NUMERICAL ERROR ANALYSIS IN FOUNDATION PHASE (GRADE 3) MATHEMATICS

By

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DECLARATION

I, Pumla Patricia Ndamase - Nzuzo sincerely declare that this dissertation is my own work and that any assistance obtained has been fully acknowledged in the text. No part of this dissertation has been previously submitted to any other University.

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ABSTRACT

The focus of the research was on numerical errors committed in foundation phase mathematics. It therefore explored: (1) numerical errors learners in foundation phase mathematics encounter (2) relationships underlying numerical errors and (3) the implementable strategies suitable for understanding numerical error analysis in foundation phase mathematics (Grade 3).

From 375 learners who formed the population of the study in the primary schools (16 in total), the researcher selected by means of a simple random sample technique 80 learners as the sample size, which constituted 10% of the population as response rate. On the basis of the research questions and informed by positivist paradigm, a quantitative approach was used by means of tables, graphs and percentages to address the research questions. A Likert scale was used with four categories of responses ranging from (A) Agree, (S A) Strongly Agree, (D) Disagree and (S D) Strongly Disagree.

The results revealed that: (1) the underlying numerical errors that learners encounter, include the inability to count backwards and forwards, number sequencing, mathematical signs, problem solving and word sums (2) there was a relationship between committing errors and a) copying numbers b) confusion of mathematical signs or operational signs c) reading numbers which contained more than one digit (3) It was also revealed that teachers needed frequent professional training for development; topics need to change and lastly government needs to involve teachers at ground roots level prior to policy changes on how to implement strategies with regards to numerical errors in the foundational phase.

It is recommended that attention be paid to the use of language and word sums in order to improve cognition processes in foundation phase mathematics. Moreover, it recommends that learners are to be assisted time and again when reading or copying their work, so that they could have fewer errors in foundation phase mathematics. Additionally it recommends that teachers be trained on how to implement strategies of numerical error analysis in foundation phase mathematics. Furthermore, teachers can use

tests to identify learners who could be at risk of developing mathematical difficulties in the foundation phase.

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DEDICATION

This dissertation is dedicated to my late mother who passed on during the early stages of this study. The loss was immense to bear but thanks; it lifted me up. I also dedicate this dissertation to my child, my daughter-in-law and my grandchildren whose affection, patience and encouragement made this work possible.

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CHAPTER 1

1.0 INTRODUCTION

The current study seeks to explore how to improve the underlying mathematical errors that learners encounter in foundation phase mathematics learning. The investigation partly involved a review of literature based on the relationships underlying mathematical error analysis concepts. The background included analysis of neurological dysfunctions and mathematical errors, the Annual National Assessment document (ANA 2011) and an international report. This was followed by the statement of the problem as well as the significance of the study, limitation of the study, and research questions. The literature review as well as the theoretical framework includes a review of; (1) cognitive dissonance theory and (2) cognitive fit theory. Methodology, including paradigm, approach to research, data-collection methods and data-analysis techniques, will eventually be discussed. Finally, the summary and discussions of the results together with the recommendations will also be discussed.

1.1 BACKGROUND OF THE STUDY

Numerical information is a dominant feature of everyday life according to Ansari (2008). Thus, basic numerical and mathematical skills play a critical role in determining an individual's life success and several possible mental representations and the neural markers which may be educationally relevant. Bettina (2000) suggested that improvement of numerical skills among learners would have benefits for the society at large, as it would lead to a stronger work-force.

At the ages of 7, 8 and 9 years it was best for learners to operate with numbers and understand that numbers could have different representations and could act as different points of reference. Quinn (2011) stated that at this stage children developed through experience, improved their ability to organize incoming information and learnt to amend their organizational schemes accordingly.

However, Van Nes and De Lange (2007) reported that in a school context, learners were the most important; they were also expected to understand what they were taught in class. Nevertheless Bornman (2010) commented that ten to sixteen per cent (10-16%) of learners were perceived by their teachers as having a learning difficulty in counting. The suggested reasons were neurological dysfunction.

1.1.1 NEUROLOGICAL DYSFUNCTIONS AND ERRORS IN FOUNDATION PHASE MATHEMATICS

With regard to neurological dysfunction, Radatz (2011) contested that in classifying errors according to a learner's individual difficulties, one could acknowledge that errors were a function of other variables in the educational processes. Nonetheless, the following were unclear: (1) which numerical errors impacted on the learning of mathematics in the foundation phase; (2) how to determine the various causes of errors that cut across mathematical content in the foundation phase; (3) how the teachers commented on and probed the mechanisms used in obtaining, processing, retaining and reproducing the information contained in mathematical tasks.

In contrast and according to Dehaene (2003) individuals with Attention Deficit/ Hyperactive Disorder (ADHD) manifested unexpected problems in mathematics that caused impairment in academic achievement and daily functioning: estimates ranged from 10% to 60%. These impairments of cognitive skills were considered to be integral features of the ADHD syndrome and hence, could cause mathematical difficulties in some of these children. These learners could exhibit different underlying mechanisms including specific deficits in basic numerical processing, as was manifested in children with developmental dyscalculia.

1.1.2 MATHEMATICAL ERRORS IN THE FOUNDATION PHASE

Hasym (2002) defined an error as a systematic deviation when a learner had not learnt something and consistently got it wrong. A mistake would be made by a learner while writing or counting when there was a lack of attention, fatigue, carelessness or another aspect of performance present. Jiang (2007) stated that errors could tell the teacher how far towards the goal the learner had progressed and consequently what remained for him or her to learn. Learner errors are valuable feedback.

The significance of this study is therefore to inform the learners about numerical errors they make and the obligation of the teachers to summarize these frequently appearing errors and remind learners of these errors as often as possible so that they could make an effort to avoid them.

Pardesi (2008) reported that when counting tens and ones, learners misapplied the procedure by counting on and treating tens and ones as separate numbers

Example 1

When asked to count collections of bundled tens and ones, such as 111

learners counted 10, 20, 30 1, 2, instead of 10, 20, 30, 31, 32.

The learner had an alternative concept of multi-digit numbers and saw them as numbers independent of place value.

Example 2

Learner read the number 32 as “thirty-two” and could count out 32 objects to demonstrate the value of the number, but when asked to write the number in expanded form, she wrote “3+2”.

Learner read the number 32 as ”thirty-two” and could count out 32 objects to demonstrate the value of the number, but when asked the value of the digits in the number, she responded that the values were “3” and ”2”.

The learner recognized simple multi-digit numbers, such as thirty (30) or 400 (four hundred) but she did not understand that the position of a digit determined its value.

Example 3 Learner mistook the numeral 306 for thirty-six. Learner wrote 4008 when asked to record four hundred eight.

The learner misapplied the rule for reading numbers from left to right.

Example 4 Learner read 81 as eighteen. The teen numbers often caused this difficulty.

The learner ordered numbers based on the value of the digits, instead of place value

Example 5 69 > 102, because 6 and 9 were larger than 1 and 2.

The learner might have known the commutative property of addition but failed to apply it to simplify the “work” of addition or misapplied it in a subtraction situation.

Example 6 Learner stated that 9+4 = 13 with relative ease, but struggled to find the sum of 4+9. Learner wrote “12-50” when he meant 50-12.

The learner might have known the commutative property of multiplication but failed to apply it to simplify the "work" of multiplication.

Example 7 Learner stated that 9x4 = 36 with relative ease, but struggled to find the product of 4x9.

The learner saw multiplication and division as discrete and separate operations. His concept of the operations did not include the fact that they were linked as inverse operations.

Example 8 Learner had reasonable ease with multiplication facts but could not master division facts. He might have known that 6x7 =42 but failed to realize that this fact also told him 42/ 7= 6. Generally children know procedures for dividing but have no idea how to check the reasonableness of their answers.

1.1.3 COGNITIVE PROCESSES

Popular views held by Quinn (2011) suggest that cognitive neuroscience extends from biology to cognitive psychology. The centrality of cognitive neuroscience is the internal representation of mental events. Also complex cognitive processes such as attention and decision making have been shown to be correlated with activity patterns of individual groups of cells in certain regions of the brain. The neural basis of cognition starts with local operations in the brains. These developments should be considered in the perspective of mathematics learning. Learning is closely connected with memory. Memory means the ability of an organism to store knowledge and to retrieve this knowledge when necessary.

Nevertheless, Ansari (2002) reveals that the problem with numbers comes in a variety of forms. Sometimes children with otherwise normal intelligence show particular problems with arithmetic. This is usually called “mathematical disability”. Despite much progress in normal number development, dyscalculia and numeracy deficits turn out to be surprisingly neglected areas of the cognitive sciences. For the normal case, the cognitive neuroscience approach has shifted from culturally-dependent aspects of numerical cognition, such as arithmetic, to an analysis of its representational primitives. The vast majority of studies of dyscalculia remains focused on higher level school-like concepts such as addition and multiplication. These general problems include the immature development of problem-solving strategies, a poor working memory-span leading to computational errors, a slow speed of processing as well as a disturbance of visuo-spatial functioning.

1.1.4. NUMERICAL REPRESENTATIONS – ARITHMETIC SKILLS

Dahaene (2003) suggests three different numerical representations (verbal, non- verbal semantic and visual-spatial) to be utilized depending on the task demands. However, children with arithmetic difficulties have problems specifically in automating basic mathematical facts which may stem from a general speed of processing which could also be connected to automatic dyslexia that relates to the cerebellum dysfunction.

Ansari (2008) reports that the acquisition of arithmetic skills represents an essential step in the development of mathematical competence and it has been repeatedly found that younger children primarily use effort and slow procedural strategies in solving mathematical problems. The current study aims at investigating the underlying numerical errors in the foundation phase, the relationships underlying numerical errors and the strategies of numerical error analysis from events of the mechanisms used in obtaining, processing, retaining and reproducing mathematical tasks as per the questions posed.

Ansari (2005) argued that one hundred and sixty arithmetic problems were presented in a pseudo-randomized order as part of an event-related FMRI design. The problems were based on all four arithmetic operation types which were included and composed of 20 small and 20 large problems each. Small problems in addition and multiplication were defined as problems with the product smaller than 25, for example “8+2”, and large problems with products larger than 25, for example “6x8”. In subtraction and division small and large problems were also defined on the basis of inverse relationships to addition and multiplication. The participants had to indicate the position of the solution by button-pressing of the corresponding index finger. The study revealed that participants had many errors. Those findings highlighted that in both structures and function the left tempo-parietal cortex played a role in mental arithmetic and particularly so when problems had a high probability of being solved via retrieval of arithmetic facts.

1.1.5 LEARNING DISABILITIES – DEVELOPMENTAL DISABILITIES

Archibald (2009) reported that specific learning disabilities are a category of developmental disabilities characterized by learning difficulties in one or more areas despite otherwise typical neurological, physical and emotional development. Dyscalculia may also have difficulties with a number sense, the ability to quickly understand, approximate and manipulate numerical quantities.

According to Holloway (2012) the research on numeracy provides a clear demonstration of the indirect effect that neuroscience research can have on education. Neuroscience research has helped researchers to gain a greater understanding of the role of such basic processes in the typical development of mathematical skills. Studies of mental arithmetic showed that the brain areas are activated in fundamental ways with chronological age during mental arithmetic. It is important to note that what we are currently lacking from the study of the neuroscience of numeracy are studies that investigate brain activity in children who are in the process of learning mathematics and how this activity is affected by different instructional conditions. In addition, there is a great need for studies that evaluate the effects of structured remedial interventions on brain function for learners with mathematical difficulties.

Thomas (2009) argues that the concept of delay is of interest, because it is widely used in the study of developmental disorders to classify children's cognitive abilities. However, it has several shortcomings. As a mechanistic explanation of slower development, delay can amount to little more than a re-description of behavioral data indicating that the disorder group produced scores and error patterns similar to those of younger typically developing controls, with no elaboration of the causal mechanisms by which this similarity may have arisen.

Duncan (2007) argues that low mathematical competence has been associated with lower indices of life-success. These factors demonstrate the fundamental significance of mathematical competence and highlight the importance of identifying sources of its variability. A potential source of individual differences in mathematical competence is the neural architecture supporting the performance of simple arithmetic problem solving. Indeed, children with mathematical learning difficulties exhibit immature procedural strategies and poor mathematical fact performance

Quinn (2011) acknowledges that neuroscience studies confirmed that incorrect internalization quickly becomes very difficult to correct. To accomplish this every assignment must be checked for correctness and learners be required to locate and correct errors in their work record. Always finding errors also provides consistent reinforcement for accuracy and good work habits

Reis (2004) reports that in the first years of mathematical education, word problems were used to enhance modeling competency in school children. Nevertheless, studies have shown that word problems are more difficult to solve than numerically presented problems and that various factors have an impact on the difficulty of a specific word problem

Barth, (2011) suggests that the neuro-cognitive mechanism that underlies the use of landmark reference points remains, however, unknown. Besides its frequent and successful use in behavioral research, the number-line task is highly ecologically relevant since number lines are introduced in schools and may help to enhance children's symbolic number representation as a result of experience when using numbers within a spatial context.

1.1.6 BASIC NUMERICAL PROCESSING

According to Kadja, (2010) deficit in basic numerical processing was identified as a central, probably causal problem in developmental dyscalculia. Two studies were conducted with 268 typically developmental and 132 dyscalculia children in Grades 2 and 3. Findings indicate that the efficiency of processing numbers and numerosities improves over time and that dyscalculia children are generally less efficient than children with typical development. Robust effects of numerical distance, size congruity and compatibility of ten and unit digits in two-digit numbers could be identified as early as the end of Grade 2. Only the distance effect for comparing symbolic representations of numerosities changed developmentally.

Furthermore, dyscalculia children in Grade 2 showed an especially strong effect of compatibility when comparing two-digit numbers. Apart from that, the patterns of numerical processing were stable across grade and arithmetic levels. These findings seem to indicate that the cognitive representation of numbers may be intact in dyscalculia and that accessing and processing these representations may be the problem rather than the representation itself.

Ansari (2008) argues that the majority of children were able to learn basic numerical concepts rather effortlessly, while some children displayed severe difficulties in acquiring basic number skills.

Cohen (2007) reported that the neural basis of cognition starts with local operations in the brain. Functional brain-imaging methods were used in the investigation of changes in the activity of the neuronal system in connection with mental processes, as well as in observing the brain system activated in particular situations. For the processes of thinking and acting, the concept of working memory is important. If we consider cognitive processes such as conversing with others, thinking about a problem, mentally calculating something or actually solving a problem, the sub-processes can be identified as the successes of the entire process.

At first, the relevant information must be identified and stored temporarily. Then this information must be opened to manipulation for interpretation in the light of knowledge stored in the long-term memory. The overall results of these processes should be suitable for communication in verbal or in written form and for storage in the long-term memory. This means that the cognitive processes require the brain systems and allow for short-term storage and manipulation of information (audio or visual). The systems are somehow connected in the long-term memory, and they are able to communicate results. All this requires overall control and monitoring.

According to Dehaene (1997) neuroscience distinguishes between different brain systems that fulfill specific functions. Different systems exist for cognitive processes and for emotional processes. The systems are located in different parts of the brain and have their own memory system with their own specific features. The emotion system has strong connections to the body system and an activated emotion system can lead to bodily reactions. Furthermore, although the emotion system works unconsciously, it is not easy to recognize the end results of an activated emotion system, for example the "feelings".

Quinn (2011) suggests that the amount of mental mathematics needed is a compromise between what learners could learn relatively easily, and how simply the examples could be contrived and still be effectively illustrated structures. The standard compromise is that the following should be fully transparent:

- Addition and subtraction of single-digit integers.
- Addition of four or five single-digit integers, or a two-digit and a three-digit integer.
- Multiplication of a single-digit and a two-digit integer.

In addition, multiplying a one-digit and a three-digit integer should not be a distraction, though it needs not be fully transparent. Multiplying two-digit integers is cognitively more complex, but should still not disrupt the train of thought. We can almost always contrive to avoid larger multiplications. Finally, the formal structure of mathematics should be automatic enough that a few symbols would not cause problems. Another important objective is to track error handling. It is well established that internal uncertainty about correctness causes delays and unusual patterns of activities. For operational purposes we call it an internalizing “bad” if it is incorrect but is so firmly embedded that it does not provoke this error-related activity. It is essential that incorrect internalization be identified and fixed before they go bad.

Training sessions should last between 30 and 60 minutes, with at least three short quizzes to reinforce learning and familiarize students with the quiz format. It should be possible to repeat at least the first sub-section if the corresponding quiz outcome is unsatisfactory.

According to Halloway and Ansari (2009) the acquisition of arithmetic competence is a crucial predictor of school and life success. The representation and processing of numerical magnitude relate to individual differences in mathematical achievement. The roles of neuronal mechanisms, sub-serving basic numerical magnitude in processing numerical magnitude in order to explain individual differences in the engagement of the left parietal cortex during numerical magnitude processing, are related to variability in children’s arithmetic achievement performance. Moreover, in this relationship between parietal brain activation during symbolic number processing and arithmetic, competence was found to be domain-specific, as no relationship between brain activation and the measure of reading achievement or intelligence was observed.

Maloney (2011:17) states that “mathematics anxiety (MA) is a condition in which individuals experience a negative effect when engaged in tasks demanding numerical and mathematical skills”. Individuals with high mathematics anxiety (HMA) have been shown to perform worse than their low mathematics anxious (LMA) peers on a wide range of numerical and mathematical tasks such as counting objects and more complex arithmetic problems involved with carrying.

Ansari (2011) states that, the brain changes with experience, the brain is not a static organ, but is changeable in response to experience. The neuronal plasticity focuses on experience-dependent changes in sensory motor brain functions in animals. These studies provided unprecedented insights into the experience-dependent changes in brain function at molecular, cellular and systems levels of description with clear clinical implications.

1.1.7 THE ANNUAL NATIONAL ASSESSMENT DOCUMENT (ANA 2011) AND THE INTERNATIONAL REPORT

The Department of Basic Education (2010).reported in its annual national assessment document (ANA 2011) that the country's schooling system performed well below in the areas of reading, writing and solving numerical problems. From the results (ANA, 2011) the Eastern Cape got the lowest percentage. According to ANA (2011) the results in Grade 6 indicated that only 30% of learners fell in the lowest category and few learners were able to achieve outstanding results, for example, only 3% of learners in the Grade 6 mathematics class could be considered as outstanding. In the distribution of scores by ANA the provinces emphasized the great challenge that the country was facing in numeracy and literacy. ANA (2011) verified that the province of the Eastern Cape had the lowest results, i.e. 84% (cf Table 1).

Table 1: Percentage of successful returns verified by ANA (2011)

Province	%
Eastern Cape	84%
Free State	88%
Gauteng province	96%
Kwazulu Natal	100%
Limpopo province	92%
Mpumalanga	92%
Northern Cape	96%
North West	95%
Western Cape	97%

On the basis of frequently undesirable mathematical standards as evidenced from national levels data, the current study explored some of the numerical errors that eventually led to national under-performance in mathematics.

In addition, the Mathematics Panel of the USA National Council of Teachers (2008) attempted to comprehend error analysis in mathematics. The USA National Council of Teachers noted that failure of American learners to master, for instance fractions, was the greatest obstacle to learning algebra.

This Mathematics Panel of USA National Council of Teachers (2008) suggested that to remedy this challenge; fewer topics should be covered in greater depth in order to prepare students for algebra and advanced mathematics, the curriculum should develop conceptual understanding, computational fluency, problem-solving skills and reading fluency in order to read problems. The USA National Research Council (NRC) (2000) also reported that Grade 2 learners who had been taught to use ‘cognitive guided instruction’ (CGI) in geometry outperformed a control group of Grade 4 learners in terms of skills in representing and visualizing three-dimensional forms.

In the light of criteria set out above the current study is geared and guided (1) to remedy challenges of numerical errors, (2) to pay attention to fewer topics, (3) to cover in greater depth to prepare learners for say algebra, and (4) to develop conceptual understanding and computational fluency by introducing problem-solving skills. These sets of standards eventually guided the development of these research questions and questionnaires.

Mathematics Panel of USA National council of Teachers (2008:8) reported that mathematics teachers in Austria often said, “weak learners are unable to understand mathematics”. Teachers and learners often agreed that the only way to learn mathematics was to rote learn rules and algorithms and to practice applying these mathematical exercises. The effects of rote learning can be illuminated by a neuro-scientific concept that is normally used to explain the concept of a perceptual representation system. Therefore this study will investigate the numerical errors that learners encounter in mathematics.

Nunan (2010) states that learner errors are systematic, rather than random and many learners tend to commit the same kind of numerical errors during the learning process. It is therefore the obligation of teachers to summarize these frequently appearing errors as often as possible so that learners can make a bold effort to avoid these errors.

1.2 STATEMENT OF THE PROBLEM

Following the contestation noted in the background, there seems to be a misunderstanding of the semantics of the mathematical text which is often the source of learner errors. Research both from ANA (2011) and the international report suggests that learner difficulties in solving mathematical problems arise from deficiencies in understanding for instance mathematical semantics and structure. It is also due to transgressing the boundary lines of mathematical sets and ignoring relevant concepts. On this foundation, the current study seeks to explore the understanding and misunderstanding of the mathematical semantics as well as irrelevant concepts in order to enhance the learning and teaching of mathematics in terms of error analysis.

1.3 RESEARCH QUESTIONS

The following research questions emerged from the aforementioned discussion.

1 The underlying numerical errors that learners encounter in obtaining, processing, retaining and reproducing information contained in mathematics in the foundation phase.

