , interviews , document analysis , assignment , games , class work , home work , pair work , individual work , worksheets. <u>Assessment Method</u> : Peer, group, self, reflection and feedback. <u>Assessment Tools</u> Rubric, checklist, observation, memorandum.	• Individual / group and whole class attention. • Pairing with abled learners to promote social or cooperative learning. • Scaffolding • Use of manipulatives. • Simple communication language. • Use of bold visual aids. • Visually impaired learners accommodated next to windows with sufficient light. • Rewarding / reinforcement of correct answers. • Remedial work and rapid feedback.

decimal fractions to at least 2 decimal places, common fractions and fractions written in percentage form. • In this phase, the learner is expected to move from counting reliably to calculating fluently in all four operations. The learner should be encouraged to memorise with understanding, multiply fluently and sharpen mental calculation skills. • Attention needs to be focused on understanding the concept of place value so that the learner develops a sense of large numbers and decimal fractions. • The learner should recognize and describe properties of numbers and operations, including identity properties, factors, multiples, and commutative, associative, and distributive properties. <u>TOPIC : CALCULATIONS WITH FRACTIONS:</u> • Addition and subtraction of common fractions in which one denominator is a multiple of another. E.g. $\frac{2}{4} + \frac{1}{2} = 1$ $\frac{4}{10} - \frac{1}{5} = \frac{2}{10}$	 +  $= \frac{4}{4}$ or a whole / 1 1.5. Learners play a fraction game where they read instructions / answers from pieces of provided papers. E.g. one quarter plus one quarter equals / I am a half and I am formed of four ………. <u>ACTIVITY 2</u> : 2.1. Learners are required to write these word problems in numerical notations and add them. E.g. $\frac{1}{4} + \frac{1}{4} = \frac{2}{4}$ / $\frac{1}{2}$ or $\frac{5}{6} - \frac{2}{6} = \frac{3}{6}$ 2.2. The teacher/researcher presents learners with word problems which are on worksheets and fraction pieces to calculate. <u>ACTIVITY 3</u> : 3.1. The researcher introduces mixed fractions to learners where the calculations involve whole numbers. E.g. $2\frac{3}{5} + 4\frac{1}{5} = 6\frac{4}{5}$ $6\frac{4}{10} - 3\frac{3}{10} = 3\frac{1}{10}$ ACTIVITY 4 : 4.1. Using the cut off fraction pieces and the fraction wall discover equivalent fractions of multiples. E.g. $\frac{1}{2} = \frac{2}{4} = \frac{4}{8} = \frac{8}{16}$ or $\frac{1}{3} = \frac{2}{6} = \frac{3}{9}$ etc.		

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- Addition and subtraction of mixed numbers in which one denominator is a multiple of another. E.g.

$5\frac{4}{6} + 4\frac{1}{3} = 9\frac{4}{6} + \frac{2}{6} = 9\frac{6}{6}$

= 9 + 1 =10.

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4.2. Learners are required to discover the common ratio/ general formula. And discover on their own that the preceding fraction is found by multiplying by $\frac{2}{2}$.

<u>ACTIVITY 5</u>

5.1. The researcher introduces denominators which are multiples of each other using equivalent fractions.

e.g.

$2\frac{4}{6} + 5\frac{1}{3} X \frac{2}{2}$

$7\frac{4}{6} + \frac{2}{6} = 7\frac{6}{6} = 8$

5.2. The learners are introduced to calculus where the denominator is raised throu8gh multiplication. E.g.

$9\frac{8}{14} - 6\frac{3}{7} = 3\frac{8}{14} - \frac{3}{7} x \frac{2}{2} =$

$3\frac{8}{14} - \frac{6}{14} = 3\frac{2}{14}$

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RESOURCES :

- Fraction pieces
- Fraction bars
- Pencils and cartridge paper
- Fraction Wall chart

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CONSOLIDATION :

Learners play a game on fractions on designed board; they alternate representations of word problems either with symbolic notations, numerical notations cyclically. Using flash cards.

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REFLECTIONS ON PLANNING:

- The lesson covers all the desired outcomes.
- It caters for the skills and knowledge to be acquired.
- The activities are structured such that they accomplish the expected learning

REFLECTIONS ON LEARNING :

- More work and thinking is done by learners.
- The teacher/ researcher assist the learners to achieve the required skills.
- The teaching instruction is constructivist in nature or self-discovery

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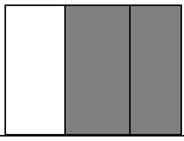
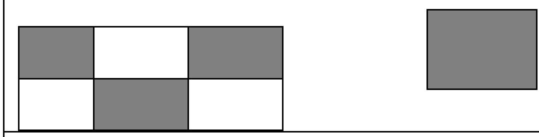
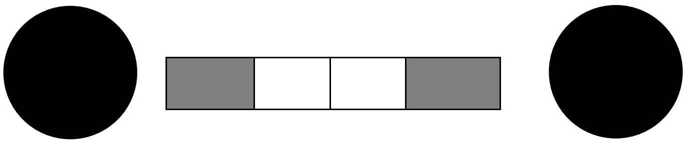
APPENDIX E – POST – TEST TASK

<u>POST – TEST ASSESSMENT TASK FOR SCHOOL A (MLUNIGSI P.S.) AND SCHOOL B (XHALANGA P.S.) – APRIL 2015</u>

<u>INSTRUCTIONS: ANSWER ALL QUESTIONS IN THE PROVIDED WORKSHEET</u>

<u>QUESTION MARKS: 20</u>

<u>WRITE THE CORRECT NUMERICAL NOTATION FOR THE SHADED PARTS IN THE FOLLOWING DIAGRAMS:</u>

SHADED DIAGRAMS	NUMERICAL NOTATION
1. 	
2. 	
3. 	
4. 	
5.  .	

