

**UNIVERSITY OF FORT HARE**

**STM 312**

**DEGREE EXAMINATIONS**

**JUNE 2023**

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**Time: 3 Hours**

**Subject: Advanced Mathematical Statistics A1**

**Marks: 100**

**This paper consists of 4 pages including cover page**

**Internal Examiner**

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**Instructions**

1. Answer all questions using black or blue pen **NOT** pencil.
2. Table sheets are provided separately.

**Question One****[11 Marks]**

1.1 Let  $X$  have the probability mass function given by  $p(x) = \left(\frac{1}{2}\right)^x$ , for  $x = 1, 2, 3, \dots$ , and zero elsewhere. Find the probability mass function of  $Y = X^3$ . [3]

1.2 If  $X$  has a uniform probability density function given by  $f(x) = \frac{1}{\pi}$ , for  $-\frac{\pi}{2} < x < \frac{\pi}{2}$ , find the probability density function of  $Y = \tan X$  [4]

1.3 Let  $X$  have a probability density function  $f(x) = 4x^3$ , for  $0 < x < 1$ , and zero elsewhere. Find the cumulative distribution function and the probability density function of  $Y = -\ln X^4$ . [4]

**Question Two****[20 Marks]**

2.1 Show that the moment generating function of a gamma distribution with parameters  $\alpha$  and  $\beta$  is given by  $\frac{1}{(1 - \beta t)^\alpha}$ ,  $t < \frac{1}{\beta}$ . [4]

2.2 Let  $X$  and  $Y$  have a bivariate normal distribution with parameters  $\mu_1 = \mu_2 = 0$ , and correlation coefficient  $\rho$ . Find the distribution of the random variable  $Z = aX + bY$  in which  $a$  and  $b$  are non-zero constants. [4]

2.3 Show that, if  $X$  has a  $N_n(\mu, \Sigma)$  distribution, where  $\Sigma$  is positive definite. Then the random variable  $W = (X - \mu)' \Sigma^{-1} (X - \mu)$  has a  $\chi^2(n)$  distribution. [5]

2.4 Let  $Y_1 < Y_2 < Y_3 < Y_4$  be the order statistics of a random sample of size 4 from the distribution having pdf  $f(x) = e^{-x}$ ,  $0 < x < \infty$ , zero elsewhere. Find  $P(Y_4 \geq 3)$ . [4]

2.5 What does it mean to say  $X_n$  converges in probability to  $X$ , for all  $\varepsilon > 0$ ? [3]

**Question Three****[24 Marks]**

3.1 Let  $W_n$  denote a random variable with mean  $\mu$  and variance  $\frac{b}{n^p}$ , where  $p > 0$ ,  $\mu$ , and  $b$  are constants (not functions of  $n$ ). Prove that  $W_n$  converges in probability to  $\mu$ . Hint: Use Chebyshev's inequality. [4]

3.2 Let  $Y_n$  denote the maximum of a random sample of size  $n$  from a distribution of the continuous type that has cdf  $F(x)$  and pdf  $f(x) = F'(x)$ . Find the limiting distribution of  $Z_n = n[1 - F(Y_n)]$ . [5]

3.3 State the central limit theorem. [3]

3.4 Prove that the random variable  $Y_n = \frac{\sqrt{n}(\bar{X}_n - \mu)}{\sigma}$  has a limiting standard normal distribution. [8]

3.5 The delta method theorem states that, let  $X_n$  be a sequence of random variables such that  $\sqrt{n}(X_n - \theta) \xrightarrow{D} N(0, \sigma^2)$ . and suppose that there exist a function  $g(x)$  that is differentiable at  $\theta$  and  $g'(\theta) \neq 0$ . Then  $\sqrt{n}(g(X_n) - g(\theta)) \xrightarrow{D} N(0, \sigma^2 (g'(\theta))^2)$ . Prove this theorem. [4]

**Question Four****[18 Marks]**

4.1 Let  $X_1, X_2, \dots, X_n$  be a random sample from a distribution with pdf

$$f(x) = \begin{cases} \frac{2\beta - 2x}{\beta^2}, & 0 < x < \beta \\ 0, & \text{elsewhere} \end{cases}$$

Use the method of moments to obtain an estimator of  $\beta$ . [6]

4.2 Suppose  $X_1, \dots, X_n$  are a random sample from an exponential distribution with parameter  $\theta$ . Find the Maximum likelihood estimator of  $\hat{\theta}$  [7]

- 4.3 Let  $X_1, \dots, X_{n_1}$  be a random sample from a  $N(\mu_1, \sigma^2)$  distribution and let  $Y_1, \dots, Y_{n_2}$  be a random sample from a  $N(\mu_2, \sigma^2)$  distribution. Show that the pooled estimator  $\hat{\sigma}^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$  is unbiased for  $\sigma^2$ , where  $S_1^2$  and  $S_2^2$  are the respective sample variances. [5]

**Question Five**

**[27 Marks]**

- 5.1 Let  $X_1, \dots, X_n$  be a random sample from the uniform distribution on the interval  $(\theta, \theta + 1)$ , such that  $\hat{\theta}_1 = \bar{X} - \frac{1}{2}$  and  $\hat{\theta}_2 = X_{(n)} - \frac{n}{n+1}$ , where  $X_{(n)}$  is the  $n^{\text{th}}$  order statistics. Find the efficiency of  $\hat{\theta}_2$  relative to  $\hat{\theta}_1$ . [11]

- 5.2 Let  $X_1, \dots, X_n$  be a random sample from the Weibull density

$$f(x) = \begin{cases} \left(\frac{2x}{\alpha}\right) e^{-\frac{x^2}{\alpha}}, & x > 0 \\ 0, & \text{elsewhere} \end{cases}$$

Find a uniformly minimum unbiased estimator for  $\alpha$ . [6]

- 5.3 Let  $X_1, \dots, X_n$  be a random sample from a normal distribution with unknown mean  $\mu$  and known variance  $\sigma^2$ . Find the maximum likelihood estimator of  $\mu$  and show that it is a function of a minimal sufficient statistic. [10]

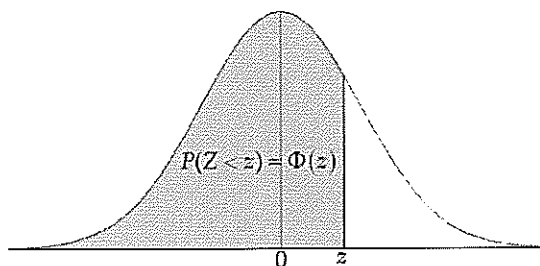
**Table 1: Area under the standard normal curve**