2 The relationships underlying numerical errors in the foundation phase.

3 The implementable strategies found in the numerical error analysis in relation with questions 1-2.

1.4 SIGNIFICANCE OF THE STUDY

The understanding of numerical error analysis may improve the underlying mathematical errors that learners encounter in the foundation phase. It may provide explanations in terms of curriculum development and strategies involved in teaching and learning mathematical semantics. The departmental policies will be unpacked so that implications can be understood in relation to the mathematical foundation and various mechanisms. Lastly, it may provide recommendations to improve the quality of learning and teaching mathematics. This research would be valuable to find learners' underlying numerical errors in the foundation phase and the relationships underlying numerical errors, which could assist the respondents with regard to numerical errors. The results of this study should serve as guidance to teachers on how to assist learners to have fewer errors in foundation phase mathematics.

1.5 SCOPE AND LIMITATION OF THE STUDY

Due to the fact that the researcher teaches in the foundation phase, the study was scoped around the classes of the foundation phase (Grade 3). However, during the research period the researcher did an onsite neighboring school visit with all the sampled (cf sample-size justification) schools for evidence-based data collection, but because of time constraints and financial constraints, the district office was used as the central point where the participants were going to fetch the questionnaires.

1.6 METHODOLOGY AND APPROACH TO RESEARCH

Research Paradigm, Approach and Design

A paradigm is simply a belief system that guides the way we do things, how we make decisions, and more importantly, how our discipline is affected (Cohen, 2007). MacMillan and Schumacher (2006) chose positivism in their study to uncover and measure patterns of behavior and to gather the respondent data to ensure maximum objectivity of numerical error analysis in foundation phase mathematics. On the basis of the research questions, the researcher chose the quantitative approach, which is generally associated with the positivist paradigm, to collect and convert data into numerical form so that statistical calculations can be made and conclusions drawn. The design which was chosen for this study was a descriptive design, concerned with finding out “what is” which could be applied to investigate research questions. The aim was to provide an accurate and valid representation of variables that are relevant to the research questions.

1.6.1 POPULATION AND SAMPLE SIZES

The data was collected from both teachers and learners in the district of East London, where the researcher works. About 80 learners participated in the study. From 375 learners who form the population of the study in the primary schools, the researcher selected by means of a simple random sample technique 80 learners as the sample size, which constituted 10% of the population. The researcher used 16 schools in East London, as they were accessible to the researcher. A simple random selection of 5 learners per school was done. Only 16 teachers were asked to answer questionnaires based on the research questions and the background of the study. Those comprised (1) remedies to challenges, (2) paying attention to fewer topics, (3) covering in greater depth in order to prepare learners for numerical errors, and (4) developing conceptual understanding and computational fluency by introducing problem-solving skills and reading fluency.

Table 2: Category of Respondents and Sample size

Respondents	Population size	Sample size	Percentage of population
Learners	375	80	10%
Teachers		16	10%

1.7 Data-collection method

Data was collected from learners by using a semi-structured instrument. This method was used to obtain a balanced view of the study based on the questions.

1.7.1 QUESTIONNAIRES

Questions were based on concepts such as (1) remedies to challenges of numerical errors (2) paying attention to fewer topics (3) covering in greater depth, and (4) developing conceptual understanding and computational fluency by introducing problem-solving skills in multiple-choice rating format, which are mathematical problems.

The underlying mathematical errors that learners encounter will be enquired by means of a rating scale (that is the Likert scale with 4 categories of responses ranging from 1 = strongly disagree to = 4 strongly agree).

1.8 VALIDITY ANALYSIS

Delport (2005) states that validity refers to the extent to which an empirical measure accurately reflects the concept it intends to measure. It refers to the truthfulness of the findings and conclusions. This study used face and content validity as the questionnaires were given to learners and teachers to comment whether the instrument would provide valid data for the purpose of the study.

1.9. ETHICAL MEASURES

A strict code of ethics was adhered to. Informed consent was requested from all participants: participants were informed about their right to withdraw at any time; their data would be kept anonymously and confidentially so as to protect the participants. The analysis and interpretation of data were trustworthy because great care was taken to safe-guard the identity of the respondents, and the participants were protected from physical and psychological harm. They were assured that they would not be identifiable.

1.10. THE STRUCTURE OF THE STUDY

Chapter One: Outlines the introduction, the research problem, the research questions, significance of the study, and limitation of the study, research methodology, data-collection method and ethical measures.

Chapter Two: Discusses the literature on numerical errors in foundation phase mathematics.

Chapter Three: Presents the methodology which is quantitative in nature.

Chapter four: Presents the findings. The findings focused on the numerical error analysis of foundation phase mathematics.

Chapter five: Discussion of the findings of the numerical error analysis of foundation phase mathematics.

Chapter six: Summary, conclusion and recommendations. It summarizes the study, the outline of the chapters and provides further recommendations.

CHAPTER 2

LITERATURE REVIEW

2.0 INTRODUCTION

In the previous chapter an overview of this study and the rationale for undertaking the study were presented. . The current study sought to explore how to improve the underlying numerical errors by learners in the foundation phase. The researcher investigated the underlying numerical errors that learners encountered, the significant relationships underlying numerical errors and what implementable strategies were available in the numerical error analysis. However, it was necessary to first provide a brief explanation about the numerical errors that learners encountered in the foundation phase, the relationships underlying the numerical errors and attention was also given to strategies of numerical error analysis in foundation phase mathematics. This chapter addresses introduction, the underlying numerical errors that learners encounter in foundation phase mathematics, the relationships underlying numerical errors and the strategies of error analysis in the foundation phase and the conclusion.

2.1 ANALYSIS OF NUMERICAL ERRORS IN THE FOUNDATION PHASE

Hasym (2002) defines numerical errors in the foundation phase as systematic deviation when a learner has not learnt something and consistently gets it wrong. An act, assertion, or belief that unintentionally deviates from what is correct or true, the condition of having incorrect or false knowledge, an incorrect belief.

Ansari (2005) reported that, counting is widely regarded as the first step towards children's exact verbal understanding of numbers. The representation and processing of numerical magnitude play a role in children's development of counting. It is clear that in order to solve calculation problems one needs to activate numerical magnitude representation.

Radatz (2005) suggests that various causes of that which cuts across mathematical content topics can be identified by examining the mechanisms used in obtaining, processing, retaining and reproducing the information contained in mathematical tasks. For many learners the learning of mathematical concepts, symbols, and vocabulary is a 'foreign language' problem. Errors are a function of other variables in the educational process. Errors in the learning of mathematics are the result of a very complex process. A sharp separation of the possible causes of a given error is often quite difficult because there is a close interaction among causes. However, this study was interested in investigating the numerical errors in the foundation phase.

2.1.1 SPATIAL STRUCTURES IN EARLY NUMERICAL PROBLEMS

Arcavi (2003) states that the spatial structure which subsequently arises can simplify early numerical procedures. When young children are asked to determine the quantity of a randomly arranged set of objects, they initially tend to count each object. As the set of objects grows, this procedure eventually confronts them with the difficulty of keeping track of which objects have already been counted and with the time-consuming process of the numbers that accompany the counting of large sets. It becomes necessary therefore to expose the errors of mathematics in the foundation phase in this study.

However, according to Warren (2005) there are many indications that an understanding of pattern and structure is important in early mathematics learning. A mathematical pattern is described as any predictable regularity, involving numerical, spatial or logical relationships. In early childhood, the patterns children experience include repeating patterns, for example 2, 4, 6, 8, ….. Repeating patterns are particularly important, since they recur in measurement (which involves the iteration of identical spatial units) and multiplication (which involves the iteration of identical numerical units). In a numerical structure, spatial relationships, provided by a certain domain, and the study of patterns and structure are embedded in a diverse range of studies of mathematical development in pre-school children and in those in the early years of schooling. Wright and Young (2002) in their studies on numeration, describe first graders' co-ordination of units of 10 and 1 in terms of the structure of collection. While the majority of 10 year-olds could solve the problems, some experienced difficulty in the structural understanding of combinatorial problems, and often had trouble explaining fully the two-dimensional structure of the problems and could rarely identify the cross-multiplication feature. They lacked a complete understanding of the combinatorial structure.

Mason (2010) believes that the roots of mathematical thinking lie in detecting sameness and difference, in making distinctions, in classifying and labeling, or simply in algorithm seeking. Young children who have learned to look for mathematical similarities and differences within and between patterns are likely to develop an understanding of the structure of those patterns. Moreover, they will tend to look for similarities and differences in new patterns and broaden their structural understanding accordingly. By contrast, learners who tend not to notice salient features of structures are likely to focus on non–mathematical features in all situations. All of these involve processes such as repetition, grouping and partitioning, and all require spatial structuring in visualizing and organizing their structure. Therefore this study is interested in investigating numerical errors in the foundation phase.

Watson (2009) reveals that mathematical thinking in early childhood can be described in terms of a growing awareness of pattern and structure, which prominently contribute to children who achieve well in mathematics in school, and inversely to those who do not progress easily or develop learning difficulties.

2.1.2 NEUROLOGICAL DYSFUNCTIONS

According to Keppler (2005) neurological dysfunction is the internal representation of mental events. Learning in all its forms entails that an individual acquires knowledge. This knowledge can influence the individual's behavior. Learning is closely connected with memory. Memory means the ability of an organism to store knowledge and to retrieve this knowledge when necessary. Cognitive neuroscience distinguishes between two basic forms of memory, the declarative and the non-declarative memory. The declarative memory is accompanied by consciousness that is its content can be reported. Neurological disorders are diseases of the brain, the spine and the nerves that connect them. There are more than 600 diseases of the nervous system, such as brain tumors, epilepsy, and strokes as well as less familiar ones. Hence this study investigated the numerical errors of mathematics in the foundation phase

Keppler (2005) explained that it is important for parents to protect their children with neurological disorders by getting them the flu vaccine. It is important that learners with these disorders are diagnosed correctly so that they are not exposed to unnecessary tests and inappropriate treatments. It is also important that persons with these disorders are open to the correct diagnoses so that they can pursue proper treatment. With treatment, the symptoms can often improve and even go away entirely.

Ansari (2011) revealed that the brain mechanism underlying calculation change as a function of learning and development has shown that the brain areas activated during mental arithmetic change in fundamental ways with chronological age. This provides a proper example of how neuro-scientific data might through methodological triangulation, complement knowledge that is obtained on the basis of behavioral data. It is important to note that most of the existing neuro-imaging studies in this domain have been carried out with adult participants. What is currently lacking are studies in the neuroscience of numeracy and arithmetic that investigate brain activity in children who are in the process of learning mathematics and how this activity is affected by different instructional conditions. In addition, there is a great need for studies that evaluate the effects of structured remedial interventions on brain function for learners with mathematical difficulties.

According to Dehaene (2003) in neuro-psychology and neuro-imaging the left angular gyres (LAG) are systematically involved in mental arithmetic. The precise role played by this brain region in calculation is however, still poorly understood. The LAG sub-serves the retrieval of arithmetic facts stored in verbal memory. Ansari (2008:22) states that, “the LAG supports the automatic mapping between mathematical symbols and their semantic referents, such as arithmetic problems and solutions, stored in memory.” In arithmetic it is assumed that over-learned arithmetic problems become higher-order symbols and that their presentation automatically activates the associated solution in long-term memory.

Ansari (2011) reported that the availability of non- invasive tools to image the human brain allowed researchers to investigate how the brain changed over the course of development and learning and to investigate the brain circuits involved in key academic skills, such as reading and arithmetic, as well as more general cognitive skills, such as working memory. There should be bi-directional and reciprocal interaction between both disciplines of neuroscience and education, and research originating for each of these traditions is considered to be compelling in its own right. This will add to a greater communication between neuroscientists, educators and educational researchers, of which the aim must be to achieve a common language to generate future research questions and translate research into concrete educational applications. For neuro-education to survive, much patience is required.

2.1.3 ERROR ANALYSIS IN FOUNDATION PHASE MATHEMATICS

Richards (2005) defines error analysis in the foundation phase as the study and analysis of the errors made by learners in mathematics. Error analysis may be carried out as an aid for teaching in order to find out how well the learners learn, how a learner learns to count mathematically and to obtain information on common difficulties in learning mathematics. Brown (2003) proposes that error analysis is a technique for identifying, for classifying and systematically interpreting the unacceptable errors made by learners in the foundation phase. Therefore, this study was interested to expose those errors that exist in the foundation phase.

According to Cranefield (2006) tests were piloted at a primary school in a city. Three experienced test administrators were trained to present the tests to 20 Grade 1 learners, 20 Grade 2 learners and 20 Grade 3 learners in three different classrooms. To Grade 1 the administrator read each question and to Grades 2 and 3 the administrator briefly explained the instructions for each test. At the end the Principal, the research supervisor, the test administrator, the teachers of Grades 1, 2 and 3 as well as the remedial teacher at the school discussed the rationale for testing at this level.

Only the results of Grade 3 were analyzed as test A, where 91.6% and 68.3% were able to answer the respective questions, which involved counting the number of turtles and counting with the use of plus and minus signs. The fact that 9.4% were unable to count the number of turtles was a concern, especially among the Grade 3 learners. Counting backwards in twenty fives also proves to be beyond the capabilities of this group since they scored 0.8%. Adding in fives with plus signs between the numbers proved to be better than counting in fives without the signs.

The word problems were poorly done, obtaining only 71.1%. Concerning most of the questions word problems were the most difficult type, and problems with visual prompts appeared to be the easiest. None of the learners displayed higher-order thinking skills. The insights gained from the test will be used in the development phase.

Bruner (2009: 47) reports that, “recent neurological research demonstrates the importance of the child’s first few years of development. The brain is well developed at birth and continues to expand rapidly in the first year, making new synaptic connections, linking neurons and developing pathways”. At about 2 years of age there is a pruning of the brain’s dendrite system and unused or rarely used connections appear to be deleted. The brain at first over-produces neural connections and then as a result of experience or learning trims them back so that only a smaller percentage of the initial connections actually ends up surviving and being used throughout life.

Pausigere (2011: 2) suggests that, “for mathematics classroom activities teachers can let learners choose any number and allow them to investigate various ways of representing that number”. In the process of such an investigation learners will explore the characteristics of their chosen number as well as how different mathematical operations could be applied to that number. For example,

$$13-6 = 7$$

$$3+4 = 7$$

$$7\text{x}1 =7$$

$$49/7 =7$$

The possible reason for the learner errors was that learners lacked knowledge of mathematical signs. Therefore this study seeks to investigate the numerical errors that exist in foundation phase mathematics.

Gate (2011) states that mathematics is all around us, it underpins much of our daily lives and is critical to our children’s future. The economic future depends on stimulating, innovating, developing technological breakthroughs, making connections between scientific disciplines. It is therefore of fundamental importance to ensure that children have the best possible grounding in mathematics during their primary years. Difficulties with using the formal algorithms often relate to learners' lack of understanding about how to record rather than how to calculate.

Fleish (2008: 108) states that, "by the end of primary school, children will have a confident grasp of counting, numbers and arithmetic, which will provide a solid platform for them to engage with algebra and other aspects of the school mathematics curriculum when they reach secondary school. Evidence suggests that the majority of children do not achieve this competency at the end of primary school, and a deficit emerges in the early years that become augmented when learners reach high school". There are three interlinked aspects to mastering foundation phase numeracy, namely (1) progression in acquiring the number concept, (2) the shift from concrete to abstract reasoning, and (3) the move from counting to calculating. This provides a platform for learners to learn to calculate using strategies, and the move beyond this is to learn to calculate using symbolic representation.

According to Gelman and Gallistel (2005) there are three parallel sets of specializing strategies in relation to mode of representation.

Counting objects

This entails mapping a number sequence onto a set of objects, which includes counting animals on a poster, counting members of a family depicted in a picture, counting out a set of objects such as counters, stones, crayon boxes, beads, dots and tallies.

Number line

This is a way to locate numbers on a number line which includes identifying numbers on a number chart, finding numbers closest to a given number, bigger or smaller than a given number, or between two given numbers. It also entails the use of expanding number cards to represent numbers. This study was to investigate errors present in foundation phase in mathematics.

2.1.4 MISUNDERSTANDING OF THE SEMANTICS OF THE MATHEMATICAL TEXT

According to Goswami (2008) a misunderstanding of the semantics of the mathematical text is often the source of learner errors. Deficits in basic prerequisites include ignorance of algorithms, inadequate mastery of basic facts, incorrect procedures in applying mathematical techniques, and insufficient knowledge of necessary concepts and symbols. Buswell (1999: 19) reports that, “errors in the learning of mathematics are simply the absence of correct answers or the result of unfortunate accidents”. These are the consequences of definite processes whose nature must be discovered. Piping (2001).suggested four types of errors that learners encounter:

- Errors of association involving incorrect interactions between single elements. For example 66+12=77, 56+15=67
- Errors of interference in which different operations or concepts interfere with each other, for example

  845

  +372

  +1217

  + 561

  1778

- Errors of assimilation in which incorrect hearing causes mistakes in counting. Such errors result from lack of attention and concentration (random or careless errors)

- Errors of negative transfer from a previous task, in which one can identify the effect of an erroneous impression obtained from a set of exercises.

Burrows (2000) states that when one attempts to go beyond the description of faulty techniques and error patterns and towards the analysis of possible causes in the learners' cognitions, the various aspects of information processing seem to offer a fruitful basis for classification. The analysis of learner errors in mathematics should be seen against the background of the observation that most errors derive from the learner's use of very individual and sensible rules or strategies.

Bornman (2010) reports that when the neurological condition that affects the ability to acquire mathematical skills is present, then the children have difficulty in understanding simple number concepts, confuse similar looking numbers (e.g. 6 and 9 or 12 and 21), lack an intuitive grasp of numbers and mathematical procedures, when they have to read three-digit numbers containing a zero, for example 1007 or 1087, or when they have to write one thousand and three, then the zero is lost and they write 103.

There are learners who have problems with copying numbers, recalling numbers in calculations and writing mathematical symbols. In addition learners experience problems with understanding the concept of weight, space, direction and time. Reading maps, tables, diagrams and measurement is extremely confusing to them, as well as arranging numbers by size (16 becomes before 17 and 74 is five more than 69).

Flavell (1999), the neuropsychologist researcher, states that the brain stem is critical in focusing on attention and consciousness. All neural messages that flow between the sections of the brain are modulated and regulated here.

Learners who have been diagnosed as hyperkinetic are unable to focus or do cognitive tasks. Therefore they are highly distractible and always make errors in mathematics. Malfunctioning of the reticular system prevents the message from coming from the sensory receptors to be screened. The malfunctioning is likely caused by a biochemical disturbance. This study investigated the numerical errors in the foundation phase that would help the learners and teachers exposed to those numerical errors.

Dehaene (2003) proposed possible causes of various sub-types for dyscalculia, *inter alia*

➢ Deficits in verbal symbolic representation, which would hinder learning of basic mathematical facts.

Shalev (2004) stated that some learners struggled with a brain disorder which interfered with normal mathematical thinking. Dyscalculia is a learning disability that exists among learners in both under-developed and developed countries, and between 3% and 8% of learners will show dyscalculia. Dyscalculia is a primary cognitive disorder of childhood that affects normal acquisition of arithmetic skills. It is not a result of reading but of problems with basic numerical processing.

Shunkoff (2000) suggests that the early years form a sensitive period for a new-born brain because it is composed of trillions of neurons. The experience in childhood determines which neurons are used to wire the circuits of the brain. Those neurons that are not used may die. Skott (2005) reported that in South Africa in the 1990’s learners were expected to develop their own methods for arithmetic operations, but most found it impossible to progress on their own from counting to actual calculating. In 2008, 79% of children in Grade 5 and 60.3 % in Grade 7 still relied on simple-unit counting to solve problems one way or another, while 38,1% relied on this method and 11% relied exclusively on this method.

2.1.5 DYSCALCULIA – A COGNITIVE DISORDER

Deheane (2003) suggests that the areas in the brain that do mathematics are fragmented into specialized systems where both sides manipulate Arabic numbers and numeric quantities. The inputs and outputs of these areas must be integrated in order for learners to calculate correctly. Lack of integration is one potential explanation for dyscalculia, the difficulty that some learners have in mathematical performance. In 2004 the Department of Education performed a Systematic Evaluation survey at the Intermediate Phase level (Grade 6). The purpose of the survey was to establish baseline data on learner achievement and conditions of learning and teaching at the Intermediate Phase level.

The survey involved a sample of approximately 35 000 learners from about 1000 schools. Findings showed in general that learners obtained the lowest scores (30%) for numeracy. This trend revealed that learners from the rural areas were found to be at a disadvantage regarding their level of performance in mathematics. Average scores ranged from high (45%) in urban schools, 26% in township schools, 23% in farm schools to 21% in rural schools, according to (Doe 2005:30).

Quinn (2011) reported concern with learning taking place during an educational activity, but the children and often the teachers do not see that. When mathematics is done by hand, a lot of activity comes as a package activated by simple goals. New methods, especially technology, cause these packages to come apart and important ways of learning may be lost. The reason for this study was to explore the numerical errors in the foundation phase.

2.1.6 INDIVIDUAL DIFFERENCES IN SCHOOL CHILDREN

Bruner (2005) reports that many errors result from substantial individual differences in learners' spatial thinking and consequent difficulties some children have in obtaining visual or spatial information in performing a mathematical task. The most common error made by most learners is reading a Venn diagram. It is clear that these difficulties in reading Venn diagrams do not arise from deficiencies in understanding symbols. These difficulties arise because of inhibitions about transgressing the boundary lines of sets and ignoring relevant lines. Fleisch (2008) showed that at the end of 2001, a large class of Grade 3 learners revealed a very poor performance in elementary mathematics. Learners achieved an average score of 30% on the numeracy task set. It also revealed that between 2002 and 2006 in the Western Cape Department of Education numeracy results dropped from 36.6% to 32%. He cited performance at Grade 3 levels to be problems with curriculum coverage, coherence, cognitive demand and pacing.