QUESTION B

DRAW SHADED DIAGRAMS TO REPRESENT THE FOLLOWING FRACTIONS

NUMERICAL NOTATION	DIAGRAMATIC REPRESENTATION
1. $\frac{4}{5}$	
2. $\frac{3}{6}$	
3. $2\frac{3}{4}$	
4. $1\frac{5}{6}$	
5. $\frac{5}{10}$	

QUESTION C

WRITE THE FOLLOWING WORD PROBLEMS IN NUMERICAL NOTATIONS

WORD PROBLEMS	SIMPLIFICATION
1. Phozisa has two and a quarter of a cake; Zola added three and a half of the same cake. How much cake do they have altogether?	
2. Veli had Five and two fifths of wire to make a car; Simphiwe stole three and two tenths of the wire. How much wire is left?	
3. Khanyisile bought two thirds of fibre to make her hair, and her mother brought three sixth of fibre, how much fibre does she have?	
4. Subtract three fifths from sixth tenths	
5. Your mother bought two and a quarter kilograms of mutton stew and your father bought seven and three quarter's kilograms of mutton. How much meat do they have?	

QUESTION D

SIMPLIFY THE FOLLOWING FRACTIONS

FRACTIONS	CALCULATIONS
1. $\frac{3}{12} + \frac{5}{6}$	
2. $3\frac{3}{4} + 6\frac{1}{4}$	
3. $\frac{7}{10} - \frac{1}{5}$	
4. $9\frac{3}{7} - 3\frac{4}{14}$	
5. $4\frac{1}{2} + 2\frac{2}{4} - 1\frac{3}{6}$	

IN LUMINE TUO VIDEBIMUS LUMEN

University of Fort Hare
Together in Excellence

ETHICAL CLEARANCE CERTIFICATE
REC-270710-028-RA Level 01

Certificate Reference Number:	THO011SLIB01
Project title:	**Errors and Misconception of Grade 6 learners in common fraction: A case study of two Primary Schools in the Queenstown Education District.**
Nature of Project:	Masters
Principal Researcher:	Babini Precious Libazi
Collaborating investigator:	N/A
Supervisor:	Mr C Thomas
Co-supervisor:	N/A

On behalf of the University of Fort Hare's Research Ethics Committee (UREC) I hereby give ethical approval in respect of the undertakings contained in the above-mentioned project and research instrument(s). Should any other instruments be used, these require separate authorization. The Researcher may therefore commence with the research as from the date of this certificate, using the reference number indicated above.

Please note that the UREC must be informed immediately of

- Any material change in the conditions or undertakings mentioned in the document
- Any material breaches of ethical undertakings or events that impact upon the ethical conduct of the research

The Principal Researcher must report to the UREC in the prescribed format, where applicable, annually, and at the end of the project, in respect of ethical compliance.

Special conditions: Research that includes children as per the official regulations of the act must take the following into account:

Note: The UREC is aware of the provisions of s71 of the National Health Act 61 of 2003 and that matters pertaining to obtaining the Minister's consent are under discussion and remain unresolved. Nonetheless, as was decided at a meeting between the National Health Research Ethics Committee and stakeholders on 6 June 2013, university ethics committees may continue to grant ethical clearance for research involving children without the Minister's consent, provided that the prescripts of the previous rules have been met. This certificate is granted in terms of this agreement.

The UREC retains the right to

- Withdraw or amend this Ethical Clearance Certificate if
  - Any unethical principal or practices are revealed or suspected
  - Relevant information has been withheld or misrepresented
  - Regulatory changes of whatsoever nature so require
  - The conditions contained in the Certificate have not been adhered to
- Request access to any information or data at any time during the course or after completion of the project.
- In addition to the need to comply with the highest level of ethical conduct principle investigators must report back annually as an evaluation and monitoring mechanism on the progress being made by the research. Such a report must be sent to the Dean of Research's office

The Ethics Committee wished you well in your research.

Yours sincerely

Gdl Wet

Professor Gideon de Wet
Dean of Research

25 May 2016

8 Nahoon Valley Place

Nahoon Valley

East London

5241

29 January 2017

TO WHOM IT MAY CONCERN

I hereby confirm that I have edited the following mini thesis using the Windows "Tracking" system to reflect my comments and suggested corrections for the student to action:

Errors and misconceptions of Grade 6 learners in common fractions: A case study of two primary schools in the Queenstown Education District by BABINI PRECIOUS LIBAZI, a thesis submitted in fulfilment of the requirements for the degree of MASTERS IN MATHEMATICS EDUCATION in the FACULTY OF EDUCATION at the University of Fort Hare.

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Professional Editor

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Disclaimer: Although I have made comments and suggested corrections, the responsibility for the quality of the final document lies with the student in the first instance and not with myself as the editor.

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