Normal probabilities to the left of Z.

$$\Phi(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^z e^{-\frac{1}{2}x^2} dx$$

Entries in the table are values of  $z \geq 0$

$$\Phi(-z) = 1 - \Phi(z)$$



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586
0.1	0.53983	0.54380	0.54776	0.55172	0.55567	0.55962	0.56356	0.56749	0.57142	0.57535
0.2	0.57926	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409
0.3	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173
0.4	0.65542	0.65910	0.66276	0.66640	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793
0.5	0.69146	0.69497	0.69847	0.70194	0.70540	0.70884	0.71226	0.71566	0.71904	0.72240
0.6	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857	0.75175	0.75490
0.7	0.75804	0.76115	0.76424	0.76730	0.77035	0.77337	0.77637	0.77935	0.78230	0.78524
0.8	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327
0.9	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891
1.0	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214
1.1	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298
1.2	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147
1.3	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149	0.91309	0.91466	0.91621	0.91774
1.4	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189
1.5	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408
1.6	0.94520	0.94630	0.94738	0.94845	0.94950	0.95053	0.95154	0.95254	0.95352	0.95449
1.7	0.95543	0.95637	0.95728	0.95818	0.95907	0.95994	0.96080	0.96164	0.96246	0.96327
1.8	0.96407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856	0.96926	0.96995	0.97062
1.9	0.97128	0.97193	0.97257	0.97320	0.97381	0.97441	0.97500	0.97558	0.97615	0.97670
2.0	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.98030	0.98077	0.98124	0.98169
2.1	0.98214	0.98257	0.98300	0.98341	0.98382	0.98422	0.98461	0.98500	0.98537	0.98574
2.2	0.98610	0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.98840	0.98870	0.98899
2.3	0.98928	0.98956	0.98983	0.99010	0.99036	0.99061	0.99086	0.99111	0.99134	0.99158
2.4	0.99180	0.99202	0.99224	0.99245	0.99266	0.99286	0.99305	0.99324	0.99343	0.99361
2.5	0.99379	0.99396	0.99413	0.99430	0.99446	0.99461	0.99477	0.99492	0.99506	0.99520
2.6	0.99534	0.99547	0.99560	0.99573	0.99585	0.99598	0.99609	0.99621	0.99632	0.99643
2.7	0.99653	0.99664	0.99674	0.99683	0.99693	0.99702	0.99711	0.99720	0.99728	0.99736
2.8	0.99744	0.99752	0.99760	0.99767	0.99774	0.99781	0.99788	0.99795	0.99801	0.99807
2.9	0.99813	0.99819	0.99825	0.99831	0.99836	0.99841	0.99846	0.99851	0.99856	0.99861
3.0	0.99865	0.99869	0.99874	0.99878	0.99882	0.99886	0.99889	0.99893	0.99896	0.99900
3.1	0.99903	0.99906	0.99910	0.99913	0.99916	0.99918	0.99921	0.99924	0.99926	0.99929
3.2	0.99931	0.99934	0.99936	0.99938	0.99940	0.99942	0.99944	0.99946	0.99948	0.99950
3.3	0.99931	0.99934	0.99936	0.99938	0.99940	0.99942	0.99944	0.99946	0.99948	0.99950
3.4	0.99966	0.99968	0.99969	0.99970	0.99971	0.99972	0.99973	0.99974	0.99975	0.99976
3.5	0.99977	0.99978	0.99978	0.99979	0.99980	0.99981	0.99981	0.99982	0.99983	0.99983
3.6	0.99984	0.99985	0.99985	0.99986	0.99986	0.99987	0.99987	0.99988	0.99988	0.99989
3.7	0.99989	0.99990	0.99990	0.99990	0.99991	0.99991	0.99992	0.99992	0.99992	0.99992
3.8	0.99993	0.99993	0.99993	0.99994	0.99994	0.99994	0.99994	0.99995	0.99995	0.99995
3.9	0.99995	0.99995	0.99996	0.99996	0.99996	0.99996	0.99996	0.99996	0.99997	0.99997
4.0	0.99997	0.99997	0.99997	0.99997	0.99997	0.99997	0.99998	0.99998	0.99998	0.99998

**Table 2: Values of the inverse normal distribution**

Entries in the table of the inverse function  $z = \Phi^{-1}(u)$  for  $u \geq 0.5$  where  $u = \Phi(z)$  denotes the standard normal distribution function Note that  $\Phi(-z) = 1 - \Phi(z) > 0.5$  when  $\Phi(z) < 0.5$