Scholar (2008) shows the dominance of concrete methods such as tally counting in solving problems in mathematics in the early grades. The efficiency of such methods, and the failure of many learners to progress to abstract from concrete representation, were offered as a significant contributor to poor mathematics achievement in South African schools. Mendoza (2000) reported that Singapore learner performance in geometry and number sense showed some clear trends. Primary Grade 4 learners outperformed Grade 3 learner girls outperformed boys because boys rather than girls tended to leave more questions blank.

The USA's Mathematical Panel (National Council of Teachers) (2008) did some error analysis in mathematics, and found that the failure of American learners to master fraction was the greatest obstacle to algebra. To remedy this challenge, they decided to cover fewer topics in greater depth.

A survey was conducted by Howie (2002) where learning attainment of Grade 4 learners was measured by means of numeracy scores achieved, and a criterion referenced approach was used. The results showed that learners in South Africa performed poorly in numeracy. South Africa (514.2) performed significantly behind Swaziland with a score of 516.5, compared to Botswana with 512.9 and Mozambique with 530.0. Therefore it was necessary for this study to explore strategies of errors that learners commit in foundation phase mathematics.

2.1.7 EARLY NUMBER CONCEPTS – S0UTH AFRICAN STUDIES

Cranfield (2006) investigated Grades 1 to 3 in South Africa, where number concepts were learnt early. Three schools were included and involved, 222, 257 and 240 learners in Grades 1, 2 and 3 respectively. The children's level of understanding was assessed by the use of four tests. An analysis of performance, misconceptions and errors made by the learners in each grade was achieved by means of an in-depth analysis of 48 learners. The results suggested that the majority of learners were unable to solve straight calculations employing the strategy of counting all and counting on, while none engaged in formal or innovative methods. There was no progression in terms of conceptual mathematical development across the foundation phase.

Pillay (2010) suggests that, the teaching of emergent numeracy skills encourages learners to construct meaning through inquiry and self-discovery of their own understanding and interpretation of a given situation. Educators need to understand that learners need plenty concrete experiences of the meaning of numeracy skills and concepts. The teaching of numeracy must be an interactive process which includes their verbal responses. Children must be able to explain why, what and how they reached a conclusion. Teachers need to be careful not to mark the children's work as wrong, as this discourages improvement. It is equally important to pay attention to individual learners to ensure that no one is left behind.

Carrey (2007:15) reports "when children understand that the purpose of counting is to determine the total number of items in a set, they can be said to have acquired a “cardinality principle”. A popular task to measure whether or not children have acquired the cardinality principle is to give a number task.

In this experiment children are given a pile of objects and asked to give different numbers. Children who have acquired the cardinality principle will use counting to give an experimenter the exact number of objects requested. However, children who do not understand the meaning of counting will typically just grab a number of objects and give them to the experimenter without counting. These “grabbers” do not yet understand the counting sequence and regard it as a meaningless verbal sequence.

Scholar (2008) suggests that the majority of learners do not achieve competency at the end of the primary school, and a deficit emerges in the early years that becomes augmented when learners reach high school. According to Taylor (2006) the significantly low numeracy competencies cause serious concern. One of the major challenges facing the education system is the provision of the right number of appropriately qualified mathematics educators in all the schools. This is probably one factor that could have contributed to the particularly low levels of competence in mathematics. Therefore this study is urgently investigating the underlying numerical errors that learners encounter in the foundation phase.

Alex (2002) suggests that early schooling in mathematics is critical for the later success of all learners. The lack of an adequate foundation for some children is impacted by the way mathematics is learnt in the early grades and which teaching strategies are used. Many elements must be acknowledged about the way mathematics is learnt early. That includes recognition of the developmental aspects of learning, the importance of building on prior mathematical understanding, and the essential fact that children learn mathematics primarily through doing, talking, reflecting, discussing, observing, investigating, listening and reasoning. For the teaching and learning processes to be successful, it is important that the child's existing conceptual understanding of mathematics be recognized. Children need to encounter concepts in an appropriate manner, at an appropriate time, and with a developmentally appropriate approach.

According to Edward (2000) many children begin school with a limited understanding of numbers and number relationships. Counting skills which are essential for ordering and comparing numbers, are an important component in the development of number sense, counting on, counting back, concepts of more and less, and the ability to recognize patterned sets, All of these mark advances in children's development of number ideas. Basic facts are mathematical operations for which some learners may not be conceptually prepared.

The following skills should be in place before children are expected to acquire basic facts:

- Learners can immediately name the number that comes after a given number from 0-9 or before a given number from 2-10.
- When shown a familiar arrangement of dots 10 on ten frames, dice, or dots cards, learners can quickly identify the number without counting.
- The concept of less tends to be more problematic for children and is related to strategies for subtraction.
- Basic facts and mental calculation strategies are the foundations for estimation. Attempts at estimation are often thwarted by the lack of knowledge of the related facts and mental mathematical strategies.
- Learners develop and use thinking strategies to recall answers to basic facts.

The development of mental computation skills needs to be a goal of any mathematical programme for two reasons. Firstly, in the day to day activities, most learners' calculation needs can be met by having well developed mental computational processes. Secondly, while technology has replaced paper-pencil as the major tool for complex computations, learners still need to have well developed mental strategies to be alert to the reasonableness of answers generated by technology. Mathematics is about patterns and relationships and many of these are numerical.

Without a command of the basic facts, it is very difficult to detect these patterns and relationships. Furthermore, nothing empowers learners with more confidence, and a level of independence in mathematics, than a command of the number facts. Understanding the base ten system of numeration is the key to developing computational fluency. At all grades, beginning with single-digit addition, the special place of the number 10 and its multiples is stressed. In addition, learners are encouraged to add to make 10 first, and then add beyond the ten. Addition of ten and multiples of ten is emphasized, as well as multiplication by 10 and its multiples. This study would like to include the above findings in the strategies of the numerical error analysis of foundation phase mathematics.

2.2 RELATIONSHIPS UNDERLYING NUMERICAL ERROR ANALYSIS

Scholar (2008) suggests that there are three principles interlinked to mastering foundation phase mathematics, namely: progression in acquiring the number concept, the shift from concrete to abstract reasoning, and related to this the move from counting to calculating. Gelman and Gallistel (2000) reported that learners can be deemed to have mastered counting when these principles are in place.

According to the principle the learner is able to mark off items in a collection with distinct markers so that one marker is used for each item. The child has to learn to co-ordinate tow processes of partitioning and marking. The two processes have to work simultaneously, beginning and ending at the same time. Three kinds of errors can arise in this, errors in partitioning (such as tagging an item more than once), errors in the use of tags (such as using tags more than once), and failure to co-ordinate the two processes adequately.

2.2.1 THE STABLE ORDER PRINCIPLE

Learners need to recognize that the tags themselves are organized in a repeatable, stable order.

2.2.2 THE CARDINAL PRINCIPLE

Here the learner needs to understand that a number, such as five, can be achieved by counting the items in a set of five objects and that it represents the total number of items in a set. This understanding is crucial to all of the child's later number reasoning.

2.2.3 THE ABSTRACTION PRINCIPLE.

Hereby is understood that counting procedures can be applied to any collection of items. There are five types of different countable items, perceptual units, (which can be seen), figural items (items not present but recallable), motor units (movement like steps), verbal units (utterances of number words) and abstract units.

2.2.4 THE PRINCIPLE OF IRRELEVANT ORDER

This principle entails the understanding that the order in which the items in a collection are counted does not affect its numerosity. The tag applied to an item is arbitrary and is not a characteristic of the item, and the same cardinal number applies to a collection irrespective of the order in which the items are counted.

Gray (2007) suggested there are types of mathematical concepts, the perception of objects, processes that are symbolized and axiomatic thinking. For example knowing things like 4 and 2 makes 6, so 6 take away 4 must be 2 and using these thing to derive knowledge such as 26 take away 4 must be 22 because 26 is just 20 and 6.

Tall (2000) reports that the shift from counting to calculations makes use of counting strategies, which occur across Grades 1 to 3 and beyond as learners move from the perceptual world (the use of fingers and counters) to using representation (the use of tallies and later number words).

Anglheri (2006) argues that acoustic (oral) counting encourages learners to memorize number sequences and number patterns. For example, counting forward and backward, such as counting in 1s, 2s, 3s and 5s, etc develops an understanding of patterns that assist in early addition and subtraction.

Counting out objects entails mapping, a number sequencing to a set of objects, for example counting of animals, counting using counters etc.

Radatz (2000) suggests that one can presume the analysis of mathematical errors is like many educational research problems in that the differences are found in educational curricula and syllabi. In classifying errors according to individual learner difficulties, one should of course acknowledge that errors are also a function of other variables in the educational process. Errors in the learning of mathematics are the result of very complex processes. A sharp separation of the possible causes of a given error is often quite difficult because there is such a close interaction among causes. However, it is very important for the learners to be made aware of the numerical errors because these suggestions could put them in a better position to make minor errors when doing mathematics. Therefore the reason for this study to be conducted was to make the teachers aware about the numerical errors that the learners make in the foundation phase.

Pipping (2005) suggested an interesting classification of these types of errors.

- Errors of perseveration, in which single elements of a task or problem predominate, for example 9x60 = 560 5x13 = 63 41+7 =47.
- Errors of association, involving incorrect interactions between single elements, for example 66 + 12 = 77 3x9 = 36 56+15 =67

- Errors of interference, in which different operations or concepts interfere with each other. The following example shows the interference between the algorithms for addition and subtraction.

```
     6845
+     372
+   35437
+  561
-------
   30375
```

To obtain the answer, the child added the digits in the unit column, getting 15; added all but the top digit in the tens and hundreds columns, getting 17 and 13, respectively; and then subtracted to get the remaining two digits in the answer. Errors of interference arise when a previously learned skill or algorithm, because of its similar processes, makes the learning of a new skill or algorithm difficult.

- Errors of assimilation where incorrect hearing causes mistakes when reading or writing. Such errors are often classified as errors resulting from a lack of attention and concentration (random or careless errors).
- Errors of a negative transfer from previous tasks, in which one can identify the effect of an erroneous impression obtained from a set of exercises or word problems. An example of an erroneous impression developed while working on a task is the following;

Task

+7

in	out
	31
	20
	79

+7

In	out
31	38
20	27
86	79

2.2.5 ERRORS DUE TO THE APPLICATION OF IRRELEVANT RULES OR STRATEGIES

Quinn (2011) says the topics of use of irrelevant rules, the development of incorrect algorithms, and the application of inadequate strategies in solving mathematical tasks and problems were especially important in Soviet research in the learning of mathematics. This kind of error often stems from experiences in successfully applying comparable rules or strategies in other content areas.

Learners' understanding of mathematics, and especially of arithmetic, as a game with arbitrary rules may provide the background for analyzing many causes of learner errors. The sense of various strategies and rules can be recognized in many errors due to each of the causes mentioned previously. Examples of errors due to the application of irrelevant rules and closely related to errors due to the deficient mastery of prerequisites are the following: 1000 / 200 = 500   96 / 16 =10   155 / 5 = 301. The five types of errors in the information-processing classification above should be seen as the elements of a model for grouping the errors. Additionally, aspects of information processing that are responsible for errors in mathematics learning include failure to consider the relevant conditions of a problem, failure to complete the solution process, loss of intermediate solution, and one-sided use of concepts.

When one attempts to go beyond the description of faulty techniques and error patterns and toward the analysis of possible causes in the learners' cognition, the various aspects of information processing seem to offer a fruitful basis for classification. The analysis of learner errors in mathematics should be seen against the background of the observation that most errors derive from the learner's use of very individual and sensible rules or strategies.

Ginsburg (2000) suggested possible causes of learner errors in mathematics. At first, errors in the learning of mathematics are not simply the absence of correct answers or the result of unfortunate accidents. They are the consequences of definite processes whose nature must be discovered. Secondly, it seems to be possible to analyze the nature and the underlying causes of errors in terms of the individual's information-processing mechanism. Thirdly, the analysis of errors offers a variety of points of departure for research into the processes by which children learn mathematics. Fourthly, the prevailing quantitative aspects of tests and similar methods for measuring achievement cannot provide sufficient criteria for efficient instructional procedures.

Consideration of the diagnostic and causal aspects of errors could provide specific help to mathematics teachers by allowing them to integrate their knowledge of curriculum content with their knowledge of individual differences in children. Finally, one should take note of some difficulties in the present state of research into learner errors. In many of the examples cited above, it is often quite difficult to make a sharp separation among the possible causes of a given error because there is such a close interaction among causes. The same error can arise from different sources, and the same error can arise from different problem-solving processes. The reason for this study to be conducted was to assist teachers with strategies regarding numerical errors in the foundation phase.

2.2.6 MISCONCEPTIONS, ERRORS AND MISTAKES

According to Pardesi (2008) there are common misconceptions about errors and mistakes made by learners in the classroom. A misconception can be understood to be what a learner thinks is incorrect with regard to his/her understanding of a concept, whereas an error is a mistake that is repeated and thought to be correct by the learner, while a mistake is something that a learner knows is wrong and allows it to be corrected.

Niss (2006) reported errors in effective situations. Mathematics is concerned with long-term processes and cognitive representation which determine the dynamics of learning processes. We need more insights into the effect of emotions during problem solving in order to explain how errors emerge. One must bear in mind that attention is a crucial prerequisite for all conscious processes. It was necessary for this study to expose the numerical errors of foundation phase learners in order to improve learning and teaching.

Errors often occur when learners think they understand, which can be seen as not incorrect knowledge, but incomplete knowledge. An error may only surface once existing knowledge is applied to new situations. An error occurs where there is little or no progression in terms of mathematical conceptual development across the foundation phase.

Ansari (2011) announces that one of the most significant events in early number development is children's development of counting skills. Representation and processing of numerical magnitude play a role in children's development of counting. The development of counting is a step towards exact verbal number representation. Thus, children appear to use areas involved in attention, working memory and cognitive control during numerical magnitude processing to a greater extent than do adults. The intra-parietal sulcus (IPS) plays a critical role in the processing and representation in the brain of humans and animals.

Fleish (2008) reports that classroom talk not only helps learners to develop conceptual understanding, but is also effective for revealing and clarifying learners' partial understanding and misconceptions. In the process of making sense of experiences, learners often generalize ideas in incomplete ways.

Campbell (2005:35) states that, "small problems are usually solved by means of fact retrieval, whereas large problems are more often solved by error-prone and time consuming quantity-based procedural strategies, such as counting or decomposing a problem into smaller problems". For example; to choose which of two incorrect answers is closer to the correct solution,

(4 + 5 = 8 or 3)

(4 + 5 = 9 or 7)

These findings are likely to reflect the greater involvement of quantity-based processes, such as estimation, which can be necessary to solve an approximate calculation problem. The similarity between effects of problem size on brain activation and the networks activated during approximate calculation, further supports the suggestion that the greater involvement of front-parietal regions while calculating problems of a larger compared to a small problem size reflects the larger involvement of quantity-based processes and the engagement of working-memory resources.

Children with mathematical difficulties when there are no general impairments to learning can have an intellectual functioning often referred to as Developmental Dyscalculia. Children with Developmental Dyscalculia (DD) activate their IPS much less than typically developing children. These findings reveal that brain regions that are known to play a critical role in numerical magnitude processing in adult participants, undergo a process of age-related specialization and that this process of specialization is disrupted in children with mathematical difficulties, such as Developmental Dyscalculia.

Halberda (2008) suggests a relationship between individual differences in numerical magnitude representations and individual levels of mathematical skills development. Moreover, numerical magnitude representation between teenagers' non-symbolic numerical magnitude discrimination abilities and their mathematical scores measured when they were younger was found to be rather specific, as these relationships were still significant after controlling for variables such as the children's working-memory scores, language skills and intelligence.

Numerical errors can tell the teacher how far towards the goal the learner has progressed and consequently that which remains for him or her to learn. Learners provide valuable feed-back. However the significance of this study was to inform the learners about numerical errors that they make and for them to learn to avoid these errors as often as possible. According to Duncan (2007) school entry mathematics predicts much stronger about later academic achievement than early reading or socio-emotional skills, and low mathematical competence is associated with lower indices of life success. Improvements in mathematical competence are related to growth of gross domestic product. These factors demonstrate the fundamental significance of mathematical competence and highlight the importance of identifying sources of its variability.

A potential source of individual differences in mathematical competence is the neural architecture supporting the performance of simple arithmetic problem solving. Arithmetic fluency, the speed and efficiency with which correct solutions to numerical computations are generated, is thought to present a scaffold upon which higher-level mathematical skills are built. Learners rely on procedural strategies, such as counting aloud, finger counting or decomposition to solve calculations. These explicit procedures are gradually replaced by more efficient strategies, such as the retrieval of solutions from memory. This shift towards memory-based calculations is a hallmark of successful arithmetic development. Children with mathematical learning difficulties exhibit mature procedural strategies and poor mathematical fact performance.

According to Malcolm (2002) these invented rules may only apply in a limited domain, even though many of those are correct.. When learners systematically use incorrect rules, or use correct rules beyond their proper domain of application, we have a misconception. Frequently a “misconception” is not wrong thinking but is a concept in embryo or local generalization that the learner made. It may in fact be a natural stage of development.

Cockburn (1999) suggests that errors in mathematics may arise for a variety of reasons. They may be due to the pace of work, the slip of a pen, a slight lapse of attention, a lack of knowledge or a misunderstanding. Some of these errors could be predicted prior to a lesson and tackled at the planning stage to diffuse or unpick possible misconceptions about why these errors may have occurred and how to unravel the difficulties for the child to continue learning. Mathematical errors may occur when teachers make assumptions about what children already know. When children are asked to complete tasks, there is a certain understanding of basic knowledge.

For example, Percy was shown a picture of 12 children and 24 lollies and asked to give each child the same number of lollies. Percy’s response was to give each child a lolly and then kept 12 for himself. Misconception may occur when a child lacks the ability to understand what is required from the task. When children make errors it may be due to a lack of understanding of which strategies / procedures to apply and how those strategies work. Mathematical errors may occur when a child’s mind, imagination or creativity, when deciding on an approach using past experience, may contribute to a mathematically incorrect answer. If the task is too difficult, errors may occur. If a task is not presented in an appropriate way, a child may become confused with what is required of him/her. This requires the child to read and interpret problems and to understand which mathematical methods to apply and to understand the language used as well.

If a teacher lacks confidence or dislikes mathematics, the amount of errors made by a child within the teaching may increase. With the pressures of teaching today, teachers may feel under pressure or rushed for time and not perform to the best of their ability. Too much teacher knowledge could result in a teacher not understanding the difficulties children have whereas too little knowledge could result in concepts being taught in a limited way. Knowledge can be gained from making mistakes. Teachers may learn about children's misconceptions by coming across those within their teaching.

Mark (2009) suggests that in order to understand the common misconceptions that occur with column addition it is important to consider the key developments of a child's addition abilities:

(1) Counting on (the first introduction to addition is usually through counting on to find one more).

(2) Memorizing facts (these include number bonds to ten. It is very important that children have a sound knowledge of such facts).

(3) Facts involving zero (adding zero, is a set with nothing in it, is difficult for young children. Children need practice with examples where zero is involved).

(4) The communicative property of addition (if children accept that order is not important it greatly reduces the number of facts they need to memorize).

(5) Facts with a sum equal to or less than 10 or 20 (it is very beneficial to children to only learn a few facts at a time).

(6) Adding tens and units (the children add units and then add tens. The process of exchanging ten units for one ten is the crucial operation).

(7) Adding mentally in an efficient way (starting with the largest number or grouping numbers to make multiples are examples of this).

(8) Adding more than two numbers (children should realize that they are only able to add two numbers at a time. 5+8+6 is calculated as 5+8=13 then13+6=19).

(9) Adding regardless of magnitude (if the children have understood the process of exchanging they will be able to carry out this operation when dealing with hundreds and thousands).

(10) Word problem (identifying when to use their addition skills and use them efficiently).

When learners in year 6 have difficulty with column addition, they probably do not have sufficient understanding of one of the areas outlined above; such ‘gaps’ in knowledge should be filled in order to provide the learners with a proper understanding of such addition.

According to Koshy (2000) a large number of misconceptions originate from reliance on rules which were not understood, forgotten or only partly remembered. For example:

H t u	h t u	h t u	h t u
760	729	534	546
+240	111	383	364
990	839	897	899

In this example the learner can add two digits accurately, but is “mixed up” with the carrying aspect of addition. He/she learnt a rule based on the place value system, where the largest number in a column is 9. The learner followed the rule correctly, but does not know what to do with the rest of the number, as he/she never understood the underlying principle behind the rule.

Children make mistakes. These might include mistakes with simple mental addition or mistakes when setting out column addition, as the place value of digits is sometimes wrong, for example:

<u>H t u</u>

160

<u>+8</u>

960

Teaching children to estimate answers is another good way of helping them to limit the number of careless mistakes. The calculation above was incorrect because of a careless mistake with the placing of a digit. However, many mistakes with column addition are caused by a fundamental weakness in a child's understanding of place value. When teaching how to add vertically, it is also useful to reinforce the principles of place value used in the operation. Children will then be more likely to relate the word "carrying" to what is actually happening rather than learn it as a rule that helps to produce correct.

According to Koshy (2000), subtraction can be described in three ways:

- Taking away (where a larger set is shown and a sub-set is removed leaving the answer, for example, 5 take away 2 leaves 3).
- The difference between (where both sets are shown and the answer is shown by the unmatched members of the larger set, for example, the difference between 5 and 3 is 2.
- Counting on (where the smaller set is shown and members are added to it).
- Make it up to the larger set, for example, 3 and 2 makes 5. It is important to remember that subtraction is the opposite of addition.

Koshy (2000) suggested learning to subtract step by step:

(1) The process of taking away involving 1 to 5
(2) Take away involving 0 to 10.