$\Phi(z)$	0.000	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.50	0.0000	0.0025	0.0050	0.0075	0.0100	0.0125	0.0150	0.0175	0.0201	0.0226
0.51	0.0251	0.0276	0.0301	0.0326	0.0351	0.0376	0.0401	0.0426	0.0451	0.0476
0.52	0.0502	0.0527	0.0552	0.0577	0.0602	0.0627	0.0652	0.0677	0.0702	0.0728
0.53	0.0753	0.0778	0.0803	0.0828	0.0853	0.0878	0.0904	0.0929	0.0954	0.0979
0.54	0.1004	0.1030	0.1055	0.1080	0.1105	0.1130	0.1156	0.1181	0.1206	0.1231
0.55	0.1257	0.1282	0.1307	0.1332	0.1358	0.1383	0.1408	0.1434	0.1459	0.1484
0.56	0.1510	0.1535	0.1560	0.1586	0.1611	0.1637	0.1662	0.1687	0.1713	0.1738
0.57	0.1764	0.1789	0.1815	0.1840	0.1866	0.1891	0.1917	0.1942	0.1968	0.1993
0.58	0.2019	0.2045	0.2070	0.2096	0.2121	0.2147	0.2173	0.2198	0.2224	0.2250
0.59	0.2275	0.2301	0.2327	0.2353	0.2378	0.2404	0.2430	0.2456	0.2482	0.2508
0.60	0.2533	0.2559	0.2585	0.2611	0.2637	0.2663	0.2689	0.2715	0.2741	0.2767
0.61	0.2793	0.2819	0.2845	0.2871	0.2898	0.2924	0.2950	0.2976	0.3002	0.3029
0.62	0.3055	0.3081	0.3107	0.3134	0.3160	0.3186	0.3213	0.3239	0.3266	0.3292
0.63	0.3319	0.3345	0.3372	0.3398	0.3425	0.3451	0.3478	0.3505	0.3531	0.3558
0.64	0.3585	0.3611	0.3638	0.3665	0.3692	0.3719	0.3745	0.3772	0.3799	0.3826
0.65	0.3853	0.3880	0.3907	0.3934	0.3961	0.3989	0.4016	0.4043	0.4070	0.4097
0.66	0.4125	0.4152	0.4179	0.4207	0.4234	0.4261	0.4289	0.4316	0.4344	0.4372
0.67	0.4399	0.4427	0.4454	0.4482	0.4510	0.4538	0.4565	0.4593	0.4621	0.4649
0.68	0.4677	0.4705	0.4733	0.4761	0.4789	0.4817	0.4845	0.4874	0.4902	0.4930
0.69	0.4959	0.4987	0.5015	0.5044	0.5072	0.5101	0.5129	0.5158	0.5187	0.5215
0.70	0.5244	0.5273	0.5302	0.5330	0.5359	0.5388	0.5417	0.5446	0.5476	0.5505
0.71	0.5534	0.5563	0.5592	0.5622	0.5651	0.5681	0.5710	0.5740	0.5769	0.5799
0.72	0.5828	0.5858	0.5888	0.5918	0.5948	0.5978	0.6008	0.6038	0.6068	0.6098
0.73	0.6128	0.6158	0.6189	0.6219	0.6250	0.6280	0.6311	0.6341	0.6372	0.6403
0.74	0.6433	0.6464	0.6495	0.6526	0.6557	0.6588	0.6620	0.6651	0.6682	0.6713
0.75	0.6745	0.6776	0.6808	0.6840	0.6871	0.6903	0.6935	0.6967	0.6999	0.7031
0.76	0.7063	0.7095	0.7128	0.7160	0.7192	0.7225	0.7257	0.7290	0.7323	0.7356
0.77	0.7388	0.7421	0.7454	0.7488	0.7521	0.7554	0.7588	0.7621	0.7655	0.7688
0.78	0.7722	0.7756	0.7790	0.7824	0.7858	0.7892	0.7926	0.7961	0.7995	0.8030
0.79	0.8064	0.8099	0.8134	0.8169	0.8204	0.8239	0.8274	0.8310	0.8345	0.8381
0.80	0.8416	0.8452	0.8488	0.8524	0.8560	0.8596	0.8633	0.8669	0.8705	0.8742
0.81	0.8779	0.8816	0.8853	0.8890	0.8927	0.8965	0.9002	0.9040	0.9078	0.9116
0.82	0.9154	0.9192	0.9230	0.9269	0.9307	0.9346	0.9385	0.9424	0.9463	0.9502
0.83	0.9542	0.9581	0.9621	0.9661	0.9701	0.9741	0.9782	0.9822	0.9863	0.9904
0.84	0.9945	0.9986	1.0027	1.0069	1.0110	1.0152	1.0194	1.0237	1.0279	1.0322
0.85	1.0364	1.0407	1.0450	1.0494	1.0537	1.0581	1.0625	1.0669	1.0714	1.0758
0.86	1.0803	1.0848	1.0893	1.0939	1.0985	1.1031	1.1077	1.1123	1.1170	1.1217
0.87	1.1264	1.1311	1.1359	1.1407	1.1455	1.1503	1.1552	1.1601	1.1650	1.1700
0.88	1.1750	1.1800	1.1850	1.1901	1.1952	1.2004	1.2055	1.2107	1.2160	1.2212
0.89	1.2265	1.2319	1.2372	1.2426	1.2481	1.2536	1.2591	1.2646	1.2702	1.2759
0.90	1.2816	1.2873	1.2930	1.2988	1.3047	1.3106	1.3165	1.3225	1.3285	1.3346
0.91	1.3408	1.3469	1.3532	1.3595	1.3658	1.3722	1.3787	1.3852	1.3917	1.3984
0.92	1.4051	1.4118	1.4187	1.4255	1.4325	1.4395	1.4466	1.4538	1.4611	1.4684
0.93	1.4758	1.4833	1.4909	1.4985	1.5063	1.5141	1.5220	1.5301	1.5382	1.5464
0.94	1.5548	1.5632	1.5718	1.5805	1.5893	1.5982	1.6072	1.6164	1.6258	1.6352
0.95	1.6449	1.6546	1.6646	1.6747	1.6849	1.6954	1.7060	1.7169	1.7279	1.7392
0.96	1.7507	1.7624	1.7744	1.7866	1.7991	1.8119	1.8250	1.8384	1.8522	1.8663
0.97	1.8808	1.8957	1.9110	1.9268	1.9431	1.9600	1.9774	1.9954	2.0141	2.0335
0.98	2.0537	2.0749	2.0969	2.1201	2.1444	2.1701	2.1973	2.2262	2.2571	2.2904
0.99	2.3263	2.3656	2.4089	2.4573	2.5121	2.5758	2.6521	2.7478	2.8782	3.0902

**Table 3: The  $t$  distribution**

Entries in table give the  $t$ -value of the Student's  $t$ -distribution as a function of the probability (areas in the right tail) and the degrees of freedom.

$df \downarrow$	0.1	0.05	0.025	0.0125	0.01	0.005	0.001
1	3.0777	6.3138	12.706	25.452	31.821	63.657	318.309
2	1.8856	2.9200	4.3027	6.2053	6.9646	9.9248	22.3271
3	1.6377	2.3534	3.1824	4.1765	4.5407	5.8409	10.2145
4	1.5332	2.1318	2.7764	3.4954	3.7469	4.6041	7.1732
5	1.4759	2.0150	2.5706	3.1634	3.3649	4.0321	5.8934
6	1.4398	1.9432	2.4469	2.9687	3.1427	3.7074	5.2076
7	1.4149	1.8946	2.3646	2.8412	2.9980	3.4995	4.7853
8	1.3968	1.8595	2.3060	2.7515	2.8965	3.3554	4.5008
9	1.3830	1.8331	2.2622	2.6850	2.8214	3.2498	4.2968
10	1.3722	1.8125	2.2281	2.6338	2.7638	3.1693	4.1437
11	1.3634	1.7959	2.2010	2.5931	2.7181	3.1058	4.0247
12	1.3562	1.7823	2.1788	2.5600	2.6810	3.0545	3.9296
13	1.3502	1.7709	2.1604	2.5326	2.6503	3.0123	3.8520
14	1.3450	1.7613	2.1448	2.5096	2.6245	2.9768	3.7874
15	1.3406	1.7531	2.1314	2.4899	2.6025	2.9467	3.7328
16	1.3368	1.7459	2.1199	2.4729	2.5835	2.9208	3.6862
17	1.3334	1.7396	2.1098	2.4581	2.5669	2.8982	3.6458
18	1.3304	1.7341	2.1009	2.4450	2.5524	2.8784	3.6105
19	1.3277	1.7291	2.0930	2.4334	2.5395	2.8609	3.5794
20	1.3253	1.7247	2.0860	2.4231	2.5280	2.8453	3.5518
21	1.3232	1.7207	2.0796	2.4138	2.5176	2.8314	3.5272
22	1.3212	1.7171	2.0739	2.4055	2.5083	2.8188	3.5050
23	1.3195	1.7139	2.0687	2.3979	2.4999	2.8073	3.4850
24	1.3178	1.7109	2.0639	2.3909	2.4922	2.7969	3.4668
25	1.3163	1.7081	2.0595	2.3846	2.4851	2.7874	3.4502
26	1.3150	1.7056	2.0555	2.3788	2.4786	2.7787	3.4350
27	1.3137	1.7033	2.0518	2.3734	2.4727	2.7707	3.4210
28	1.3125	1.7011	2.0484	2.3685	2.4671	2.7633	3.4082
29	1.3114	1.6991	2.0452	2.3638	2.4620	2.7564	3.3962
30	1.3104	1.6973	2.0423	2.3596	2.4573	2.7500	3.3852
31	1.3095	1.6955	2.0395	2.3556	2.4528	2.7440	3.3749
32	1.3086	1.6939	2.0369	2.3518	2.4487	2.7385	3.3653
33	1.3077	1.6924	2.0345	2.3483	2.4448	2.7333	3.3563
34	1.3070	1.6909	2.0322	2.3451	2.4411	2.7284	3.3479
35	1.3062	1.6896	2.0301	2.3420	2.4377	2.7238	3.3400
36	1.3055	1.6883	2.0281	2.3391	2.4345	2.7195	3.3326
37	1.3049	1.6871	2.0262	2.3363	2.4314	2.7154	3.3256
38	1.3042	1.6860	2.0244	2.3337	2.4286	2.7116	3.3190
39	1.3036	1.6849	2.0227	2.3313	2.4258	2.7079	3.3128
40	1.3031	1.6839	2.0211	2.3289	2.4233	2.7045	3.3069

Table 3: The  $t$  distribution (*continued*)

$df \rightarrow$	0.1	0.05	0.025	0.0125	0.01	0.005	0.001
41	1.3025	1.6829	2.0195	2.3267	2.4208	2.7012	3.3013
42	1.3020	1.6820	2.0181	2.3246	2.4185	2.6981	3.2960
43	1.3016	1.6811	2.0167	2.3226	2.4163	2.6951	3.2909
44	1.3011	1.6802	2.0154	2.3207	2.4141	2.6923	3.2861
45	1.3006	1.6794	2.0141	2.3189	2.4121	2.6896	3.2815
46	1.3002	1.6787	2.0129	2.3172	2.4102	2.6870	3.2771
47	1.2998	1.6779	2.0117	2.3155	2.4083	2.6846	3.2729
48	1.2994	1.6772	2.0106	2.3139	2.4066	2.6822	3.2689
49	1.2991	1.6766	2.0096	2.3124	2.4049	2.6800	3.2651
50	1.2987	1.6759	2.0086	2.3109	2.4033	2.6778	3.2614
51	1.2984	1.6753	2.0076	2.3095	2.4017	2.6757	3.2579
52	1.2980	1.6747	2.0066	2.3082	2.4002	2.6737	3.2545
53	1.2977	1.6741	2.0057	2.3069	2.3988	2.6718	3.2513
54	1.2974	1.6736	2.0049	2.3056	2.3974	2.6700	3.2481
55	1.2971	1.6730	2.0040	2.3044	2.3961	2.6682	3.2451
56	1.2969	1.6725	2.0032	2.3033	2.3948	2.6665	3.2423
57	1.2966	1.6720	2.0025	2.3022	2.3936	2.6649	3.2395
58	1.2963	1.6716	2.0017	2.3011	2.3924	2.6633	3.2368
59	1.2961	1.6711	2.0010	2.3000	2.3912	2.6618	3.2342
60	1.2958	1.6706	2.0003	2.2990	2.3901	2.6603	3.2317
61	1.2956	1.6702	1.9996	2.2981	2.3890	2.6589	3.2293
62	1.2954	1.6698	1.9990	2.2971	2.3880	2.6575	3.2270
63	1.2951	1.6694	1.9983	2.2962	2.3870	2.6561	3.2247
64	1.2949	1.6690	1.9977	2.2954	2.3860	2.6549	3.2225
65	1.2947	1.6686	1.9971	2.2945	2.3851	2.6536	3.2204
66	1.2945	1.6683	1.9966	2.2937	2.3842	2.6524	3.2184
67	1.2943	1.6679	1.9960	2.2929	2.3833	2.6512	3.2164
68	1.2941	1.6676	1.9955	2.2921	2.3824	2.6501	3.2145
69	1.2939	1.6672	1.9949	2.2914	2.3816	2.6490	3.2126
70	1.2938	1.6669	1.9944	2.2906	2.3808	2.6479	3.2108
71	1.2936	1.6666	1.9939	2.2899	2.3800	2.6469	3.2090
72	1.2934	1.6663	1.9935	2.2892	2.3793	2.6459	3.2073
73	1.2933	1.6660	1.9930	2.2886	2.3785	2.6449	3.2057
74	1.2931	1.6657	1.9925	2.2879	2.3778	2.6439	3.2041
75	1.2929	1.6654	1.9921	2.2873	2.3771	2.6430	3.2025
76	1.2928	1.6652	1.9917	2.2867	2.3764	2.6421	3.2010
77	1.2926	1.6649	1.9913	2.2861	2.3758	2.6412	3.1995
78	1.2925	1.6646	1.9908	2.2855	2.3751	2.6403	3.1980
79	1.2924	1.6644	1.9905	2.2849	2.3745	2.6395	3.1966
80	1.2922	1.6641	1.9901	2.2844	2.3739	2.6387	3.1953
90	1.2910	1.6620	1.9867	2.2795	2.3685	2.6316	3.1833
95	1.2905	1.6611	1.9853	2.2775	2.3662	2.6286	3.1782
100	1.2901	1.6602	1.9840	2.2757	2.3642	2.6259	3.1737
120	1.2886	1.6577	1.9799	2.2699	2.3578	2.6174	3.1595
$\infty$	1.2816	1.6448	1.9600	2.2414	2.3263	2.5758	3.0902

**Table 4: The Chi-square ( $\chi^2$ ) distribution**

Entries in this table give the critical values of  $\chi^2$  for the specified number of degrees of freedom and areas to the right tail.