(3) Subtraction in the range of numbers 0 to 20 (using a range of vocabulary to phrase questions such as fifteen take away eight).
(4) Difference (the formal approach known as equal addition, is not a widely used method but it involves finding a number difference).
(5) Subtraction of tens and units (this is where common misconceptions occur because of the decomposition method).
(6) Subtraction by counting on (this method is more formally known as complementary addition. The procedure is to add on mentally in steps to the next ten, the next hundred etc.).
(7) Word problems (identifying when to use their subtraction skills and use them efficiently).
(8) Decomposition with larger numbers.

According to Ginsburg (2000) by considering the development of subtraction and by consulting a school’s agreed through routes we should be able to see where common misconceptions are likely to occur. It seems that to teach in a way that avoids learners creating any misconception is not possible, and we have to accept that learners will make some generalizations that are not correct and many of those misconceptions will remain hidden unless the teacher makes specific efforts to uncover them.

Many of the mistakes children make with written algorithms are due to their misconceptions relating to the place value of numbers. When considering this aspect it is worth pointing out that children tend to make more mistakes with subtraction than with any other operation. Some children carry out an exchange of a ten for ten units when this is not required and some forget they have carried out an exchange. Children should realize that in most subtractions the smaller number is subtracted from the larger.

Solving problems is one of the most important activities in mathematics. Therefore, in order to solve problems children will need to know general strategies. General strategies are methods or procedures that guide the choice of which skills or knowledge to use at each stage in solving a problem. Problems in mathematics can be familiar or unfamiliar. When a problem is familiar the learner did something like it before and should remember how to go about solving it. This is when general strategies are useful, for they suggest possible approaches that may lead to a solution (Ernest 2000: 15).

Such general strategies may include:

- Representing the problem by drawing a diagram
- Trying to solve a simpler approach, in the hope that a method may be identified
- Making a table of results
- Searching for a pattern amongst the data
- Thinking up a different approach and trying it out
- Checking or testing results

According to Koshy (2000) there are 7 steps of solving problems:

(1) Read the question (underline key words to help you solve the problem).
(2) Decide which operation to use.
(3) Write down the calculation you are going to do.
(4) Work out the approximate answer.
(5) Decide if you will use a calculator.
(6) Do the calculation and interpret the answer.
(7) Does the answer make sense?

The greatest benefit is that children learn to apply the mathematics they learn in school to real-life situations. Thus, by realizing the importance and relevance of a subject that unfortunately is often boring to many learners, children will also learn to:

- Interpret instructions more effectively.
- Modify their behaviour to teach the best group solution.
- Evaluate what their own group and others do constructively.
- Gain confidence in solving problems.

Koshy (2000) suggests multiplication and division are clearly fundamental skills that need to be mastered if a child is to be considered a proficient mathematician. Key steps in learner development within multiplication will start with patterns of repeated addition. Children will move on to learning multiples tables, by building sets of objects, looking at patterns, grouping the facts for learning and playing related games. It is at this stage, when children are secure with the concept, and have a sound understanding of multiplication facts, that written methods are slowly introduced.

Ansari (2008) says the key steps for the learners with regard to division will start with informal practical activities, where they are required to carry out equal sharing tasks, distributing items among a group. Before children attempt formal division calculations, they need to understand the process of repeated subtraction, or 'the measurement model'. This model is vitally important as later written methods will be based upon it. For example, a boy was once asked, "how many times can you subtract 4 from 20?" He was busy for some time filling a whole page of his book:

20	20	20	
<u>-4</u>	<u>-4</u>	<u>-4</u>	
16	16	16	………

Finally he returned with the answer "I have subtracted 4 from 20 over 60 times, and it keeps coming to 16!" Many children hold that 2/4 is equal to 4/2. The notion is that the smallest number always divides into the largest, and confusion of how to read division sentences all contributed to predict the misconceptions which are likely to occur here. The identification of misconceptions in learners' work is a vital part of the process to move towards a focus on learning rather than teaching. Teachers need to predict the misconceptions which are likely to occur in particular pieces of work.

According to Louise (2007) teachers of mathematics need to analyze written work so that error patterns in children's work do not go unnoticed.

There are two distinct types of errors:

- Errors as a result of carelessness or wrong recall of number facts, this may not affect all questions, and may appear quite "random".
- Errors caused by applying a defective algorithm. These are of greater consequence, and are those which the teacher needs to address.

The nature of errors which occur in children's mathematics work will vary greatly and will depend on different factors.

2.3 STRATEGIES OF ERROR ANALYSIS IN THE FOUNDATION PHASE

Siegler and Thompson (2005) reckon if one considers the case of arithmetic, it is clear that in order to solve calculation problems one needs to activate numerical magnitude representation. According to Thompson (2005) children can solve arithmetic problems by retrieving the measures that are thought to capture individual differences in the fundamental representation and processing of numerical magnitude and are related to school relevant measures of mathematical achievement.

This raises the possibility that tasks such as number comparison and number line estimation can be used as educational measurements to characterize both large cohorts of children as well as provide a deeper understanding of the performance of individual children. These tests can be very useful as tools for the characterization of a learner's level of preparedness, which will in turn inform the kind of information that is being taught in school. Further, more such tests can be used to identify children who might be at risk of developing mathematical difficulties, such as developmental dyscalculia.

Wright (2000) reports that combining teaching methods and media in the classroom can be powerful because the simultaneous input of the visual, aural and tactile from different sources can create a better learning experience and help to minimize those problems more realistically for young learners. Combinations of methods like discussion, co-operative learning presentation, demonstration discovery, drill-and, problem solving, simulation, tutorial and games on the computer can also result in the educator reaching more learners in the classroom. The variation brought about by combined teaching methods can support foundation learners to achieve the learning objectives. Educators should therefore choose and combine appropriate teaching methods, resources and strategies to bring the learning matter to the learners in a realistic way.

Scholar (2008) avers that by the time they leave primary school, children will have a confident grasp of counting, numbers and mathematics, which will provide a solid platform for them to engage with algebra and other aspects of the school mathematics curriculum when they reach secondary school. If the majority does not achieve this competency at the end of primary school, a deficit emerges in the early years which become augmented when learners reach high school.

According to Abadzi (2006) there are three principles interlinked to mastering foundation phase numeracy: progressing in acquiring the number concept, the shift from concrete to abstract reasoning, and related thereto the move from counting to calculating. Gray (2008) suggests that a significant leap in understanding is achieved when a child moves away from regarding numbers as reflecting numerosity, to objects which can be manipulated according to certain laws. They understand that counting numbers does reflect numerosity.

Ansari (2008) suggests that errors in the mathematical model bodies are often approximated by spheres when calculating their properties. Another typical source of error is error in the input data. This may arise for instance, from physical measurements that are never infinitely accurate, thus it may occur that when a given problem is numerically carefully solved, the resulting solution would not quite match the observation of the phenomenon being examined. However, both should be taken into account, for instance when the accuracy to which the numerical problem should be solved, is determined.

Cotton (2010) is of the opinion that children should be introduced to the process of calculation through practical activities and they should talk about their calculation strategies before being introduced to more formal recording systems to carry out calculations on paper. Initially children will use “jottings” to record their mental methods. Children should leave primary school with:

- A secure knowledge of a number facts and a good understanding of the four basic operations.
- An ability to carry out calculations mentally and apply general strategies when using one-digit numbers and particular strategies to special cases involving bigger numbers.
- The ability to use diagrams and informal notes when using mental methods that generate more information than can be kept in their heads.
- Efficient, reliable and compact methods of calculations of the four basic operations that they can apply with confidence.
- The ability to use a calculator effectively.

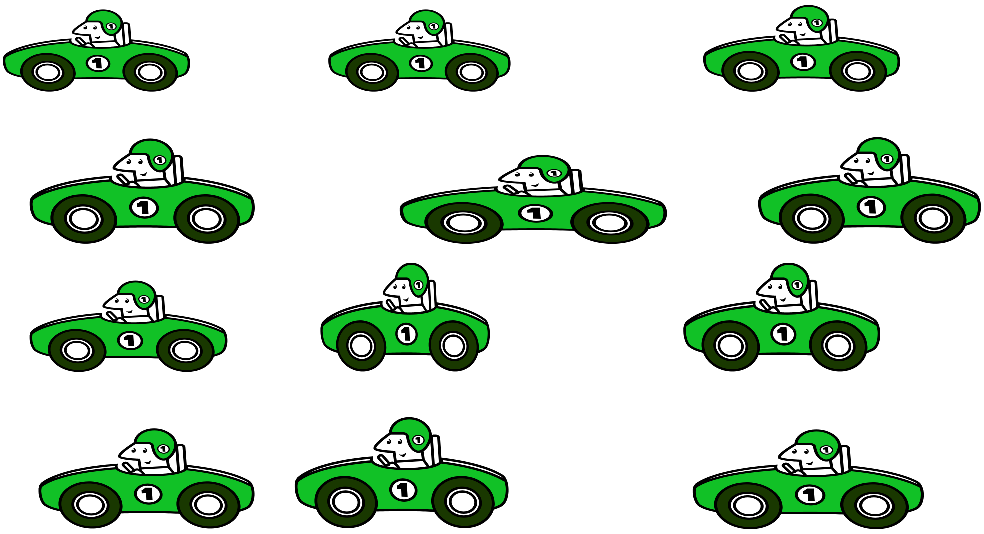
Blackwell (2004:135) states that in foundation stage:

“Counting is at the heart of calculating and in the foundation stage children will be counting objects. They will use language such as “more and less” which introduces them to ideas of addition and subtraction.” They will also compare different groups of objects to find out how many more or less there are in each group. They will also start to explore sharing by splitting groups of objects into equal groups.

Year 1: Children start to formalize ideas of addition by being introduced to the idea of counting on. They will also realize that addition can be carried out in either order. Similarly, they will be able to carry out subtraction by finding the difference between two numbers through “counting up “. This also shows them how addition and subtraction are inverse or opposite operations. Educators will support them in using the vocabulary related to addition and subtraction and work towards recording their addition and subtraction calculations using number sentences. Educators will introduce the symbols to record their practical activities rather than ask them to calculate from number sentences such as 7 + 5 =? Educators will also develop grouping activities they engage in the foundation stage by asking them to combine groups of 2, 5 or 10 objects and by sharing into equal groups. By the end of year 1 they will be able to add or subtract one-digit numbers or multiples of 10 to one-digit or two-digit numbers. They may use informal written methods to support them in these calculations.

Year 2: During year 2 educators will be expected to introduce children to four symbols, +, -, x and / as well as =. By the end of the year children would be able to find unknown numbers in number sentences such as 20 –? = 12. Mentally they would be able to add and subtract one-digit numbers or multiples of 10 from two-digit numbers and they could use practical or informal methods to add and subtract two-digit numbers. Children can then understand that addition and subtraction are inverse operations and will be able to represent multiplication as an array, for example 4x3 is the same as:

Table 3 Multiplication table using the objects



They can also use practical and informal methods to support multiplication and division including finding remainders.

Year 3**:** By the end of year 3 children will be expected to add or subtract one-digit and two-digit numbers mentally as well as use a range of written methods to record and explain addition and subtraction of two-digit and three-digit numbers. Educators will introduce them to practical and informal methods to multiply and divide two-digit numbers and teach them that multiplication and division are inverse operations. In year 3 learners will start to find unit fractions. The next stage which is introduced by the strategy is the use of partitioning, for example:

39 + 52 = 30 + 50 + 9 +2 = 80 + 11= 91.

In order to move towards formal written methods it is important to offer children practice in the second form of partitioning. This can then be represented in a more formal way as:

39 =30 + 9

+52 = 50 + 2

= 80 + 11 = 91

This leads children into the expanded column method which records the calculation as above.

Subtraction can be viewed as take away or as difference. Sometimes children will model a subtraction calculation such as 15-6 by placing 15 objects in a box and taking out 6 and then counting how many remain. This will give them 9. The final stage offered is the expanded layout. An example of this is:

657 – 289:

600 + 50 +3 500 + 140 +13

-200 + 80 + 9 _ 200 + 80 + 9

300 + 60 + 4

In moving from one calculation to the other, the partition is as follows:

600 = 500 + 100

50 = 40 + 10

3 = 3

It is important to let children develop their own informal methods to record multiplication as they did with addition and subtraction. The first formal stage therefore would be the grid method. 38 x 7 as

X	7
30	210
8	56
	266

The next step is as follows:

38

X7

210

56

266

The grid method can be extended to multiply a two-digit number by a two-digit number, for example

Table: 4 Multiplication of two-digit numbers

X	20	9	Total
30	600	180	780
7	140	42	182
			962

It is important to remember that by the end of primary school the most complex calculation that learners would be expected to carry out is a two-digit basic operation.

This is not something that many learners should be exploring before year 6. Hopefully, if the progression up to this stage is carefully managed learners will not have the same fear of long division. It is important to allow children to explore mental methods and also their understanding of division as the inverse of multiplication. For example

84 / 7

70 + 14 / 7 = (70 /7) = 10 + 2 = 12

Martin (2007) suggests that children's errors in calculations are often not because they do not understand the mathematical concepts, but rather that they get confused with how the written symbolism relates to the calculations they are carrying out. It is a problem of translation rather than a problem of mathematics. It is important to work with learners and ask them to say the number sentence as they complete it, and in this way they will begin to see the + sign as simply shorthand for saying “add”. Another activity which helps children begin to see how the multiplication sign “x” can describe an array as simply an orderly arrangement of objects can help them to understand multiplication by lining up objects in rows and columns.

- To remediate the educator should allow learners to tell him/her about their answer. The educator should provide opportunities for them to identify their own misconceptions
- The educator should not expect to teach a strategy to the entire class and have them all understand
- Identify the error.
- Determine the cause
- Collect the cause
- Establish any common links
- Identify relationships if any

Most of the misconceptions will remain hidden unless teachers make specific efforts to uncover them. A misconception is merely an opportunity for the delivery of effective teaching. (Ansari, 2011: 234)

Goswami (2008) suggests three types of mathematical concept: (1) based on the perception of objects, (2) based on the processes that are symbolized and conceived dually as process or object, and (3) based on a list of properties that acts as a concept. The perceptual method divides between those who cling to the comfort of counting procedures that at best enable them to solve simple problems by counting and those who develop a more flexible form of arithmetic in which the symbols can be used dually as processes or as concepts to manipulate mentally.

Ansari (2008) reckons perceptual thinking occurs when counting procedures are compressed into number concepts with a rich connection – for example, knowing things like 4 and 2 makes 6, so 6 take away 4 must be 2 and by using these things to derive new knowledge such as 26 take away 4 is 22 because 26 is just 20 and 6. Until learners can make a shift from process to concept, they will not be able to understand that 10 is a concept, and they will not be able to comprehend two-digit numbers and a place value.

This emerging understanding of numbers and of mathematics requires that teachers assist learners to deepen notions of counting, develop flexible and powerful means of representing numbers using apparatus such as beads, number lines and empty number lines and so on, so that learners gain confidence in using counting as a means of calculating. Counting strategies lay the basis for learning addition and subtraction.

Booth and Siegler (2005) report that many teachers may not know that even a very young infant can process non-symbolic numerical magnitudes and that we share representation for the processing of numerical magnitude. Such knowledge will allow teachers to reappraise the abilities of the children in their classrooms and think about ways in which to harness the skills that learners bring to their classrooms. Early education in numerical and mathematical processing should focus on strengthening children’s ability to process numerical magnitudes through activities that might involve estimation, comparison both with symbolic and non-symbolic representations of numerical magnitude.

Gray (2008) reports that educators need to devote time to strengthen children’s understanding of numerical magnitudes and the relationships between them. Snakes and ladders lead to improvements in children’s numerical magnitude representation. During those games children have to relate to magnitudes to one another and move up and down the number line. Such activities are effectively engaging for children and thus the strengthening of numerical magnitude processing skills is a by-product of game-like activities. Such games will invariably involve culturally-appropriate materials.

Furthermore, the reason for this study to be conducted was due to the numerical errors that the learners make in the foundation phase.

2.3.1 Understanding of the numerical magnitude and challenges

Ansari (2005) says that one of the key challenges in learning about numerical magnitudes is developing understanding that numerical magnitudes are abstract and that different external formats of presentation can all refer to the same internally represented numerical magnitude. Thus part of early mathematics should focus on helping children to integrate across various formats for the representation and processing of numerical magnitudes in order to build an increasingly abstract representation of numerical magnitude.

In this context educational activities to help children understand that symbolic stimuli such as digits refer to numerical magnitudes that can be represented non-symbolically are important. This could be achieved by educational activities in which children are asked to match different external ways of representing numerical magnitudes with one another (such as matching digits with dots). Another way is to ask children to identify the odd number. Specifically, in such an exercise children are presented with 7 dots, the numeral 7, 7 marks on a page, 7 pictures of fruit and the numeral 10 and they have to identify that the numeral 10 is the odd one out (since it differs from the other stimuli in numerical magnitude). Goswami (2008) states that current standardized tests of numerical and mathematical processing do not routinely contain items that tap into the basic representations of numerical magnitude. Most of the available standardized tests consist of either calculation or so-called 'Mathematical Reasoning' tests. For the calculation tests, children are presented with a series of calculation problems in all four arithmetic operations and are required to solve these problems under timed or untimed conditions.

For the tests of ‘Mathematical Reasoning’ children are asked to solve diverse items such as judging the order of items of varying sizes or solving illustrated word problems. What is currently lacking from educational assessments of children’s numerical and mathematical skills are ”processing measures”.

In other words, that which is lacking are measures that capture something fundamental about the basic representation children require in order to process numerical information in the context of calculation and ‘Mathematical Reasoning’. Given that standardized measures of numerical magnitude processing are currently, to the best of Goswami’s knowledge, not available, there are some design issues that need to be taken into account. One important design choice is whether to use computers or pencil and paper as the response modalities.

One clear advantage of computerized measurement is that reaction-time data can be acquired. However, computers are expensive and require specialized software. Furthermore, when working with individuals who may have had only limited exposure to computers, the use of computers to administer tests may lead to difficulties which may preclude the accurate measurement of children’s competencies. While pencil and paper tests do not allow the measurement of item by item reaction times, they can nevertheless be used to measure how many items are completed within a given time period.

According to Ansari (2008) most of the current research into the neuroscience of numbers is focused on the neural correlates of the basic representation of numerical quantity in the brain. There is also a growing exploration of the relationship between numerical processing, children who exhibit normal intelligence but present a specific and persistent difficulty with calculation and mental arithmetic, have what is called developmental dyscalculia. In this case there is a pattern of structural and functional alterations in the intra-parietal sulcus (IPS) and the prefrontal cortex in the brain of children who had low birth weight and who exhibit calculation difficulties as compared with developmental dyslexia in the brain of children of similar low birth weight but with normal scores in a calculation test.

Children with specific calculation difficulties were found to have grey matter in the left IPS. Children with developmental dyscalculia have less right-parietal grey matter and grey matter abnormalities in regions of the frontal cortex. Furthermore, to establish that a brain region is specifically involved in a particular cognitive process, one needs to compare the engagement of that region during the cognitive process of interest with its activation in response to an infinite number of other stimuli categories and processes.

Ginsburg (2000) is of the opinion that children arrive at school with a variety of backgrounds and experiences with more mathematical knowledge than was previously thought, regardless of different socio-economic situations. Children may not immediately communicate this understanding, but children begin the process of making sense of their world at a very young age, and this includes mathematical sense. One of the difficulties in trying to improve the teaching of early mathematics is that teachers tend to underestimate the capabilities of young children when it comes to mathematics and may not have the knowledge to focus on important mathematical experiences. For example, teachers may not realize that developing a solid understanding of the quantitative value of a number is as important as number identification and counting features.

Children need to see mathematics as sensible, and they do so when the mathematics they are learning in school connects with their intuitive sense of mathematics and with the understanding of mathematics that they bring with them to the classroom. If we could help children to build upon and extend the intuitive modeling skills that they apply to basic problems as young children, we would have accomplished a great deal by way of developing problem-solving abilities in children in the primary grades.

Halloway (2010) reports on complex mathematical skills that are formally taught in school such as arithmetic, and particularly about how individual differences in arithmetic competence modulate the neural processes recruited by solving arithmetic problems. Being fluent and efficient in performing basic calculations has been regarded as an important building block for the development of mathematical skills. Some children are known to have difficulties in mastering these basic number combinations, but the neural correlates of these developmental impairments remain unknown.

Pipping (2005) argues that errors arising from negative transfer are well-known in the literature on error analysis in mathematics education. They have become of special importance in studies of the problem-solving process. Inadequate flexibility in decoding and encoding new information often means that experience with similar problem solving will lead to habitual rigidity of thinking. In such cases learners develop cognitive operations and continue to use them even though the fundamental conditions of the mathematical task have changed. Some aspects of content or of the solution process persist in the mind, inhibiting the processing of new information. The role of this study was to inform learners about the numerical errors that they encounter in foundation phase mathematics.

2.3.2 An effective mathematical learning environment

According to Ginsburg (2000) an effective mathematics learning environment is an environment that:

- Promotes positive beliefs and attitudes towards mathematics;
- Values prior knowledge;
- Makes connections between that knowledge and the world of the child;
- Encourages the establishment of a community of mathematics learners;
- Focuses on important mathematical concepts or big ideas;
- Explores concepts through problem solving;

- Includes a variety of learning resources;
- Is supported by the strong roles of teacher, principal, and senior administrator;
- Is supported by the home.

Thomas (2001) suggests that a sense of mathematical community is fostered when teachers design classroom activities and groupings that promote intellectual sharing and value different ways of solving problems. Then learners can build on one another's knowledge through demonstrations, models, and questioning. When children have the freedom to talk, they can explore concepts, have their concepts challenged, and then make their own decisions about their learning. The community of the classroom must value these opportunities, and the teacher promotes community building by providing guidance about how such interactions can occur and by facilitating these interactions. The environment for learning must allow students to feel safe and willing to take risks as well.