$df \rightarrow$	0.995	0.99	0.975	0.95	0.9	0.1	0.05	0.025	0.01	0.005	0.001
1	0.0000	0.0002	0.0010	0.0039	0.0158	2.7055	3.8415	5.0239	6.6349	7.8794	10.8276
2	0.0100	0.0201	0.0506	0.1026	0.2107	4.6052	5.9915	7.3778	9.2103	10.5966	13.8155
3	0.0717	0.1148	0.2158	0.3518	0.5844	6.2514	7.8147	9.3484	11.3449	12.8382	16.2662
4	0.2070	0.2971	0.4844	0.7107	1.0636	7.7794	9.4877	11.1433	13.2767	14.8603	18.4668
5	0.4117	0.5543	0.8312	1.1455	1.6103	9.2364	11.0705	12.8325	15.0863	16.7496	20.5150
6	0.6757	0.8721	1.2373	1.6354	2.2041	10.6446	12.5916	14.4494	16.8119	18.5476	22.4577
7	0.9893	1.2390	1.6899	2.1673	2.8331	12.0170	14.0671	16.0128	18.4753	20.2777	24.3219
8	1.3444	1.6465	2.1797	2.7326	3.4895	13.3616	15.5073	17.5345	20.0902	21.9550	26.1245
9	1.7349	2.0879	2.7004	3.3251	4.1682	14.6837	16.9190	19.0228	21.6660	23.5894	27.8772
10	2.1559	2.5582	3.2470	3.9403	4.8652	15.9872	18.3070	20.4832	23.2093	25.1882	29.5883
11	2.6032	3.0535	3.8157	4.5748	5.5778	17.2750	19.6751	21.9200	24.7250	26.7568	31.2641
12	3.0738	3.5706	4.4038	5.2260	6.3038	18.5493	21.0261	23.3367	26.2170	28.2995	32.9095
13	3.5650	4.1069	5.0088	5.8919	7.0415	19.8119	22.3620	24.7356	27.6882	29.8195	34.5282
14	4.0747	4.6604	5.6287	6.5706	7.7895	21.0641	23.6848	26.1189	29.1412	31.3193	36.1233
15	4.6009	5.2293	6.2621	7.2609	8.5468	22.3071	24.9958	27.4884	30.5779	32.8013	37.6973
16	5.1422	5.8122	6.9077	7.9616	9.3122	23.5418	26.2962	28.8454	31.9999	34.2672	39.2524
17	5.6972	6.4078	7.5642	8.6718	10.0852	24.7690	27.5871	30.1910	33.4087	35.7185	40.7902
18	6.2648	7.0149	8.2307	9.3905	10.8649	25.9894	28.8693	31.5264	34.8053	37.1565	42.3124
19	6.8440	7.6327	8.9065	10.1170	11.6509	27.2036	30.1435	32.8523	36.1909	38.5823	43.8202
20	7.4338	8.2604	9.5908	10.8508	12.4426	28.4120	31.4104	34.1696	37.5662	39.9968	45.3147
21	8.0337	8.8972	10.2829	11.5913	13.2396	29.6151	32.6706	35.4789	38.9322	41.4011	46.7970
22	8.6427	9.5425	10.9823	12.3380	14.0415	30.8133	33.9244	36.7807	40.2894	42.7957	48.2679
23	9.2604	10.1957	11.6886	13.0905	14.8480	32.0069	35.1725	38.0756	41.6384	44.1813	49.7282
24	9.8862	10.8564	12.4012	13.8484	15.6587	33.1962	36.4150	39.3641	42.9798	45.5585	51.1786
25	10.5197	11.5240	13.1197	14.6114	16.4734	34.3816	37.6525	40.6465	44.3141	46.9279	52.6197
26	11.1602	12.1981	13.8439	15.3792	17.2919	35.5632	38.8851	41.9232	45.6417	48.2899	54.0520
27	11.8076	12.8785	14.5734	16.1514	18.1139	36.7412	40.1133	43.1945	46.9629	49.6449	55.4760
28	12.4613	13.5647	15.3079	16.9279	18.9392	37.9159	41.3371	44.4608	48.2782	50.9934	56.8923
29	13.1211	14.2565	16.0471	17.7084	19.7677	39.0875	42.5570	45.7223	49.5879	52.3356	58.3012
30	13.7867	14.9535	16.7908	18.4927	20.5992	40.2560	43.7730	46.9792	50.8922	53.6720	59.7031
31	14.4578	15.6555	17.5387	19.2806	21.4336	41.4217	44.9853	48.2319	52.1914	55.0027	61.0983
32	15.1340	16.3622	18.2908	20.0719	22.2706	42.5847	46.1943	49.4804	53.4858	56.3281	62.4872
33	15.8153	17.0735	19.0467	20.8665	23.1102	43.7452	47.3999	50.7251	54.7755	57.6484	63.8701
34	16.5013	17.7891	19.8063	21.6643	23.9523	44.9032	48.6024	51.9660	56.0609	58.9639	65.2472
35	17.1918	18.5089	20.5694	22.4650	24.7967	46.0588	49.8018	53.2033	57.3421	60.2748	66.6188
36	17.8867	19.2327	21.3359	23.2686	25.6433	47.2122	50.9985	54.4373	58.6192	61.5812	67.9852
37	18.5858	19.9602	22.1056	24.0749	26.4921	48.3634	52.1923	55.6680	59.8925	62.8833	69.3465
38	19.2889	20.6914	22.8785	24.8839	27.3430	49.5126	53.3835	56.8955	61.1621	64.1814	70.7029
39	19.9959	21.4262	23.6543	25.6954	28.1958	50.6598	54.5722	58.1201	62.4281	65.4756	72.0547
40	20.7065	22.1643	24.4330	26.5093	29.0505	51.8051	55.7585	59.3417	63.6907	66.7660	73.4020
50	27.9907	29.7067	32.3574	34.7643	37.6886	63.1671	67.5048	71.4202	76.1539	79.4900	86.6608
55	31.7348	33.5705	36.3981	38.9580	42.0596	68.7962	73.3115	77.3805	82.2921	85.7490	93.1675
60	35.5345	37.4849	40.4817	43.1880	46.4589	74.3970	79.0819	83.2977	88.3794	91.9517	99.6072
65	39.3831	41.4436	44.6030	47.4496	50.8829	79.9730	84.8206	89.1771	94.4221	98.1051	105.988
70	43.275	45.442	48.758	51.739	55.329	85.527	90.531	95.023	100.425	104.215	112.317
75	47.206	49.475	52.942	56.054	59.795	91.061	96.217	100.839	106.393	110.286	118.599
80	51.172	53.540	57.153	60.391	64.278	96.578	101.879	106.629	112.329	116.321	124.839
85	55.1696	57.6339	61.389	64.749	68.777	102.079	107.522	112.393	118.236	122.325	131.041
90	59.1963	61.7541	65.647	69.126	73.291	107.565	113.145	118.136	124.116	128.299	137.208
100	67.3276	70.0649	74.222	77.929	82.358	118.498	124.342	129.561	135.807	140.169	149.449

**Table 5: The F distribution:  $\alpha = 0.025$**

The table returns the inverse of the F probability distribution.  $\nu_1$  is the numerator degrees of freedom and  $\nu_2$  is the denominator degrees of freedom.

$\nu_2$	$\nu_1 = 1$	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	$\infty$
1	647.8	799.5	864.2	899.6	921.8	937.1	948.2	956.7	963.3	968.6	976.7	984.9	993.1	997.2	1001.4	1005.6	1009.8	1014.0	1018.2
2	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.39	39.40	39.41	39.43	39.45	39.46	39.46	39.47	39.48	39.49	39.50
3	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.47	14.42	14.34	14.25	14.17	14.12	14.08	14.04	13.99	13.95	13.90
4	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90	8.84	8.75	8.66	8.56	8.51	8.46	8.41	8.36	8.31	8.26
5	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68	6.62	6.52	6.43	6.33	6.28	6.23	6.18	6.12	6.07	6.02
6	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52	5.46	5.37	5.27	5.17	5.12	5.07	5.01	4.96	4.90	4.85
7	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82	4.76	4.67	4.57	4.47	4.41	4.36	4.31	4.25	4.20	4.14
8	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36	4.30	4.20	4.10	4.00	3.95	3.89	3.84	3.78	3.73	3.67
9	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03	3.96	3.87	3.77	3.67	3.61	3.56	3.51	3.45	3.39	3.33
10	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78	3.72	3.62	3.52	3.42	3.37	3.31	3.26	3.20	3.14	3.08
11	6.72	5.26	4.63	4.28	4.04	3.88	3.76	3.66	3.59	3.53	3.43	3.33	3.23	3.17	3.12	3.06	3.00	2.94	2.88
12	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44	3.37	3.28	3.18	3.07	3.02	2.96	2.91	2.85	2.79	2.72
13	6.41	4.97	4.35	4.00	3.77	3.60	3.48	3.39	3.31	3.25	3.15	3.05	2.95	2.89	2.84	2.78	2.72	2.66	2.60
14	6.30	4.86	4.24	3.89	3.66	3.50	3.38	3.29	3.21	3.15	3.05	2.95	2.84	2.79	2.73	2.67	2.61	2.55	2.49
15	6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12	3.06	2.96	2.86	2.76	2.70	2.64	2.59	2.52	2.46	2.40
16	6.12	4.69	4.08	3.73	3.50	3.34	3.22	3.12	3.05	2.99	2.89	2.79	2.68	2.63	2.57	2.51	2.45	2.38	2.32
17	6.04	4.62	4.01	3.66	3.44	3.28	3.16	3.06	2.98	2.92	2.82	2.72	2.62	2.56	2.50	2.44	2.38	2.32	2.25
18	5.98	4.56	3.95	3.61	3.38	3.22	3.10	3.01	2.93	2.87	2.77	2.67	2.56	2.50	2.44	2.38	2.32	2.26	2.19
19	5.92	4.51	3.90	3.56	3.33	3.17	3.05	2.96	2.88	2.82	2.72	2.62	2.51	2.45	2.39	2.33	2.27	2.20	2.13
20	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84	2.77	2.68	2.57	2.46	2.41	2.35	2.29	2.22	2.16	2.09
21	5.83	4.42	3.82	3.48	3.25	3.09	2.97	2.87	2.80	2.73	2.64	2.53	2.42	2.37	2.31	2.25	2.18	2.11	2.04
22	5.79	4.38	3.78	3.44	3.22	3.05	2.93	2.84	2.76	2.70	2.60	2.50	2.39	2.33	2.27	2.21	2.14	2.08	2.00
23	5.75	4.35	3.75	3.41	3.18	3.02	2.90	2.81	2.73	2.67	2.57	2.47	2.36	2.30	2.24	2.18	2.11	2.04	1.97
24	5.72	4.32	3.72	3.38	3.15	2.99	2.87	2.78	2.70	2.64	2.54	2.44	2.33	2.27	2.21	2.15	2.08	2.01	1.94
25	5.69	4.29	3.69	3.35	3.13	2.97	2.85	2.75	2.68	2.61	2.51	2.41	2.30	2.24	2.18	2.12	2.05	1.98	1.91
28	5.61	4.22	3.63	3.29	3.06	2.90	2.78	2.69	2.61	2.55	2.45	2.34	2.23	2.17	2.11	2.05	1.98	1.91	1.83
30	5.57	4.18	3.59	3.25	3.03	2.87	2.75	2.65	2.57	2.51	2.41	2.31	2.20	2.14	2.07	2.01	1.94	1.87	1.79
34	5.50	4.12	3.53	3.19	2.97	2.81	2.69	2.59	2.52	2.45	2.35	2.25	2.13	2.07	2.01	1.95	1.88	1.80	1.72
35	5.48	4.11	3.52	3.18	2.96	2.80	2.68	2.58	2.50	2.44	2.34	2.23	2.12	2.06	2.00	1.93	1.86	1.79	1.70
40	5.42	4.05	3.46	3.13	2.90	2.74	2.62	2.53	2.45	2.39	2.29	2.18	2.07	2.01	1.94	1.88	1.80	1.72	1.64
48	5.35	3.99	3.40	3.07	2.84	2.69	2.56	2.47	2.39	2.33	2.23	2.12	2.01	1.94	1.88	1.81	1.73	1.65	1.56
50	5.34	3.97	3.39	3.05	2.83	2.67	2.55	2.46	2.38	2.32	2.22	2.11	1.99	1.93	1.87	1.80	1.72	1.64	1.55
60	5.29	3.93	3.34	3.01	2.79	2.63	2.51	2.41	2.33	2.27	2.17	2.06	1.94	1.88	1.82	1.74	1.67	1.58	1.48
80	5.22	3.86	3.28	2.95	2.73	2.57	2.45	2.35	2.28	2.21	2.11	2.00	1.88	1.82	1.75	1.68	1.60	1.51	1.40
100	5.18	3.83	3.25	2.92	2.70	2.54	2.42	2.32	2.24	2.18	2.08	1.97	1.85	1.78	1.71	1.64	1.56	1.46	1.35
120	5.15	3.80	3.23	2.89	2.67	2.52	2.39	2.30	2.22	2.16	2.05	1.94	1.82	1.76	1.69	1.61	1.53	1.43	1.31
$\infty$	5.02	3.69	3.12	2.79	2.57	2.41	2.29	2.19	2.11	2.05	1.94	1.83	1.71	1.64	1.57	1.48	1.39	1.27	1.00