In a classroom community of mathematics learners, learners will

- See, hear, and feel mathematics;
- Learn mathematics through doing and talking about it;
- Have enough time to solve problems and share with others;
- Be active and enthusiastic about learning mathematics.

Ansari (2008:85) states that teachers and parents need to learn together and work together to support each child's educational journey. Teachers can discuss effective mathematics strategies with parents, who can then better understand how their children learn and can gain confidence when interacting with their children. Furthermore, parents who are helped to understand the content and pedagogy of today's mathematics teaching and learning are in a better position to assist their children's growth and development.

Teachers and principals need to find ways to increase school-home partnerships and increase parental involvement in the education of their children. Some suggestions by Ginsburg for strengthening the home-school partnership include:

- Determining the child's prior home mathematical experiences.
- Suggesting activities for parents, such as games that encourage success and practice mathematics.
- Helping parents understand the content and pedagogical processes of today's mathematics classroom.
- Providing ongoing communication regarding student progress.
- Preparing take-home mathematics kits that may include activities, literature, software, and manipulative focused on a particular topic.
- Hosting school / family mathematics sessions emphasizing mathematics activities for families.
- Providing a calendar outlining mathematics activities for families.
- Promoting the distribution and use of a parent guide on helping children learn mathematics.
- Involving libraries and book stores by asking them to promote mathematical literature.

2.3.3 THE IMPORTANCE OF TIMING – MEASURES OF BASIC SYMBOLIC AND NON-SYMBOLIC NUMERICAL MAGNITUDE PROCESSING

Ansari & Holloway (2009) reported that, there are two issues that are important to carefully consider when designing measures of basic symbolic and non-symbolic numerical magnitude processing. The first one is timing. In the domain of reading it is well recognized that fluency is an important predictor of literacy development and tests, where a number of words or letters are read in a limited time window in assessments. The same is true of mathematics, where the efficient ability to access numerical magnitude information from numerical symbols is likely to aid calculation efficiency.

Thus, tests that assess numerical magnitude processing in children should test these abilities under timed conditions. When symbolic representations of numerical magnitude are used, timed tests tap into the ability of children to quickly access the semantic meaning (numerical magnitude) that the symbols represent and then to perform a comparison of the semantic representation of the two numerals. In non-symbolic numerical magnitude comparison, a timed test assesses the efficiency with which children can deploy their approximate number system to generate approximate number representations and compare them. If such tests are administered in an untimed fashion they run the risk of introducing large strategic variability between children and ceiling (perfect levels). Thus, it is important to pay attention to a number of items correctly/timed, rather than a number of items correctly alone. This is particularly true for the number-comparison or dot-estimation type of items.

Reaction times can also be used to construct within-task measures of individual differences such as individual differences in the numerical distance effect. In other words, instead of simply using a number of items correctly solved/timed, one could separately compare a number of items correctly solved/timed for small and large distances. If computer technology is available the reaction times for each comparison should be measured to be able to quantify the effect of reaction time for individual participants. Having made the point that timing the test administration is important, this may not be true for all items on a test. For example, number line estimation does not require timing, since the main variable is the difference between the veridical position of a numerical magnitude on the number line and the position indicated by participants. Furthermore, if counting is assessed children should not speed, otherwise they resort to estimation strategies.

Another important consideration when designing tests that require speed/time responses is to acknowledge that reaction times vary dramatically across chronological age and differ between literate and illiterate populations and thus careful piloting needs to be undertaken to decide what the optimal time limit is in any particular population to avoid ceiling and floor effects. This study intends to explore the strategies of numerical error analysis that would assist the teachers of the foundation phase to analyze errors in mathematics.

2.3.4 THE IMPORTANCE OF CHOOSING ITEMS – EARLY MATHEMATICS INCLUDES NON-SYMBOLIC ITEMS

Abdzi (2006) suggests that the key important thing when designing an assessment of early numerical abilities is to carefully choose items and to ensure that enough items are presented to tap into particular competencies in a valid and reliable fashion. This is the key that tests early mathematics which includes non-symbolic items that tap into children's ability to match and compare non-symbolic numerical magnitudes. This is important for the following reasons:

Non-symbolic representation is foundational and can be found even in young infants and non-human pirates. It is therefore essential to ascertain that children have these foundational competencies.

When working with children who are either illiterate or have had little access to formal schooling, non-symbolic representation of numerical magnitude can help to assess their level of understanding of numerical magnitude independent of their knowledge of cultural, symbolic representation.

Another important aspect to consider, according to Bornman (2010), is how the difficulty of items is manipulated. In most standardized tests difficulty increases as a function of the test trial, such that simpler items are presented before more difficult ones. There is a need to have a sufficient number of items to be able to assess children's ability to use measures that go beyond mean accuracy. For example, in number line estimation enough items need to be presented in order to establish whether a child's mapping of numbers onto space can be best characterized by a linear or a logarithmic function. A final point concerns the importance of choosing items on the basis of peerreviewed evidence. Only items should be included that have been found to be reliable correlates or predictors of individual differences in children’s numerical and mathematical understanding.

Goddard (2005) reckons the representation of numerical magnitude might be innate and therefore detectable early in infants. There is a large and growing body of evidence demonstrating that very young human infants as well as newborns are sensitive to numerical magnitude. In such studies infants are typically presented with a series of slides that display a certain number of objects. Throughout the experiment the infant’s looking time or in very young babies sucking time is measured. After a certain number of repetitions of the same non-symbolic numerical magnitude, the infant will start looking away from the display.

This is called habituation. The infant has become habituated to the repeated display of a particular numerical magnitude. When this happens the experimenter will present the infant with an alternative display of a new, different non-symbolic numerical magnitude to the old numerical non-symbolic magnitude to which the infant became habituated.

Harlberda (2008) suggests what the researcher wants to uncover in such an experiment is whether infants look longer at the new compared to the old non-symbolic numerical magnitude. If infants do indeed look longer at the new numerical magnitude then they can say to have noticed a change in the number of objects displayed. In the early experiment of this nature, infants were found to be able to discriminate between small numbers of dots, such as two and three dots. Moreover, infants were also found to be sensitive to simple arithmetic transformation, such as adding one doll to another doll or removing one doll from a display of two dolls.

Dehaene (2003) reports that brain-damaged patients indicated, when the inferior parietal lobes are damaged, specific deficits in number processing. Functional brain-imaging showed the parietal regions to be consistently in numerical tasks. One functional anatomical model of numerical cognition, the “triple code model” posits that modality-specific codes of numerical information are converted into an abstract and a modal code for number and that these representations are held within the parietal lobes. Furthermore, this model posits that exact arithmetic facts are stored in a verbal format in left-hemisphere perisylvian areas of the brain. Thus, according to the triple code model, there are at least two routes to solving mental arithmetic problems: a direct route involving rote retrieval of arithmetic facts (like simple addition and multiplication) from the language-related frontal regions and an indirect semantic route involving the quantity code in the parietal lobes for solving subtraction and complex addition.

Ansari et al (2007) suggest that in estimation tasks participants assess children’s ability to approximate a quantity. Similar to the symbolic magnitude comparison task, children need to have an approximate understanding of the non-symbolic magnitude expressed by the symbolic magnitude in order to successfully complete estimation tasks. It is suggested that being more skillful in approximating magnitudes limits the range of possible answers from which children need to choose, which in turn might decrease the likelihood of making mistakes and increase arithmetic proficiency.

According to Bornman (2010) this data suggests that the precision of the approximate representation of numerical magnitude increases over the early months of life. This strengthens the notion that we come equipped to life with foundational competencies in order to allow children to learn higher-level, cultural mathematical skills such as mental arithmetic. This study was interested to investigate the strategies in numerical errors analysis to assist foundation phase learners in the errors they encounter in mathematics.

2.3.5 SHORT-TERM (OR WORKING) MEMORY

Bligh (2000) reports that memories are connected through a central processor called ‘central executive’ which together with two ‘slave systems’ make up the ‘working memory’. One of the slaves systems is an articulator loop and phonological store, the other slave system is a comparable one for visual information. When various tasks are practiced enough, items are lumped or ‘chunked’ together, so the working memory passes them off as one. The association and quick connection of items form the basis of automaticity.

When various items or tasks are recalled automatically, the mind makes little effort to bring them up. Thus, attention and the limited working memory span can be directed towards problem solving. Working memory has an ‘upper limit’. The amount of information a person can learn at any given time is relatively small. If one tries to put too much information in short-term memory, some of the information will ‘fall off the table ’and never make it to long-term memory. In order for information to pass from short-term memory to long-term memory, one has to go through the process of rehearsal and memorization of small amounts of information at a time. The upper limit for working memory increases as the child ages.

Ginsburg (2000) says that children who are 2 years old can remember two ‘chunks’ of information. Pre-adolescents can handle five chunks of information. Working memory is especially critical to mathematics learning because mathematics lessons place frequent demands on working memory. Good working memory has therefore been shown to be correlated with successful mathematics learning. Those who have poor mathematics skills have problems with working memory. Using multiple strategies such as sorting required grocery items into related groups and using this clustering to aid recall, make children able to recall even more information. These strategies on how to implement numerical error analysis would help other researchers who encounter mathematical problems in the foundation phase.

2.3.6 META COGNITION

According to Goswami (2008) ‘meta cognition’ can be understood either as knowledge about or regulation of cognition. Knowledge about cognition means to have relatively stable information about one’s own cognitive processes. This knowledge develops with age and performance on many tasks and is positively correlated with the degree of one’s meta knowledge. This includes the planning before beginning to solve a problem and the monitoring and assessing thereof. The presence of this has a positive impact on intellectual performance and the absence a strong negative effect. It also includes meta memory. The concept of meta cognition was integrated into Singapore’s primary mathematics curriculum. It is identified as a key component of mathematical problem solving. The teachers can help their learners find their best way of working if the teachers:

- Begin each lesson with an overall picture of its contents, using both oral and visual stimuli.
- Thoroughly explain the logic behind each method
- Offer alternative methods
- Put the work into a familiar context, or relate it to the learners’ own experiences and existing knowledge.

Similarly, Singapore’s Ministry of Education (2006: 356) gives the following list of activities which may be useful to develop a learner’s meta cognition:

- Expose learners to general problem-solving skills, thinking skills and heuristics, and how these skills can be applied to solve problems.
- Encourage learners to think aloud the strategies and methods they use to solve particular problems.
- Provide learners with problems that require planning and evaluation
- Encourage learners to seek alternative ways of solving the same problem and to check the appropriateness and reasonableness of the answer.
- Allow learners to discuss how to solve a particular problem and to explain the different methods that they use for solving the problem.

Ansari (2005) argues that the concept of helping learners monitor their own capacity to learn and to engage in mathematical thinking received little attention thus far. Teachers are typically not taught to enhance this skill. However, developing meta cognition and meta memory even in young children seems to be a way to enhance the learning and reasoning skills of the children. However, this study will assist teachers to be able to identify learners with numerical errors in the foundation phase

According to Caldwell (2008:78) human beings are geared to think mathematically and all children are born with a mathematical mind. In the early grades children can learn emergent numeracy skills through play by manipulating concrete objects and they can get a sense of colours, size and texture. Learning concepts at a concrete level (use of the five senses) must happen first before reading and writing of a mathematical sum happens. Children must be encouraged to:

1. Sort out and match objects in accordance with colour, size, length
2. Count freely – no limitations
3. Play freely with 3D objects (that allows them to learn the geometric language) i.e. top, under, above, next to and opposite
4. Sing, count and rhyme
5. Have a sense of mass and volume by being encouraged to engage in planned sand and water activities.

Ansari (2008: 256) suggests mathematics-specific strategies for teachers to assist learners experiencing difficulty in mathematics and these strategies include:

- Rewrite problems in simpler language
- Point out key words
- Minimize copying from boards by using handouts
- Have learners do math on the board
- Have learners repeat instructions

- ❖ Use boxes, circles, lines and so forth to separate one problem from another
- ❖ Relate to real life situation
- ❖ Use colour coding whenever possible
- ❖ Reinforce mathematics in other contexts
- ❖ Use games such as dominoes and dice.

For effective intervention and remediation to occur, teachers should:

- ➢ Recognize that all children can learn mathematics
- ➢ Provide a supportive environment that fosters positive attitudes towards mathematics
- ➢ Build on the individual's prior knowledge
- ➢ Ensure the early identification of learning challenges in mathematics and provide interventions to meet the needs of children with learning needs.
- ➢ Use assessment strategies to determine individual learner needs and develop appropriate action plans
- ➢ Use strategies that address the needs of exceptional learners
- ➢ Use a wide range of remediation strategies
- ➢ Participate in professional development activities that address the special needs of children
- ➢ Involve families in supporting mathematics activities to further enhance positive attitudes towards mathematics.

Wessels (2008: 35) suggests how teachers can teach data handling by:

1. Collecting the data to answer questions
2. Sorting the data while giving reasons
3. Using representations like pictures, cluster and bar graphs, and number line plots
4. Describing a collection of information explaining reasons for grouping and answering question about the collection
5. Reading and interpreting data in tables and lists.

Traditional instruction in data handling creates the problems of too much focus on procedural knowledge that is not integrated, resulting in learners learning isolated and fragmented skills. Data handling should be taught with the view to enable learners to think statistically and interpreted data critically, whilst being able to communicate their opinions and reasons.

According to Kuhne (2008) “solving problem” is a very important skill to be developed in all learners. There is a difference between the terms “learning to solve problems” and” solving the problems” The former involves learners engaging in finding various strategies that could be used to solve a problem while the latter is basically an attempt to solve problems which involves learners learning what is intended for them to learn. Problems are used as vehicles for learning the intended mathematics. In problem solving the focus shifts from the solution of the problem to what mathematics is learnt, while wrestling with the problem. Facilitating the transition from concrete and informal ways of solving mathematical problems to more efficient, abstract and formal ways of teaching happens through four distinct developmental stages:

Stage 1: Emergent numeracy
Stage 2: Counting and calculating
Stage 3: Number-based calculating
Stage 4: Advanced calculating – use of algorithms

2.3.7 Learning in the early grades

Van Heerden (2006) reckons that the need to develop critical and creative skills is fundamental in the early grades. Teaching involves many challenges, among those is meeting the needs of different children. In order to achieve this, teachers need to define thinking strategies.

Learning in the early grades is based on **:**

1. Reality principles
2. Activity principles
3. Interaction principles
4. Integration principles
5. Guidance principles
6. Starting with real life context
7. Learning by doing
8. Listening, sharing and reflecting
9. Levels of understanding

According to Ansari (2009) it is important to have a strong sense of numbers in order to understand the place value, column sums and use of algorithms. Young learners must be given ample opportunity to engage in counting activities. Learning in the early grades involves a lot of concrete experience where learners may count fingers, concrete objects and use illustrations to get the answer. Gradually they must be introduced to advanced methods such as breaking and building numbers (that is the two-digit range) by using a counting frame or number cards. They must be able to make sense that 28 = 20 + 8 and not 2 + 8. Once learners have had enough experience in counting beyond 100 and 1000, breaking up and recomposing 2- and 3-digit numbers, their understanding of place value would be extended. A solid foundation in learning decades 10's, 100's and 1000's will allow children to understand and make sense of column arithmetic and the use of algorithms which they will encounter in the intermediate phase.

Koch (2007) suggests that the teaching of numeracy must be an interactive process. Children must be given adequate opportunity to express their responses verbally. Children must be able to explain why, what and how they have reached a conclusion. Teachers need to be careful not to mark the children's work as wrong, as this discourages chances to improve. Therefore this study would like to help teachers who need to implement numerical error analysis in their schools.

According to the Department of Education (2007) mental mathematics means different things to different people. It is important that children develop a strong sense of numbers. In other words, mental mathematics means having a strong sense of numbers, without having to resort to paper and pencil.

2.3.8 Mental mathematics strategies include

- Rote counting
- Rational counting
- Skip counting
- Breaking up numbers
- Checking answers
- Making own sums
- Playing mathematical games, for example by using games on the computer or learners can play mathematics games with the help of their teachers.

Bornman (2010) suggests that word problems also help extend a learner's ability to develop a strong sense of numbers. The building up and breaking down of numbers help learners to understand the concept of doubling, halving and the four basic computational skills, being addition, subtraction, multiplication and division. Doubling lays a foundation for multiplication by 2, whilst halving lays the foundation for division by 2. Doubling lays the basis of multiplying and dividing by 2. Learners need a lot of practice to be able to double, and later halve the numbers more easily. Learners may use their fingers and initially later work in pairs if more than two hands are needed.

The building up and breaking down of numbers assist learners to:

1. Visually see the value of each number
2. See how the number symbol is created by the use of groupings

3. See what the number symbols look like
4. Understand what ‘1’ stands for in numbers and in ’11 to 19’
5. Practice writing numbers

Essential mathematical skills to be taught in the foundation phase are:

1. Counting
2. Calculating
3. Reasoning
4. Estimation
5. Problem solving
6. Investigating
7. Interpretation
8. Describing
9. Analysis
10. Communicating

These skills are to be demonstrated by using examples. Learners need to understand numbers and have a good number sense to understand how numbers are used in their everyday life. This study seeks to provide teachers with the possible strategies to determine numerical errors in the foundation phase.

Murray (2012: 109) suggests that, “mental mathematics is important to develop a variety of mental strategies that can help the children to be confident mathematicians. Learners who can do mathematical computations quickly and accurately in their heads are seen as being good at mathematics”. According to Ginsburg (2000) poorer performing learners use less efficient strategies, and the development of mental computation makes use of a less efficient strategy approach to help them move forward. Learners with a number sense know the relative size of numbers and use a variety of computation strategies to solve a problem.

Clotty (2011:45) States that, “this enables them to approach mathematics with a real sense of what they are being asked to do, making them more confident and presents them with a greater ability to work mathematically”. For example, the brain is able to calculate and reason. Learners develop the ability to use both sides of the brain when regularly doing mental mathematics and mental mathematics improves mental co-ordination.

According to Ansari (2005),

(1) Mathematical skills require:

- ❖ Understanding (what you are doing)
- ❖ Applying (using computational skills to solve a problem)
- ❖ Reasoning (reflect and think on how they solved the problem)
- ❖ Engaging (children who engage use their understanding of the mathematics learnt to solve a problem)
- ❖ Computing (performing procedures for basic operations)

(2) Benefits of mental mathematics:

- Both hemispheres of the brain are stimulated
- Mental mathematics develops speed, accuracy and gradually the ability to do calculations involving higher-order numbers.
- It stimulates a learner’s interest in mathematics.
- It Increases a learners’ confidence, reasoning, creativity and organizational ability.

Sousa (2008:13) reports that, “the volume of the brain continues to grow until puberty. At puberty, when the brain is nearly at its full adult size, gray-matter volume begins to decrease because unneeded and unhealthy neurons are pruned away.” Parts of the brain associated with basic functions mature early. Motor and sensory functions (taste, smell and vision) mature first, followed by areas involved in spatial orientation, speech and language development, as well as attention (upper and lower parietal lobes).

Nunan (2010) Suggested that, later to mature are those areas involved in executive functions, creativity, problem solving, reflection, analysis, attention and motor co-ordination (frontal lobe). Most areas of the temporal lobes mature early. These areas are involved mainly in the auditory processing. Maturing lastly is a small section of the temporal lobe involved in the integration of memory, audiovisual association, and recognition of objects.

According to Goldin (2008) children do not always learn what teachers think they learn while working with concrete materials. The reason is that a decontextualised representation is not the same as a genuine abstraction. Children received many kinds of 'manipulative' (concrete materials) with which to play such as toys and blocks. When children sort the toys, their activities are described by using set theory terms such as 'finding the intersection of two sets'. Ansari (2005) revealed that, learners who learn mathematics using manipulative materials usually outperform learners who do not use manipulative. This may be partly because moving manipulative may create groups which can facilitate the learner's understanding.

Caldwell (2008) suggested that, this result holds across grade levels and mathematics topic areas. Manipulative also improve the learner's attitudes to mathematics and increase their performance on problem-solving tasks. However, the use of manipulative does not automatically guarantee success; how manipulative are used matters. Teachers must be aware of whether or not learners are reflecting properly on their actions while they use the manipulative and not just use them in a rote fashion.

Ginsburg (2000) suggests that manipulative (concrete materials) provide learners with tactile experiences to help them model, describe and explore mathematics. Manipulative include such things as pattern blocks, base-ten blocks, interlocking cubes, and many others. They provide concrete ways for learners to give meaning to new knowledge, and also help the learners to describe mathematics and make the dialogue between learners and the teacher more accessible. Children's mathematical thinking becomes more transparent when teachers observe children using manipulative and listen to children's conversation.

According to Ginsburg (2000) guidelines need to be established on how manipulative should be selected for classroom use, and teachers also need guidance on how to best use manipulative effectively in the early mathematics programme. Good lessons using manipulative do not just happen; they need to be thoughtfully prepared.

When selecting manipulative, teachers should

- Be certain that the manipulative chosen support the selected mathematics concepts and the big idea;
- Have enough of the manipulative available so that all learners can become active participants in the activity;
- Provide initial opportunities for learners to become familiar with the manipulative;
- Communicate classroom procedures to learners.

Ansari (2008) reported that, when planning activities with manipulative, teachers should

- Use a manipulative in such a way that learners use it as a "thinking tool" enabling them to think about and reflect on new ideas;
- Recognize that individual learners may use the manipulative in different ways to explore mathematics;
- Avoid activities that simply ask children to copy the actions of the teacher;
- Allow learners to use manipulative to justify their solution as well as solve the problem;
- Take time to become familiar with the manipulative chosen;
- Provide opportunities for learners to explore the same concept with a variety of manipulative.

Goldin (2008) suggests that the successful classroom implementation of manipulative is related to:

(a) Professional development: Teachers to have extensive training in the use of manipulative.
(b) Design of original lessons: Teachers design their own lessons –- this is an indicator of the amount of time a teacher spends preparing the lesson.
(c) Strategic classroom planning: Teachers assign learners to groups and arranged manipulative after considering how their learners would use the manipulative within their particular group situation. However, this study intended investigating the strategies of numerical errors in the foundation phase.

Jarvin (2009) suggests that, given the mixed results of the effectiveness of manipulative, teachers should

(a) Minimize the use of manipulative that are highly concrete and rich in perce ptual detail
(b) Minimize the use of toys or other manipulative that are familiar to the learners because they may not serve the expected purpose.

According to Halberda (2008) the use of these types of manipulative might engage the learners only at the level of entertainment and at the expense of learners developing a deep understanding of a mathematical concept. One could summarize that manipulative rich in perceptual detail add another ‘chunk’ of information into a very limited working memory space when the child is learning a new concept. Furthermore, when manipulative are used in the classrooms, teachers should take the time to help learners make a connection between their intuitive sense of mathematics and the formal language of mathematics. Some manipulative are more effective in facilitating the transfer of mathematical knowledge than others, like the use of strings with beads in counting and the number line. Learners can manipulate the beads by moving them backwards or forward or moving them from left or right on the number line. This study intended to assist teachers struggling to implement strategies of numerical errors analysis in mathematics of the foundation phase.

2.3.9 THINKING STRATEGIES FOR LEARNERS

According to Ansari (2008) when introducing a new strategy, it is best to use the chalkboard or overhead projector to provide learners with an example of a computation for which the strategy works. Encourage them to explain the strategy to the class with your help. If not, share the strategy yourself. Explaining the strategy should include anything that will help learners to see its pattern, logic, and simplicity. It can constitute concrete materials, diagrams, charts, or other visuals. The teacher should also "think aloud" to model the mental processes used to apply the strategy and discuss situations where it is most appropriate and efficient as well as those in which it would not be appropriate at all.

Hasym (2002) states that, in the initial activities involving a strategy, the teacher should expect to have learners do the computation the way the teacher modeled it. Later, however, the teacher may find that some learners employ their own variation of the strategy, and if it is logical and efficient for them, so much the better. The teacher's goal is to help learners broaden their repertoire of thinking strategies and become more flexible thinkers, not to prescribe what they must use.

According to Murray (2011) the teacher may find that there are some learners who have already mastered the simple addition, subtraction, multiplication and division facts with single-digit numbers. Once a learner mastered these facts, there is no need for them to learn new strategies. It is the frequency rather than the length of practice that fosters retention. Thus daily, brief practices of 5 to 10 minutes are most likely to lead to success. Once a strategy has been taught, it is important to reinforce it. The reinforcement of practice exercises should be varied in type, and focus as much on the discussion of how learners obtained their answers as on the answers themselves.

Mark (2009) suggested that, the selection of appropriate exercises for the reinforcement of each strategy is critical. Exercises should be presented with both visual and oral prompts and the oral prompts that the teacher presents should expose learners to a variety of linguistic descriptions of the operations. For example, 5 + 4 could be described as:

- The sum of 5 and 4
- 4 added to 5
- 5 add 4
- 5 plus 4
- 4 more than 5
- 5 and 4.

Allow learners more time as they learn to apply new strategies, and reduce the time as they become more proficient. Allow more time, and gradually decrease the waiting time until learners attain a reasonable time frame. In other situations, it may be more effective when all learners participate simultaneously and the teacher has a way of checking every one's answer at the same time.

There will be some learners who experience considerable difficulty with the strategies assigned to their grade and who require special consideration.

Ginsburg (2000) suggests that the teacher may decide to provide these learners with alternative questions to the ones others are expected to do, perhaps involving smaller or more manageable numbers. Alternatively, the teacher may just have the learners complete fewer questions or provide more time. There may be learners who do not have command of the basic facts.

Pardesi (2008) revealed that, for the teacher, this may mean going back to strategies at a lower grade level to build success, and accelerating them vertically to help learners catch up. For example, if the learners are in Grade 3 and yet do not know the addition facts, the teacher can find the strategies in the Grade 2 curriculum guide. For the learners who are however, more intellectually mature, the teacher can immediately apply those same strategies to tens, hundreds and to estimation of whole numbers. The reason for this study to be conducted was to investigate the strategies of numerical analysis in the foundation phase mathematics.

According to Murray (2011: 28-29) “the components of an effective professional development program is for teachers of mathematics in all grades. Effective professional development must”.

- Connect with the learner and the curriculum; (professional development should provide experiences that will enable teachers to determine what learners need to know, followed by processes that help learners to effectively construct this knowledge. In other words, professional development should connect effectively with the curriculum. Teachers need to know how to choose and create resources in a manner consistent with the curriculum. They should also be encouraged to look critically at resources that synthesize mathematical concepts and bring out the big ideas. To a significant extent, good mathematics rests in the teacher ability to transform the mathematics content into forms that are accessible to learners). Abadzi (2006) reported that;
- Value the teacher as “learner” and help to foster a community of learners; (professional learning for teachers should model the characteristics of good learning for learners. These include: valuing a teacher’s prior knowledge, mathematics education, learning through sharing, and allowing time to practice and reflect. In a collaborative learning culture, teachers should be supported to work together within their schools and boards to acquire a richer knowledge base and develop leadership skills).
- Allow professional development to take place both through structured and informal interaction and through group discussions among teachers. Frequently however, these discussions need to be stimulated by an external resource person or consultant who has knowledge of mathematics teaching and learning and who has experience in the elementary context to help guide

the discussion. Discussion among teachers who teach the same grade and share many experiences and issues can help teachers make sense of their experiences and feel less isolated.

- Connect with mathematics content and pedagogy; (the ability to make mathematics accessible to learners relies on a teacher's awareness of those aspects of mathematical content that are particularly relevant to his/her teaching. For improving the teaching and learning of mathematics and increasing a teacher's pedagogical content knowledge, teachers need to know the mathematics that they teach as well as why they teach it. Teachers who are going to engage their learners in mathematical problem solving need to fully understand the mathematics that may arise as learners explore different types of solutions.(Koch 2007 states that).

According to Warren (2005) professional development should ;

- Use a variety of models of professional development;

(1) Coaching and mentoring
(2) Action research teams
(3) Study groups
(4) Case studies
(5) Lesson studies
(6) Professional networks
(7) Book clubs

According to Goswami (2008) effective professional development material should;

(1) Provide background material knowledge;
(2) Highlight the big ideas and key concepts of each activity;
(3) Offer suggestions on how to integrate strands;
(4) Explain mathematical concepts clearly;
(5) Demonstrate effective teaching strategies;
(6) Include ways to intervene in order to support learners at both ends of the spectrum;
(7) Include ideas for ongoing assessment and evaluation.

- Provide a plan that will promote sustainability; (professional development should be long-term, with several short-term, realistic, manageable goals in mind. Long-term, sustained professional development facilitates change and improvement. Furthermore, professional development program need to be continually evaluated and revised according to well considered indicators.

2.4 THEORETICAL FRAMEWORK

Creswell (2003) states there are three critical framework elements to consider when designing a study. These elements are philosophical assumptions about what constitutes knowledge claims, strategies of enquiry and methods. To situate this study within the quantitative research approach, the theoretical framework is discussed from the viewpoint of the strategy/method of inquiry and the procedure for obtaining collected data and analysis in the way this study undertook.

The researcher chose (1) Cognitive Dissonnance Theory and (2) Cognitive Fit Theory to focus theoretical perspective when interpreting the available data. Cognitive Dissonnance Theory applies to all situations involving attitude formation and change.

Kaufman (2008) reported that it is relevant to decision-making and problem solving. It is a discomfort caused by holding conflicting cognitions (ideas, beliefs, values reactions). Becoming aware of how conflicting beliefs impact on the decision-making process is a greater way to improve the ability to make faster and more accurate choices.

For example, consider someone who buys an expensive car but discovers that it is not comfortable on long drives. Dissonance exists between their beliefs that they bought a good car and that a good car should be comfortable. Dissonnance could be eliminated by deciding that it does not matter since the car is comfortable when used mainly on short trips (reducing the importance of the dissonant belief). The study of the mind focuses on how information is presented, processed and transformed within the nervous system.

Flavell (1979), a neuropsychological researcher, states that the brain stem is critical as it focuses on attention and consciousness, and specifically as the brain stem conjoins with the reticular formation, or the reticular system. All neural messages flow between the brain and the effects (glands and muscles) and are registered in the reticular system. The messages are also modified and regulated here. Children, who have been diagnosed as hyperkinetic, are unable to focus attention or do cognitive tasks. The learners are likely to make numerical errors when they are exposed to the problem of calculating mathematics. They are highly distractible, as a malfunction in the reticular system prevents the screening of inappropriate messages coming in from the sensory receptors. The malfunction is caused more likely by a biochemical disturbance than by brain stem pathology.

According to Botha and Kruger (2005:12) “a neuron consists of a cell body, an axon and dendrites, where the cell body consists of a membrane, filled with protoplasm. It contains a nucleus, which determines the survival and functions of the neuron.” Therefore this study was interested to explore the effect of neorology on numerical errors that the learners encounter in the foundation phase. Cohen (2007) reported that, the cell body has offshoots on both sides. On one side, these offshoots (called dendrites) look like the dead branches of a tree. On the other side, there is only one offshoot that is much longer and thicker than others. This is the axon. The axon also has dendrites called telodendria at its end. They differ from the dendrites of the cell body as they have thickenings at their ends, called terminal buds.

According to Goldin (2000:87) “the most important function of the nerve cells is to generate and conduct nerve impulses. That is, they send “messages” to and from the brain and other parts of the body. To send a message from one neuron to another, the dendrites of the axon of the first neuron come into contact with the dendrites of the next body cell.” The whole neuron and its offshoots are enclosed in a membrane. This helps to ensure that the messages stay on track while being transmitted to the next neuron. This study was interested in the identification and strategies of the numerical error disorders of the learners.

Clarkson (2000) reports on all these approaches to our understanding of the development of numbers in the early years. This project needs to provide evidence of the efficacy of this pathway on teacher development and learner performance. To do this a baseline study, which measures the performance of foundation phase learners, was done. The aim of this study is to show a productive, efficient and sustainable way to lift learner’s performance in mathematics .However this study was also interested to equip teachers with strategies regarding numerical errors. The obligation of teachers is to summarize these frequently appearing errors and remind learners of those errors as often as possible so that the learners can make more of an effort to avoid those numerical errors.

2.4 1 COGNITIVE FIT THEORY

Weber (1990:30) reported that, “cognitive theory is concerned with the presentation and tasks of the individual characteristics. A fit theory must therefore address these components, using technology as a tool and technology as representation”. Duncan (2007:8) suggests that, “technology as a tool provides the physical interface for manipulating the technology as representation. Representation implies a model of the real world tasks for example a graphical representation of a document or a mental model of a user, which will guide the researcher in identifying the numerical disorder which creates a problem to the learners.”

Richards (2005) distinguishing between technology as a tool and as representation is useful. It can help organize various literatures addressing behavior with information technology. When considering fit with technology, the question thus arises as to whether it is fit with a tool, the representation, or indeed the fit between the tool and the representation. Any definition of fit must be able to capture both roles of technology: tool and representation.

Wood (1986) defines the task as the three comprising components: products, required acts, and information cues. Products are ends, required acts and information cues are means for achieving the ends or goals. Specification of products or goals should detail the level of performance, acts to be carried out, and information cues to be used will vary with the level of performance required in the task product.

Silver (2000) considers an example of task hierarchy by producing a sales forecast. There is the substantive task for which the product is the sales forecast. The required acts and information cues for making the forecast could involve collating and modeling a sizable time series of past sales. According to Mouton (2005) the size of the time series (the information cues) and the collating and modeling efforts (required acts) are contingent on the level of performance at which the product (sales forecast) is defined. Using technology support to produce a forecast involves running the time-series data against some forecasting model. Thus, there is a task that has as its product a computer model of sales (a sub-task of the substantive task), which involves the technology as representation. These more operational tasks involve the use of technology as a tool rather than as a representation, and are a primary concern of human computer interaction research.

The type of fit and relevant measures of performance vary with the level at which a task is defined. It is necessary to distinguish among three key levels in the task hierarchy. There is the level of the super-ordinate or substantive task that motivates the whole exercise (e.g. producing a sales forecast). There is also the level of computer modeling task, which involves the technology as representation (e.g. conceptual development and manipulation of sales forecasting model inside the computer).

Finally, there is the level of the more operational tasks in the hierarchy that involves the use of technology as a tool. It is evident that different users will work on different and multiple tasks and sub-tasks at different stages of interaction with an information system. Fit theory is defined as an agent's potential to achieve a given performance level at a given task. Changing the agent in question provides a basis for generating different types of fit. Fit then characterizes the degree to which an agent's expertise (a) reflects the requirements for success in performing tasks,. (b) is in accordance with the structure of available task information.

According to Goswami (2008) knowledge about cognition means to have relatively stable information about one's own cognitive processes. This knowledge develops age and performance on many tasks and is positively correlated with the degree of one's meta knowledge. Therefore this study investigated the strategies of numerical error analysis and how to improve the culture of teaching and learning.

Ansari (2005) reports that before children learn to count verbally, children need to exhibit sensitivity to numerical magnitude. In other words, there are basic mechanisms for the processing and representation of non-symbolic numerical magnitude that can be measured very early on in development. Moreover, young children become exposed to cultural, symbolic representations of numerical magnitude over the course of learning and development.

According to Scholar (2008) having a basic understanding of the neurological underpinnings of cognitive development as it relates to mathematical thinking can foster instructional efficiency. An understanding of neurological development specifically helps educators to better plan developmentally appropriate mathematics instruction. Such instruction will, in turn reduce instructional wastage resulting from attempting to introduce concepts to a child before a child is ready to learn. This study investigates the underlying numerical errors in the foundation phase that the learners encounter to explore and to improve the numerical errors which are becoming a problem in the foundation phase.

2.5 CONCLUSION

The numerical error analysis in foundation phase mathematics shows empirical evidence reviewed and provides a link between the analysis of numerical errors that learners encounter, the significant relationships underlying numerical errors and the strategies that can improve numerical errors. These findings open up the possibility of using indices of numerical magnitude processing skills as measures of children's numerical abilities across the globe. By doing so, such measures could serve as indicators of the level of foundational skills exhibited by children who will enter formal education or are within the first few years of their formal education. In closing, it should be noted that mathematical development is multifaceted and involves the complex interactions between multiple factors, such as working memory, strategy use, attention and reasoning capabilities.

Lastly, mathematical confusion and errors arrive from not knowing enough about numbers themselves. Seeing the world through mathematical eyes is important as it helps children to see beyond the classroom and recognize types of numbers and their relevance to our environment, culture and day to day lives. Numbers are quite literally everywhere and getting to know them is a major stepping stone in developing mathematical confidence.

CHAPTER THREE
RESEARCH METHODOLOGY

3.0 INTRODUCTION

In the preceding chapter various approaches of literature on numerical errors in the foundation phase were explored. In this chapter 3.1 the research paradigm is discussed. The sampling instrument was developed in 3.2. Data collection and processing was described and discussed in section 3.3. Questionnaires were discussed in 3.4, the data analysis was explained in 3.5, the reliability and the validity were also discussed in 3.6 and the ethical consideration was explained in 3.7. The assessment instrument was designed for numerical error analysis and administered to approximately 80 learners in Grade 3. In addition, questionnaires were handed to learners and teachers to complete.

3.1 RESEARCH PARADIGM

According to Clotty (2011) a paradigm is used to situate a study and will inform the theoretical study, the conceptualization and the methodology. Mackenzie (2006) suggests a paradigm as a way of seeing the world that frames a research topic and the way in which the particular problem exists and a set of agreements on how such problems can be investigated. Creswell (2003:103) states that “the theoretical perspective behind methodology in the research question is post–positivistic which reflects a deterministic philosophy in which it is assumed that causes probably determine effects or outcomes”. Thus the methods used in this study were derived from using a quantitative method that included analysis of data from questionnaires and learner achievement scores.

The current study adopts a post-positivist paradigm or scientific method that involves the manipulation of reality with a single independent variable so as to identify numerical errors and identify strategies to improve teaching and learning of mathematics

3.1.1 RESEARCH APPROACH

An approach is a research methodology when a study is being conducted. It is based on two approaches, namely the qualitative and the quantitative approach. This research adopted the quantitative approach.

Nunan (2010) reports that quantitative methods are research techniques used to gather quantitative data dealing with numbers and anything that is measurable. In other words, a quantitative method is a systematic process in which numerical data is controlled and measured to address the accumulation of facts and then to obtain information about the numerical errors in foundation phase mathematics. This research was interested to investigate the numerical errors found in the foundation phase.

This research cites three research question approaches to the analysis of numerical errors in mathematics: the underlying numerical errors that learners encounter, the relationships underlying numerical errors and which implementable strategies are provided in numerical error analysis. The quantitative approach is a systematic process in which numerical data is controlled and measured to address the analysis of numerical errors in foundation phase mathematics because the quantitative approach is statistically reliable and allows results to be analyzed and compared with similar studies.

3.1.2 RESEARCH DESIGN

According to De Vos (2005) a research design is a logical strategy for gathering evidence about knowledge desired. It is one of the many choices made in the construction of any study's research design.

3.2 SAMPLING AND SAMPLE SIZE

According to Mouton (2005) sampling involves people, settings, events, behavior and social processes. There are probability and non-probability sample techniques.

The stratified random sampling was employed. This is a type of probability sampling. In probability sampling subjects are drawn from a larger group (which was 375) in such a way that the probability of selecting each member of the population is known. McMillan. and Schumacher (2001) mention that each member of the population as a whole has the same chance of being selected as other members in the same group. Thus, probability sampling is used to ensure adequate representation of the population.

The population sampling of all Xhosa-speaking foundation phase learners in the sixteen schools was 80 learners and each school was represented by five learners per class. As the total population size was 375, a table of random numbers was used to select the sample. The general rule in determining the sample size is to obtain a sufficient number to provide a credible result, according to McMillan and Schumacher (2006). However, in situations where a random sample is selected from a large population, a sample size that is only a small percentage of the population can approximate the characteristics of the population satisfactorily. Strydom (2005: 55) states that in most cases a 10 percent sample should be sufficient for controlling sample errors.

Only 16 teachers were asked to answer the questionnaires based on the implementable strategies of numerical error analysis and the background of the study. These strategies are (1) remedies to challenges of numerical errors, (2) paying attention to fewer topics and covering those in greater depth, and (3) developing conceptual understanding and computational fluency by introducing problem-solving skills.

According to Macmillan (2006) the sample size is to obtain a sufficient number in order to provide credible results. However, in this situation a random sample was selected from a larger population.

Table 5: The sample size of respondents

Respondents	Population size	Sample size	Percentage of population
Learners	375	80	10%
Teachers		16	10%

3.3 DATA COLLECTION

The data was collected from both teachers and learners in the vicinity of the researcher's work in the district of East London. About 80 learners were selected to participate in the study. The population of the study was 375 learners but the researcher engaged only 10%, which is 80 learners, as the sample size. The researcher used only 16 schools in East London as they were accessible to the researcher. A simple random sample of 5 learners per school was selected. This justified the sample size of 80 respondents (learners).

Permission was granted by the school principals to distribute the questionnaires. Each learner and each teacher in the sample were given a questionnaire to complete. Questionnaires were personally distributed to and collected from each school. Instructions were explained to the sample to avoid confusion.

INSTRUMENTATION

The unique test questions (cf appendix 4) used for numerical error analysis in foundation phase for this research were taken from the examination papers the learners wrote as their final examination. All five learning outcomes in mathematics were included. Those were numbers, operations and relationships, patterns, functions and algebra, space and shape, measurement and data handling. Using this final examination paper was an appropriate method for this study because all the learning outcomes (cf appendix 4 and chapter 4: section A) had been taught by this time and the learners were ready to produce what they were taught. It is vital to note that the numerical error analysis used in this study focused especially on errors or mistakes to understand why they occur in the foundation phase.

3.4 QUESTIONNAIRES

The research instrument which is the questionnaire was specifically constructed to obtain information on the numerical errors in foundation phase mathematics. McMillan and Schumacher (2006) reckon a questionnaire is the most widely used technique for obtaining information from subjects. A questionnaire is relatively economical, has the same questions for all subjects and can ensure anonymity. The questionnaire comprised a few questions, all of which are relatively clear and simple in their meaning.

According to Delport (2005) the basic objective of a questionnaire is to obtain facts and opinions about a phenomenon from people who are informed on a particular issue. Questionnaires are probably the most generally used instruments of all. The advantage of a questionnaire is that it is less time consuming and the researcher may not influence these replies by revealing his or her opinions, as may be the case in interviews.

There were two questionnaires, one for the learners and one for the teachers. Each questionnaire consisted of two sections, that is sections A and B, and were administered to the respondents. The respondents were expected to answer all the questions. The questionnaires took 20-30 minutes to complete under supervision of the researcher. Completed questionnaires were collected and kept. A record was kept on a recording sheet.

Questionnaires with rating scale questions using Likert scales were used to collect the data from the respondents (Grade 3 learners of the 16 participating schools in the district of East London). The respondents' ages ranged from 9 to 11.

The learner questionnaire consisted of two sections, namely A and B. Section A (cf appendix 4 section A questions 1) consisted of their final examination test and Section B of scaled responses – numerical errors that learners encounter.