**Table 6: The F distribution:  $\alpha = 0.05$**

The table returns the inverse of the F probability distribution.  $\nu_1$  is the numerator degrees of freedom and  $\nu_2$  is the denominator degrees of freedom.

$\nu_2$	$\nu_1 = 1$	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	$\infty$
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	245.9	248.0	249.1	250.1	251.1	252.2	253.3	254.3
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.36
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
34	4.13	3.28	2.88	2.65	2.49	2.38	2.29	2.23	2.17	2.12	2.05	1.97	1.89	1.84	1.80	1.75	1.69	1.63	1.57
35	4.12	3.27	2.87	2.64	2.49	2.37	2.29	2.22	2.16	2.11	2.04	1.96	1.88	1.83	1.79	1.74	1.68	1.62	1.56
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
48	4.04	3.19	2.80	2.57	2.41	2.29	2.21	2.14	2.08	2.03	1.96	1.88	1.79	1.75	1.70	1.64	1.59	1.52	1.45
50	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07	2.03	1.95	1.87	1.78	1.74	1.69	1.63	1.58	1.51	1.44
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
80	3.96	3.11	2.72	2.49	2.33	2.21	2.13	2.06	2.00	1.95	1.88	1.79	1.70	1.65	1.60	1.54	1.48	1.41	1.32
100	3.94	3.09	2.70	2.46	2.31	2.19	2.10	2.03	1.97	1.93	1.85	1.77	1.68	1.63	1.57	1.52	1.45	1.38	1.28
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
$\infty$	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00

**Table 7: The F distribution:  $\alpha = 0.01$**

The table returns the inverse of the F probability distribution.  $v_1$  is the numerator degrees of freedom and  $v_2$  is the denominator degrees of freedom.

$v_2$	$v_1 = 1$	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	$\infty$
1	4052.2	4999.5	5403.4	5624.6	5763.6	5859.0	5928.4	5981.1	6022.5	6055.8	6106.3	6157.3	6208.7	6234.6	6260.6	6286.8	6313.0	6339.4	6365.7
2	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39	99.40	99.42	99.43	99.45	99.46	99.47	99.47	99.48	99.49	99.50
3	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35	27.23	27.05	26.87	26.69	26.60	26.50	26.41	26.32	26.22	26.13
4	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	14.55	14.37	14.20	14.02	13.93	13.84	13.75	13.65	13.56	13.46
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05	9.89	9.72	9.55	9.47	9.38	9.29	9.20	9.11	9.02
6	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.72	7.56	7.40	7.31	7.23	7.14	7.06	6.97	6.88
7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.47	6.31	6.16	6.07	5.99	5.91	5.82	5.74	5.65
8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.67	5.52	5.36	5.28	5.20	5.12	5.03	4.95	4.86
9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.11	4.96	4.81	4.73	4.65	4.57	4.48	4.40	4.31
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.71	4.56	4.41	4.33	4.25	4.17	4.08	4.00	3.91
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.40	4.25	4.10	4.02	3.94	3.86	3.78	3.69	3.60
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.16	4.01	3.86	3.78	3.70	3.62	3.54	3.45	3.36
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	3.96	3.82	3.66	3.59	3.51	3.43	3.34	3.25	3.17
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94	3.80	3.66	3.51	3.43	3.35	3.27	3.18	3.09	3.00
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.67	3.52	3.37	3.29	3.21	3.13	3.05	2.96	2.87
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.55	3.41	3.26	3.18	3.10	3.02	2.93	2.84	2.75
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.46	3.31	3.16	3.08	3.00	2.92	2.83	2.75	2.65
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	3.37	3.23	3.08	3.00	2.92	2.84	2.75	2.66	2.57
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.30	3.15	3.00	2.92	2.84	2.76	2.67	2.58	2.49
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.23	3.09	2.94	2.86	2.78	2.69	2.61	2.52	2.42
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31	3.17	3.03	2.88	2.80	2.72	2.64	2.55	2.46	2.36
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.12	2.98	2.83	2.75	2.67	2.58	2.50	2.40	2.31
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21	3.07	2.93	2.78	2.70	2.62	2.54	2.45	2.35	2.26
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.03	2.89	2.74	2.66	2.58	2.49	2.40	2.31	2.21
25	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22	3.13	2.99	2.85	2.70	2.62	2.54	2.45	2.36	2.27	2.17
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03	2.90	2.75	2.60	2.52	2.44	2.35	2.26	2.17	2.06
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.84	2.70	2.55	2.47	2.39	2.30	2.21	2.11	2.01
34	7.44	5.29	4.42	3.93	3.61	3.39	3.22	3.09	2.98	2.89	2.76	2.61	2.46	2.38	2.30	2.21	2.12	2.02	1.91
35	7.42	5.27	4.40	3.91	3.59	3.37	3.20	3.07	2.96	2.88	2.74	2.60	2.44	2.36	2.28	2.19	2.10	2.00	1.89
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.66	2.52	2.37	2.29	2.20	2.11	2.02	1.92	1.80
48	7.19	5.08	4.22	3.74	3.43	3.20	3.04	2.91	2.80	2.71	2.58	2.44	2.28	2.20	2.12	2.02	1.93	1.82	1.70
50	7.17	5.06	4.20	3.72	3.41	3.19	3.02	2.89	2.78	2.70	2.56	2.42	2.27	2.18	2.10	2.01	1.91	1.80	1.68
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.50	2.35	2.20	2.12	2.03	1.94	1.84	1.73	1.60
80	6.96	4.88	4.04	3.56	3.26	3.04	2.87	2.74	2.64	2.55	2.42	2.27	2.12	2.03	1.94	1.85	1.75	1.63	1.49
100	6.90	4.82	3.98	3.51	3.21	2.99	2.82	2.69	2.59	2.50	2.37	2.22	2.07	1.98	1.89	1.80	1.69	1.57	1.43
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.34	2.19	2.03	1.95	1.86	1.76	1.66	1.53	1.38
$\infty$	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.18	2.04	1.88	1.79	1.70	1.59	1.47	1.32	1.00