Section A Is a summary of a controlled test, to test the numerical errors in mathematics.

The teacher questionnaire is a summary of table 2 (cf appendix 5) consisting of two sections, A and B.

Section A: The biographical data of teachers

Section B: (cf appendix 5) The implementable strategies of the numerical error analysis in mathematics.

- Section: A (cf appendix 5) Biographical data of teachers

Information was obtained on the variables of gender, age, marital status, professional qualifications, years of teaching experience, the area where the school was located, race, the kind of school. These aspects are important in order to determine the background of a teacher.

- Section: B (cf appendix 5) Scaled responses

A scaled question is a type of multiple-choice question in which response categories are designed in such a way that respondents mark a certain point on a scale. Delport (2005) reports that a Likert scale is one in which the item includes a value or directions and the respondents indicate agreement or disagreement with the statement. The value of a Likert scale is that individuals have the option of expressing the degree to which they disagree or agree with a particular statement. The scaled questions were used to establish the numerical errors that learners encounter in mathematics.

3.5 DATA ANALYSIS AND INTERPRETATION

Data analysis involves working to uncover patterns and trends in data sets, and data interpretation involves explaining those patterns and trends. The techniques scientists use to analyze and interpret the data enable other scholars to both review the data and use it in future research. Egger (2008) reported that, error analysis is used both as a method of analyzing data and a theory. It is a technique for identifying, classifying and systematically interpreting the unacceptable forms of numerical errors and the accompanying understanding of those errors.

Data was collected by using structured questionnaires on numerical errors in the foundation phase. The analysis was based on numerical errors in the foundation phase. The data gleaned from the questionnaires was subjected to statistical analysis to determine the reason for numerical errors in the foundation phase and also to establish the teachers' view regarding the implementable strategies to analyze numerical errors.

3.5.1. DATA ANALYSIS TECHNIQUE

In this study the researcher used the quantitative approach to analyze the data. At first, predetermined categories were used from the following sources:

The research questions

The research instrument (questionnaire)

Concepts and themes from the literature review

Personal experience of an educator and researcher

This study used the topics embedded in the questionnaire. At first, the five pre-established categories from the learner questionnaire (biographical data), unique test questions for the learners and three questions for the teachers provided a frame for the next data analysis.

Secondly, each category was divided into sub-categories when the data was analyzed.

Thirdly, each participant's answers were used according to the above categories.

Fourthly, relationships were determined from among categories by discovering patterns in the data. To find regularities within the data, the researcher compared different source educators, learners and methods to see whether the same patterns kept recurring.

Fifthly, in seeking for patterns in the data, the researcher also searched for negative and discrepant evidence that modified a pattern.

Finally, the developed patterns and themes were used to report the experiences of the participants. This study ensures that the numerical errors that learners encounter in the foundation phase were thoroughly analyzed for the benefit of teachers, learners and researchers.

ANALYSIS OF RESEARCH QUESTION ONE

In research question one the underlying numerical errors in foundation phase mathematics were unpacked and analyzed by means of a questionnaire. The role of this study in this questionnaire was to investigate the errors of learners present in foundation phase mathematics.

ANALYSIS OF RESEARCH QUESTION TWO

In this study the relationships underlying numerical errors in foundation phase mathematics were analyzed in chapter 4 section A (cf. appendix 1 question 2) of this research question in order to unpack the relationships in underlying numerical errors that can assist learners to deal with those mathematics errors

ANALYSIS OF RESEARCH QUESTION THREE

In this research question teachers were presented with strategies and methods of numerical error analysis adopted from Ansari (2005) in chapter 2. The aim of this study in this research question was to assist teachers with the strategies of numerical error analysis in the foundation phase.

3.6 VALIDITY AND RELIABILITY

Mcmillan and Schumacher (2006) reports that scientific methods produce findings are valid. Validity has to do with whether your methods, approaches and techniques actually relate, or can be measured. Reliability relates to how well research has been carried out. Quantitative methodology derived from this, aims to improve validity by careful sampling. Reliability, using quantitative methods involves choosing measures that demonstrate consistency and are replicable, over time, over instruments and over groups of respondents.

3.6.1 VALIDITY

According to Delport (2005) the extent to which an empirical concept measures accurately reflects the concept it is intended to measure. It reflects to the truthfulness of findings and conclusions. McMillan & Schumacher (2006) say validity refers to whether the measurements measure what they are supposed to measure. This research study used two measures of validity: face and content validity.

3.6.2 FACE VALIDITY

Delport (2005) refers to face validity as the value of a measurement procedure and the relevant question in this regard is: Does the measurement technique look as if it measures the variable that it claims to measure?

Face validity was established by giving letters to parents of the learners for their permission to the researcher to give their children questionnaires.

3.6.3 CONTENT VALIDITY

Delport (2005:21) states that, “content validity is concerned with the sampling adequacy of an instrument”. According to Rosnow (1999) content validity means that the test or questionnaire items represent the kind of material they are supposed to represent. Thus, a test or a questionnaire with good content validity covers all major aspects of the relevant content areas.

To determine content validity, two questions were asked:

- Is the instrument really measuring the concept we assume it is measuring?
- Does the instrument provide an adequate sample of items that represent that concept?

In this research the numerical error analysis was validated by experts (cf chapter 1) and by the literature review in chapter 2 to sample the concept adequately.

3.6.4 RELIABILITY

According to Delport (2005) reliability measurement procedure is a stability or consistency of the measurement. This implies that if the same variable is measured under the same conditions, a reliable measurement procedure will produce identical or nearly identical procedures.

Thus to establish both reliability and validity the questionnaires were tested before being used in the study.

3.7 ETHICAL CONSIDERATION

According to McMillan (2006) ethics is considered to deal with beliefs about what is wrong or right, good or bad. Ethics is a set of moral principles which is suggested by an individual or group, which offers rules and behavioral expectations about the most correct conduct towards experimental subjects and respondents, employers, sponsors and other researchers, assistants and students,(cf appendix 3) as per Strydom (2005). Thus ethical guidelines include policies regarding informed consent, deception, confidentiality, anonymity, privacy and caring, according to McMillan and Schumacher (2006).

3.7.1 INFORMED CONSENT

The advantages and disadvantages and dangers of the respondents may be exposed. For this study the researcher obtained permission from the Department of Education in the district of East London (cf appendix 1). The letters of permission from principals of all 16 schools for the study to be conducted (cf appendix 2) were granted.

3.7.2 CONFIDENTIALITY AND ANONYMITY

Confidentiality means that subjects' disclosures are protected against unwarranted access. In this study great care was taken to safeguard the privacy and identity of the respondents and research sites. Respondents were assured of anonymity in the covering letters and that confidentiality would be maintained in this study.

3.7.3 PRIVACY AND CARING

A sense of caring and fairness was part of the researcher's thinking and actions. Therefore the respondents were monitored under supervision of the class teacher but the respondents did not write their names on the questionnaires.

Permission was granted by the school principals to distribute the questionnaires. Each learner from the 16 sampled schools was given a questionnaire to answer. Each of the 16 teachers who volunteered for this study received a questionnaire to answer. The five learners which were selected randomly in the classroom were given 1 hour to answer the questionnaire with the help of the class teacher.

CHAPTER 4

DATA ANALYSIS AND INTERPRETATION OF RESULTS

4.0 INTRODUCTION

The previous chapter focused on a discussion of the empirical research design paradigm used to investigate the research questions. In this chapter the findings and interpretation of the empirical investigation are presented. The aim of this is to provide answers to the research questions. A questionnaire was administered to the research sample in order to collect responses to the research questions. By the use of descriptive statistics the current study presents the major findings of the study, starting with underlying numerical errors that learners encounter, the relationships of underlying numerical errors and which implementable strategies are present in the numerical error analysis of learning and teaching foundation phase mathematics.

4.1 INTRODUCTORY ANALYSIS OF MAIN RESEARCH QUESTIONS

The following research questions explored and reported the numerical errors in foundation phase mathematics.

Research question 1

- What are the underlying numerical errors that learners encounter in obtaining, processing, retaining and reproducing information contained in mathematical tasks?

  To address research question 1 the data was collected from the learners by means of a questionnaire (cf appendix 4). Test questions for the learners were used to identify the numerical errors that the learners encounter in foundation phase mathematics. The researcher used quantitative data analysis to measure the balanced view on the numerical errors.

- What are the relationships underlying numerical errors?

A Likert-scale, questionnaire (cf. appendix 5) was used to foundation phase Grade 3 learners to obtain a balanced view on the study determined by agree, strongly agree, disagree, and strongly disagree. However (cf appendix 5) one teacher in each of the 16 schools helped by reading and guiding the learners concerning the statements in the questionnaire. A quantitative analysis was used to unpack the identification processes: (1) the underlying numerical errors that learners encounter and (2) the underlying relationships in connection with numerical errors in the foundation phase to improve the learners' ability to do mathematics successfully.

- What are the implementable strategies in numerical errors analysis?
  Quantitative data analysis was used to find the strategies to measure shared variables that are believed to attribute to numerical errors in foundation phase mathematics. Teachers were expected to answer the statements in a questionnaire (cf appendix 5) and they were expected to answer true or false, certain or uncertain and agree or disagree.

4.2 ANALYSIS OF RESEARCH QUESTION 1

What are the underlying numerical errors that learners encounter when obtaining, processing, retaining and reproducing information in mathematical tasks? This section provides an illustration, interpretation and discussion of the research question. The data was collected via questionnaires (cf appendix 4). The questionnaires were used to obtain information regarding numerical errors in the foundation phase. Eighty (80) questionnaires were administered to the subjects in all sixteen (16) schools and eighty (80) questionnaires were returned. 16 questionnaires were administered to the teachers and returned.

The researcher believes that to investigate the numerical errors in foundation phase mathematics is of paramount importance to identify the numerical errors.. By the use of descriptive statistics, this section presents the major findings of the study. Two questionnaires were employed, one for the learners and one for the teachers. The learner questionnaire had test questions, for example to count backwards and forwards, to count in twenties, and to show calculations which learners were expected to be able to answer. The teachers' questionnaire (cf appendix 5) consisted of statements with strategies of error analysis in the foundation phase that will assist teachers to improve on errors.

Section A

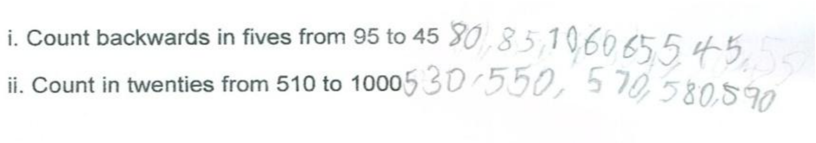
4.2.1 (i) Count backwards in fives from 95 to 45

Learners were expected to count backwards in section A question (i). 60 respondents had the correct answers, and they were able to count backwards. 20 learners were unable to count backwards

Case in point: learner 1 and learner 2 had the following respective answers.

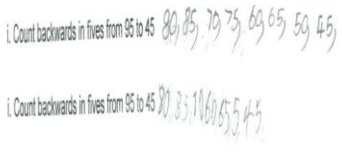
Learner 1

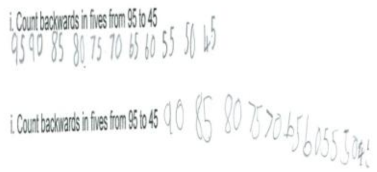
Learner 2



Learner 3

Learner 4





Learner 2 number-counted correctly which shows that they do understand how to count forwards and backwards. Learner 1 still needs help because a problem with numbers was evident in that case. The learner who could not count backwards or forwards committed errors in this exercise which suggests that they need help. Perhaps the causes are sequencing, mathematical language, lack of concentration and word sums. The learners confuse numbers, for example 21 is taken as 12, 96 as 69. They also confuse the sequencing of numbers, as they do not understand the difference between hundreds and tens. These are the usual numerical errors when counting in mathematics.

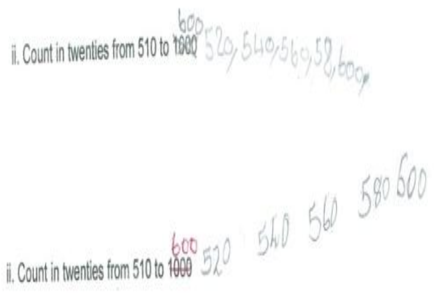
(ii) Count in twenties from 510 to 600

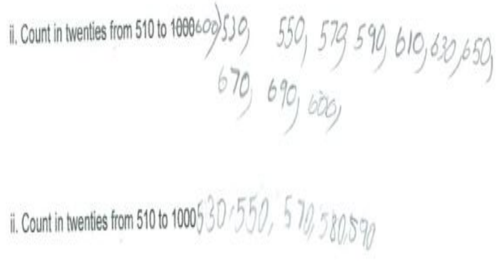
In section A question (ii) only 70 learners were able to count in twenties between 510 to 600, and 10 learners were unable to count backwards in twenties from 510 to 600.

In section A question (ii) learners were expected to count in 20's (120 140, 160,-,-,-,-260..). They were supposed to add twenty to each number. 75 learners managed to give the correct answer and only 5 respondents were unable to do so.

In section A question (ii) learners were expected to count in 10's, from 1000, 990, 980, - - - - - - 800. 70 learners were able to give the correct answer and only 10 learners were unable to give the correct answer as they confused the numbers.

I = Learner 1 2 = Learner 2

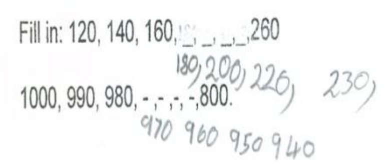


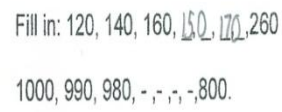


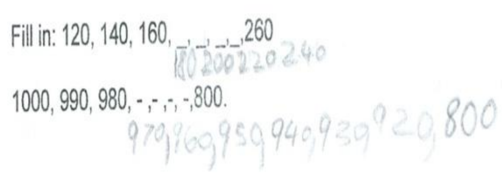
Learner 1 was able to count in twenties in this exercise, which shows that learners manage to give the correct answer, and Learner 2 responded wrong because they could not add twenty to the number they were given, which shows that they experience a problem with numerical numbers which can be caused by a lack of sequencing, disability to count, basic numerical processing or otherwise they were unable to grasp the concept of counting in earlier Grade 1 or 2 classes.

Fill in: 120, 140, 160, 180, 200, 220, 240, 260,
1000, 990, 980, 970, 960, 950, 940, 930, 920, 910, 900

Learner 1 Learner 2







Learner 3 Learner 4

Learner 1 was able to calculate correctly but had numerical errors because the respondents were supposed to add twenty to the number they were given, maybe the respondents still need help in patterns and sequencing. Learner 2 did not manage to give the correct answer, which suggests a lack of sequencing with numbers, a lack of basic counting with numbers and counting number patterns orally which should have been done in the early grades. These operations will assist in early subtraction and addition. Therefore this suggests that the learners need assistance because of the numerical errors and the evidence that they do not understand patterns.

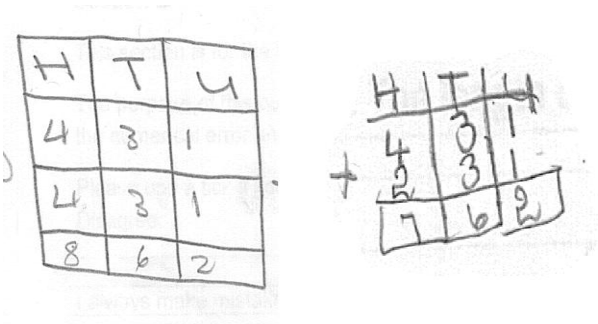
4. 2.2 Show all calculations

In section A questions (a),(b),(c) and (d) learners were expected to show all the calculations of the four basic operations, namely addition, subtraction, multiplication and division. The reason was that this study investigated whether learners were able to do calculations.

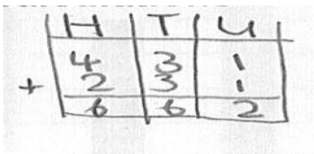
(a) 431+ 231

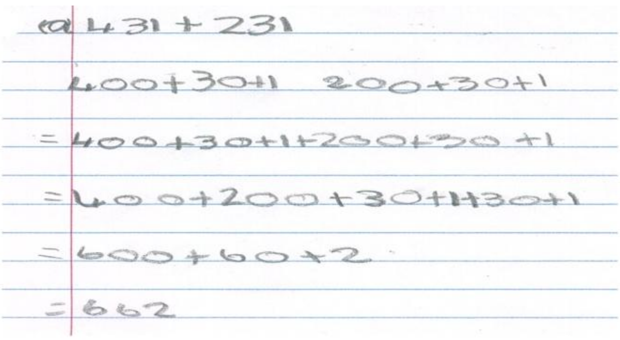
The respondents were expected to calculate according to hundreds, tens and units to show that they understood the difference between three-digit numbers, but in (a) only 30 learners were able to put each number under its digital number and add correctly. 50 learners were unable to put the numbers according to their digital numbers, therefore they calculated inaccurately as the digits were not placed correctly. This proves that the respondents need assistance with the place value of the four basic operations.

Learner 1

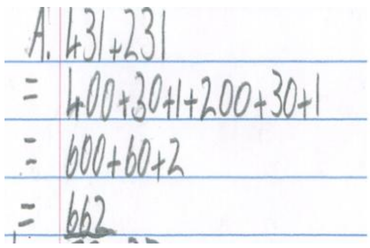


Learner 2





Learner3



Learner 4

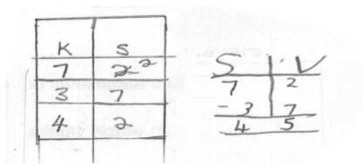
This respondent was able to calculate correctly and he/she used an own style of calculating, by adding the hundreds together, the tens together and then the units. This shows that the respondent managed to calculate correctly.

Learner 1 was supposed to calculate by using hundreds, tens and units, but the respondent could not count correctly, because he/she had problems with copying the numbers, recalling numbers with calculations and writing mathematical symbols. Some learners struggle with mathematical signs, the place value and mathematical language.

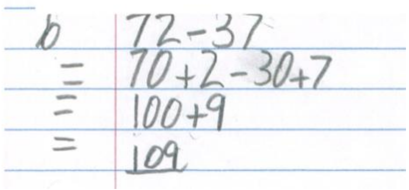
Learner 2 could count because the answer was correct, which means he/she can understand the difference between hundreds, tens and units, and can add and put the numbers correctly according to their place values.

Therefore learner 1 still needs assistance, because the work suggests that he/she does not understand the place value of addition, and still needs to know the basic counting of numbers and the place value of numbers, which were supposed to have been made clear in the early grades. It is very important when using the four basic operations, namely addition, subtraction, multiplication and division to know the place value of the numbers.

(b) Learner 1



(b) Learner 2



Section A question 1(b) (cf appendix 4) of the questionnaire was very difficult for the respondents. 65 respondents could not answer this question as they confuse the mathematical signs, and instead of subtracting they added and some did not borrow a ten from the tens. Only 15 learners were able to calculate correctly. 30 learners confused the digit numbers, they took the unit number to be tens and the tens to be units, while 35 learners managed to put the numbers in the right place value but they added instead of subtracting and did not borrow a ten from the tens to make 12, as 2 was smaller than 7 and 2 was above 7 and not under 7. The reason was that the respondents did not understand the place value because they put the tens under units and units under the tens and did not know how to subtract.

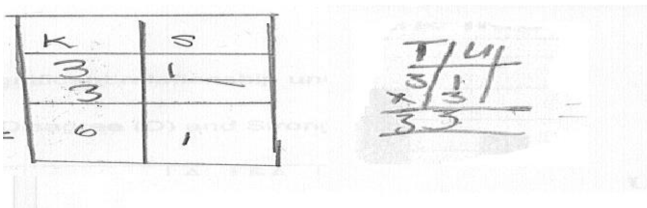
Learner 1 could count because the calculation was correct and they used different methods to subtract, so the answer was the same. This learner also understood if the number on top was smaller in the units they needed to borrow from the tens because the number on top in the units had to be bigger than the number at the bottom, to get the correct answer. This suggests that the respondent understands the mathematical procedures and can count properly.

Learner 2 still needs help because they could not see the difference when subtracting and they added instead of subtracting. This learner did not even understand that if the number on top in the units was smaller than the number at the bottom they needed to borrow a ten from the tens. They still did not understand the place value of numbers. This means that they need help as the place value is very important in addition and subtraction. Before calculating the learner must be able to first arrange the numbers according to their places. These are the core underlying numerical errors that learners encounter and this study found these results very interesting.

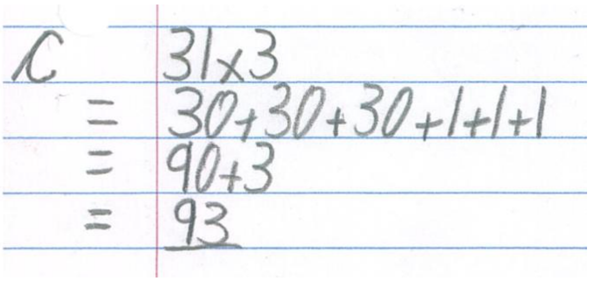
(c) 31 x 3

This question confused the respondents even more because they showed that they still did not understand the multiplication sign. Only 10 learners were able to put the place value correctly and to get the correct answer. 22 learners confused the place value and they added instead of multiplying, 40 learners were unable to put the place value and could not multiply, 8 learners did not even try to answer the question, the rest left it blank.