Table 8: The  $F$  distribution:  $\alpha = 0.1$

$\nu_2$	$\nu_1 = 1$	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	$\infty$
1	39.9	49.5	53.6	55.8	57.2	58.2	58.9	59.4	59.9	60.2	60.7	61.2	61.7	62.0	62.3	62.5	62.8	63.1	63.3
2	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38	9.39	9.41	9.42	9.44	9.45	9.46	9.47	9.47	9.48	9.49
3	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24	5.23	5.22	5.20	5.18	5.18	5.17	5.16	5.15	5.14	5.13
4	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94	3.92	3.90	3.87	3.84	3.83	3.82	3.80	3.79	3.78	3.76
5	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32	3.30	3.27	3.24	3.21	3.19	3.17	3.16	3.14	3.12	3.10
6	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96	2.94	2.90	2.87	2.84	2.82	2.80	2.78	2.76	2.74	2.72
7	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72	2.70	2.67	2.63	2.59	2.58	2.56	2.54	2.51	2.49	2.47
8	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56	2.54	2.50	2.46	2.42	2.40	2.38	2.36	2.34	2.32	2.29
9	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44	2.42	2.38	2.34	2.30	2.28	2.25	2.23	2.21	2.18	2.16
10	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35	2.32	2.28	2.24	2.20	2.18	2.16	2.13	2.11	2.08	2.06
11	3.23	2.86	2.66	2.54	2.45	2.39	2.34	2.30	2.27	2.25	2.21	2.17	2.12	2.10	2.08	2.05	2.03	2.00	1.97
12	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21	2.19	2.15	2.10	2.06	2.04	2.01	1.99	1.96	1.93	1.90
13	3.14	2.76	2.56	2.43	2.35	2.28	2.23	2.20	2.16	2.14	2.10	2.05	2.01	1.98	1.96	1.93	1.90	1.88	1.85
14	3.10	2.73	2.52	2.39	2.31	2.24	2.19	2.15	2.12	2.10	2.05	2.01	1.96	1.94	1.91	1.89	1.86	1.83	1.80
15	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09	2.06	2.02	1.97	1.92	1.90	1.87	1.85	1.82	1.79	1.76
16	3.05	2.67	2.46	2.33	2.24	2.18	2.13	2.09	2.06	2.03	1.99	1.94	1.89	1.87	1.84	1.81	1.78	1.75	1.72
17	3.03	2.64	2.44	2.31	2.22	2.15	2.10	2.06	2.03	2.00	1.96	1.91	1.86	1.84	1.81	1.78	1.75	1.72	1.69
18	3.01	2.62	2.42	2.29	2.20	2.13	2.08	2.04	2.00	1.98	1.93	1.89	1.84	1.81	1.78	1.75	1.72	1.69	1.66
19	2.99	2.61	2.40	2.27	2.18	2.11	2.06	2.02	1.98	1.96	1.91	1.86	1.81	1.79	1.76	1.73	1.70	1.67	1.63
20	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96	1.94	1.89	1.84	1.79	1.77	1.74	1.71	1.68	1.64	1.61
21	2.96	2.57	2.36	2.23	2.14	2.08	2.02	1.98	1.95	1.92	1.87	1.83	1.78	1.75	1.72	1.69	1.66	1.62	1.59
22	2.95	2.56	2.35	2.22	2.13	2.06	2.01	1.97	1.93	1.90	1.86	1.81	1.76	1.73	1.70	1.67	1.64	1.60	1.57
23	2.94	2.55	2.34	2.21	2.11	2.05	1.99	1.95	1.92	1.89	1.84	1.80	1.74	1.72	1.69	1.66	1.62	1.59	1.55
24	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91	1.88	1.83	1.78	1.73	1.70	1.67	1.64	1.61	1.57	1.53
25	2.92	2.53	2.32	2.18	2.09	2.02	1.97	1.93	1.89	1.87	1.82	1.77	1.72	1.69	1.66	1.63	1.59	1.56	1.52
28	2.89	2.50	2.29	2.16	2.06	2.00	1.94	1.90	1.87	1.84	1.79	1.74	1.69	1.66	1.63	1.59	1.56	1.52	1.48
30	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85	1.82	1.77	1.72	1.67	1.64	1.61	1.57	1.54	1.50	1.46
34	2.86	2.47	2.25	2.12	2.02	1.96	1.90	1.86	1.82	1.79	1.75	1.69	1.64	1.61	1.58	1.54	1.50	1.46	1.42
35	2.85	2.46	2.25	2.11	2.02	1.95	1.90	1.85	1.82	1.79	1.74	1.69	1.63	1.60	1.57	1.53	1.50	1.46	1.41
40	2.84	2.44	2.23	2.09	2.00	1.93	1.87	1.83	1.79	1.76	1.71	1.66	1.61	1.57	1.54	1.51	1.47	1.42	1.38
48	2.81	2.42	2.20	2.07	1.97	1.90	1.85	1.80	1.77	1.73	1.69	1.63	1.57	1.54	1.51	1.47	1.43	1.39	1.34
50	2.81	2.41	2.20	2.06	1.97	1.90	1.84	1.80	1.76	1.73	1.68	1.63	1.57	1.54	1.50	1.46	1.42	1.38	1.33
60	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74	1.71	1.66	1.60	1.54	1.51	1.48	1.44	1.40	1.35	1.29
80	2.77	2.37	2.15	2.02	1.92	1.85	1.79	1.75	1.71	1.68	1.63	1.57	1.51	1.48	1.44	1.40	1.36	1.31	1.24
100	2.76	2.36	2.14	2.00	1.91	1.83	1.78	1.73	1.69	1.66	1.61	1.56	1.49	1.46	1.42	1.38	1.34	1.28	1.21
120	2.75	2.35	2.13	1.99	1.90	1.82	1.77	1.72	1.68	1.65	1.60	1.55	1.48	1.45	1.41	1.37	1.32	1.26	1.19
$\infty$	2.71	2.30	2.08	1.94	1.85	1.77	1.72	1.67	1.63	1.60	1.55	1.49	1.42	1.38	1.34	1.30	1.24	1.17	1.00