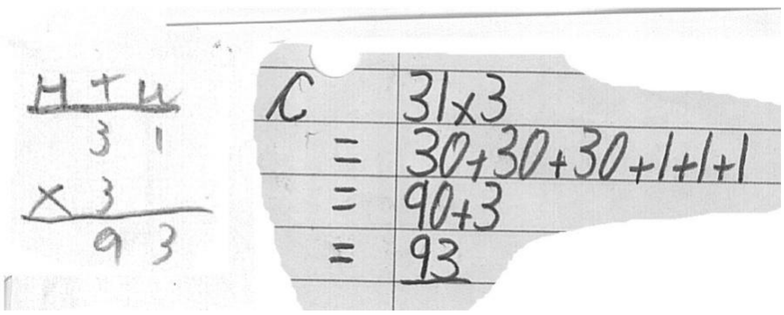
Learner 1



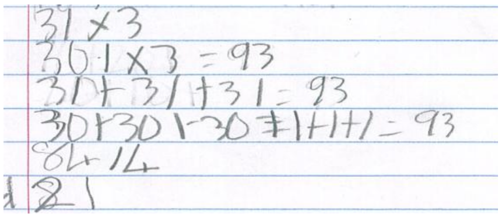
Learner 2



Learner 3



Learner 4

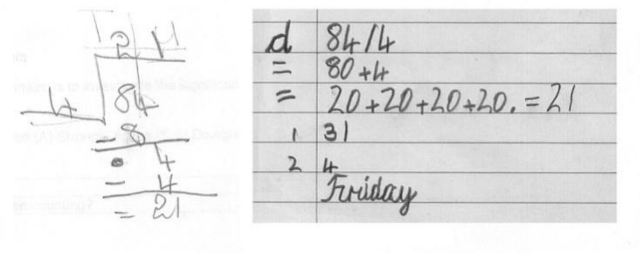


Learner 1 failed to calculate correctly because they did not understand multiplication and how to multiply, so they added instead of multiplying. They obviously could not differentiate between multiplication and addition. Secondly, the respondent did not know the place value when doing multiplication. According to Shalev (2004) they lack basic numerical processing and they are confused with the mathematical symbols, even though they could do the place value. Therefore the learners showed that they need help as they did not grasp the basic method of multiplication.

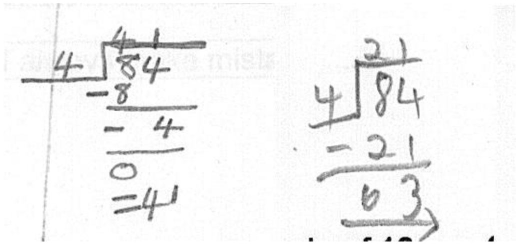
Learner 2 proved to understand multiplication because he/she used two different methods to calculate with multiplication. Even if the methods were different the answers were the same. The other one made a mistake when placing the calculations underneath the text itself. This learner can solve mathematical problems and differentiate between mathematical symbols. The aim of this study was to identify these kind of numerical errors.

(d) 84 / 4

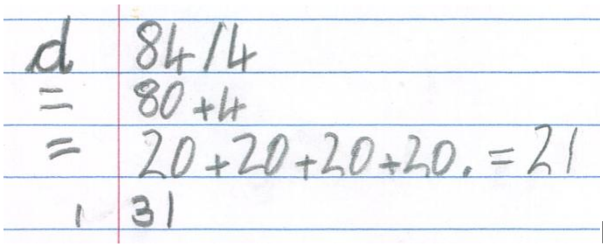
In section A question (d) the respondents were expected to do long division whereby they divided and subtracted. Only 5 were able to calculate correctly, they managed the mathematical sign and were able to do the place value when dividing, as well as division and subtraction. 35 learners were able to divide and had the right answer even while employing different methods to divide, but did not do long division where they were expected to subtract. 23 learners were able to put the division sign correctly but did not know how to divide, 17 learners did not even attempt to answer the question proving that they did not understand.



Learner 1



Learner 2



Learner 3

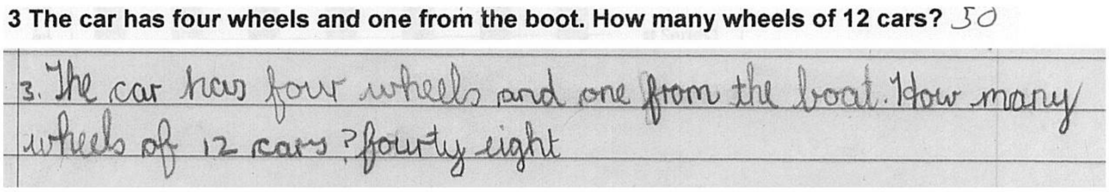
Learner 1 proved an ability to calculate and understood division and mathematical symbols. Two different methods of division were used resulting in the same answer which was correct. This learner showed an understanding of mathematical symbols and the place value. Scholar (2008).reported that children would have a confident grasp of counting by the time they leave primary school.

Learner 2 still needs help because of a problem in division and how to follow the correct steps when dividing. Division involves almost all the mathematical signs and division calculation is the most difficult basic mathematical operation. Scholar (2008) reported that a child would find it difficult if division is not mastered by the end of primary school. In order to master division a learner must be able to understand the three basic operations, namely addition, subtraction and multiplication, and be able to know the numbers very well. In this study these learners showed that they need assistance in order for them to calculate correctly.

4.2.3 The car has four wheels and one in the boot. How many wheels do 12 cars have?

The respondents were expected to count the number of wheels of one car and thereafter count the wheels of 12 cars. 40 learners were able to count the wheels for 12 cars correctly, 18 learners did not count the number of wheels for the first car, they only counted for 12 cars and forgot to count the wheel in the boot. 22 learners forgot to add the wheel in the boot and their answer was wrong.

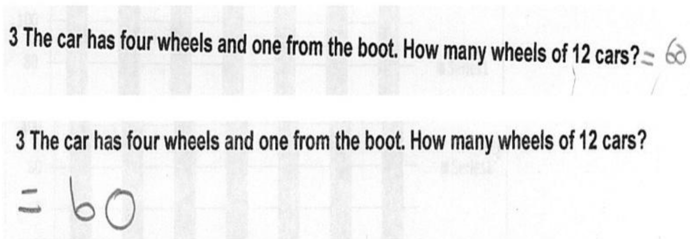
Learner 1



3 The car has four wheels and one from the boot. How many wheels of 12 cars? 30

3. The car has four wheels and one from the boat. How many wheels of 12 cars? fourty eight

Learner 2

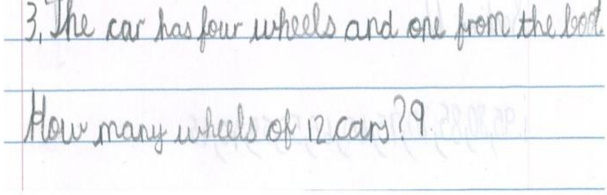


3 The car has four wheels and one from the boot. How many wheels of 12 cars? = 60

3 The car has four wheels and one from the boot. How many wheels of 12 cars?

= 60

Learner 3



3. The car has four wheels and one from the boot.

How many wheels of 12 cars? 29

Learner 1 could not get the correct answer. The respondents were supposed to calculate the number of wheels of 12 cars if one car has four wheels and one in the boot. Here is evidence that the respondents need help in order for them to be able to do problem-solving calculations. They need to know multiplication and addition. Bligh (2000) reports that, when various tasks are recalled automatically the mind makes little effort to bring them up. Thus attention and the limited working memory span can be directed towards problem solving.

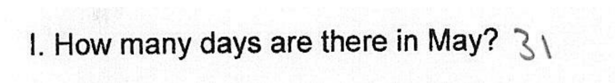
Learner 2 proved that they did understand problem solving calculation without being assisted because their answer was correct. They are capable and independent to count for themselves. The word sums can be difficult as the learners must understand   reading and writing, therefore learners must first be able to read in order to answer word sums. When learners show evidence that they cannot read the words and the numbers it shows that they need assistance to avoid numerical errors encountered in the foundation phase.

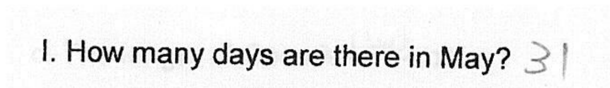
Table 6: The calendar for May

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
		1	2	3	4	5
6	7	8	9	10	11	2
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

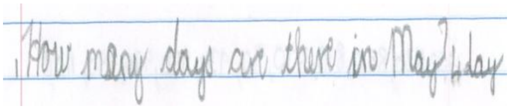
1. **How many days are there in May?**

The learners were expected to say the last number which is in the calendar but only 70 learners were able to answer the question, 10 learners were not able to read the sentence as they could not read and understand the sentence.


I. How many days are there in May? 31


I. How many days are there in May? 31

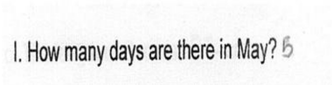
Learner 1

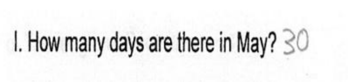

How many days are there in May? 4 day

Learner 2


I. How many days are there in May?

Learner 3


I. How many days are there in May? 5


I. How many days are there in May? 30

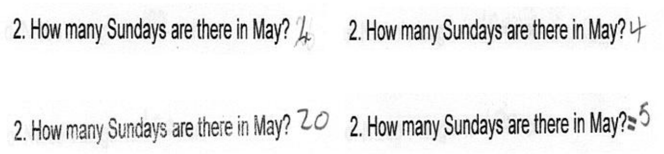
Learner 4

Learner 1 could get the correct answer, which meant they were able to read and understand the question they were asked about the calendar. They also understood the language of the calendar and they were familiar with the numbers.

Learner 2 proved that help in reading mathematical concepts was required, because in order for them to know the calendar they should be able to read. If they could not read it was difficult for them to understand the calendar. Therefore these learners need help as they did not master the mathematical concepts.

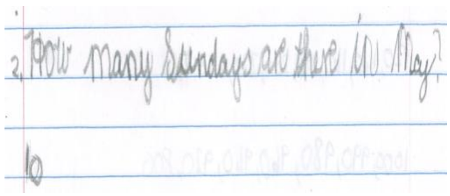
2. How many Sundays are there in May?

The respondents were expected to count the numbers of Sundays in the calendar. 50 learners managed to count the number of Sundays which was 4, 30 learners could not understand the language because they could not read and were unable to count the number of Sundays in the calendar.



Learner 1

Learner 2



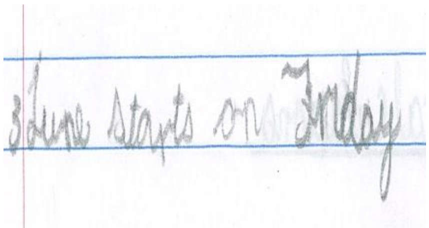
Learner 3

Learner 1 had no problem reading the words and the numbers because they were able to get the correct answer. The reason is because they can read and can count.

Learner 2 showed they had a problem because they gave the wrong answer and on top of that they had a problem reading numbers and words. This proves the respondents still need help in reading mathematical language. Bornman (2010) states that learners with dyscalculia may experience difficulty when they have to read numbers, while others did not understand the basic concept they should have learnt in previous lower classes. In order for the learners to be able to master the word sums they must be able to read and count, and this is basic information they should get in lower grades in order to overcome the numerical errors.

3. June starts on -Friday

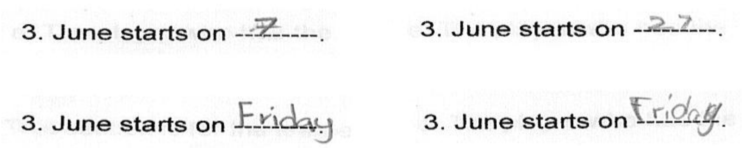
The respondents were expected to write the first day after the thirty-first which was on Thursday. 40 learners were able to answer the question which was Friday, 25 wrote only the number 1 on which the month started, 10 learners could not give the right answer as they did not understand because they could not read and 5 learners decided not to answer the question.


3June starts on Friday

Learner1


3. June starts on 7.

Learner 2

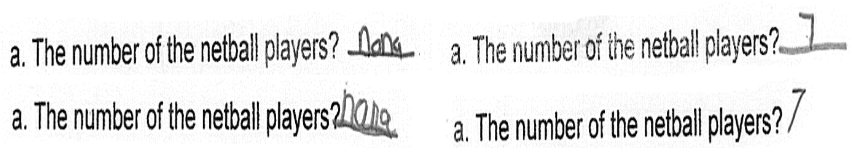

3. June starts on 7.
3. June starts on 27.
3. June starts on Friday.
3. June starts on Friday.

Learner 3

Learner 1 did not have the correct answer as the respondent could not read the calendar and therefore was not familiar with the mathematical concepts. This outcome showed they still need help with the basic information of mathematical concepts in the lower classes.

Learner 2 was able to count and to read the calendar and produced the correct answer, which proved that they could read and write.

Learner 1 Learner 2


a. The number of the netball players? nana
a. The number of the netball players? nana
a. The number of the netball players? 7
a. The number of the netball players? 7

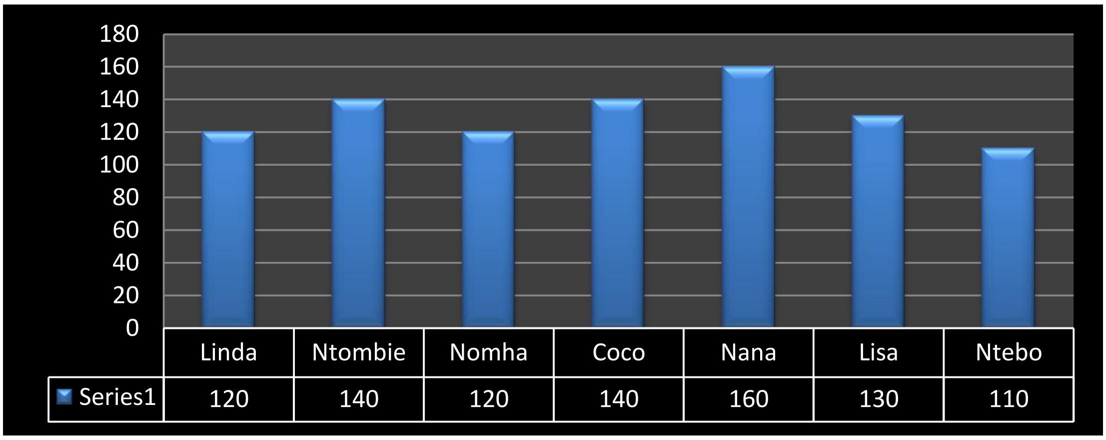
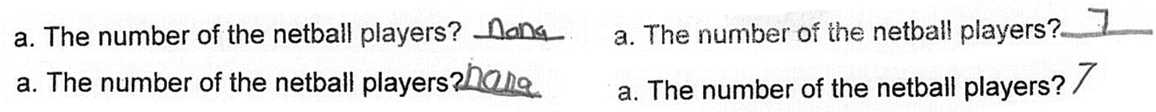


Fig 4.1: Number of netball players?

The respondents were expected to count the number of players in the graph. 55 (68%) learners were able to answer the question that there were 7 players, 15 learners were unable to get the right answer because they could not read and understand the question. 10 (13%) learners decided not to answer the question

Learner 1



Learner 2

Learner I had the wrong answer because the learner was unable to read the graph and so did not understand the language of the graph.

Learner 2 was capable of reading the graph and understood what was being interpreted by the graph. The mistakes are common and minor. According to Pardesi (2008) learners have just the misconception that a concept is incorrect according to their understanding, whereas the error is a mistake that is repeated while the learner thinks it is correct.

b. The tallest netball player

The respondents were expected to read from the graph which player was taller than other players by looking at the length of the players. 60 (75%) learners managed to read from the graph the tallest player was Nana. 16 (20%) learners did not understand the question and gave the wrong answer, and 4 (6%) learners decided not to answer the question

Learner1	Learner 2
b. The tallest netball player? Nana	b. The tallest netball player? Nana
b. The tallest netball player? 140	b. The tallest netball player? Ntombie

b. The tallest netball player? Liso

b. The tallest netball player? Nana

Learner 1 had the correct answer which proves that they could read the statements in the graph and could understand the language of the graph.

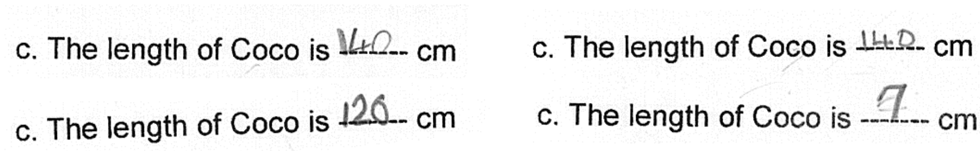
Learner 2 proved a lack of understanding by giving a wrong answer. Therefore the learner needs help because he/she did not understand and could not read and give the correct answer when working with graphs.

c. The length of Coco is 140 cm

The respondents were expected to read from the graph the length of Coco which was 140cm. 30 (38%) learners were able to read by looking at the name Coco and his length, 43 (54%) learners were unable to answer the question because they could not read and understand the question, 7(9%) learners did not answer the question.

Learner1


c. The length of Coco is 140--- cm


c. The length of Coco is 140--- cm
c. The length of Coco is 140- cm
c. The length of Coco is 120-- cm
c. The length of Coco is --7--- cm


c. The length of Coco is ---7---- cm

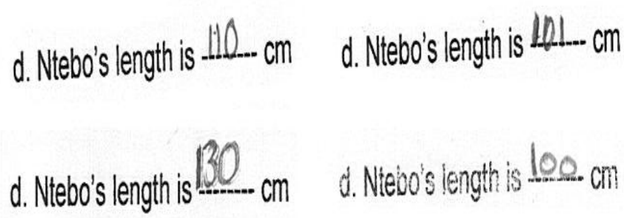
Learner 2

Learner 1 had the correct answer. The learner could read the graph and understand the question. There were only a few mistakes.

Learner 2 proved that the learner did not understand and could not read the graph, because the wrong answer was produced. This means some learners need help to understand how to read a graph.

d. Ntebo's length is 110 cm

The respondents were expected to read the length of Ntebo from the graph which is 110 cm. 20 (25%) learners were able to read that Ntebo's length was in the middle of 120 and 100 because some answered 100 cm, others 120 cm. But 15 (19%) learners did not even try to attempt the question as they could not read and understand what was expected of them.


d. Ntebo's length is -110--- cm
d. Ntebo's length is 101--- cm
d. Ntebo's length is 130------- cm
d. Ntebo's length is 100- cm

Learner 1

d. Ntebo's length is 160 cm

Learner 2

d. Ntebo's length is 100 cm

Learner 1 gave a correct answer but another learner made a mistake by writing 101 instead of 110 convinced that he wrote 110. However, they were capable of reading and answering the questions asked.

Learner 2 gave the wrong answer, which proves that the learner could not read and understand the graph. The relevant learners need help to be able to understand and read the graph.

e. The player who has the same length as Coco is - Ntombie.

The respondents were expected to read from the graph the player with the same length as Coco. 58 (73%) learners were able to read that Ntombie had the same length as Coco. 13 (16%) learners were unable to give the correct answer as they had a reading problem, and 9 (11%) learners decided not to answer the question and left it blank.

e. The player who has the same length as Coco is ntombie.

Learner1

e. The player who has the same length as Coco is 140.

Learner2

Learner I had the correct answer to this question which proves that the learner could count and read and understand the graph. Only minor help with careless mistakes is required.

Learner 2 did not give the correct answer. The other learner gave the correct answer but the spelling was wrong as a number instead of the name of the player with the same length as Coco was given. This proves that learners have problems with reading and understanding of graphs. In order for the learners to be able to give correct answers to graph questions he/she should correlate and be able to read.

SUMMARY OF RESEARCH QUESTION 1

In this research question, learners counted correctly in section A question 1 (cf. appendix 4). Only a few learners could not count, and those learners did not understand the sequencing. In section A question 2 (cf appendix 4) learners were expected to show all calculations. The majority of the respondents could not understand the place value, and confused the mathematical signs, instead of division learners added and in multiplication they subtracted and in subtraction learners added the numbers. In section A questions 3, 4 and 5 (cf appendix 4) the majority of learners had numerical errors, the reason being a lack of understanding of the mathematical language, an inability to read the graph and even the inability to spell in some cases. This study wanted to investigate the underlying numerical errors learners encounter in the foundation phase.

4.3 ANALYSIS OF RESEARCH QUESTION 2

The relationships underlying numerical errors in foundation phase mathematics.

This section provides a descriptive analysis of the investigation in the relationships underlying numerical errors in the foundation phase. The learners were expected to use a tick on the Likert scale to agree (A), Strongly Agree (SA), Disagree (D) and Strongly Disagree (SD).

Statements numbered 1, 2, 3, 4, 5, 6, and 7 of section A question 2 (cf appendix 4) were assisted by one teacher per school in all 16 schools used as the target population. The questionnaires were designed to test the respondents whether they understood the relationships underlying the numerical errors they always made when counting. These statements help to understand their numerical errors.

4.3.1. DESCRIPTIVE ANALYSIS

I always make mistakes when counting

In response to statement number one which stated that learners always made mistakes when counting, 50 (63%) learners agreed, 18 (23%) learners strongly agreed, 7 (9%) learners disagreed and 5 (7%) learners strongly disagreed that they made mistakes. The majority of the respondents showed that they agreed that they made mistakes when counting using numbers, which proves that they need help.

I have a problem in reading the numbers which contain more than one digit

In response to statement number two which stated that learners had a problem in reading numbers which contained more than one digit .25 (31%) learners agreed, 32 (40%) learners strongly agreed, 11 (14%) learners disagreed and 12 (15%) learners strongly disagreed that they had a problem with numbers which contained more than one digit. Therefore the majority of the learners strongly agreed that they had problems in reading the numbers which contain more than one digit.

I have problems when copying the numbers

The respondents were expected to state in statement number three whether they had a problem when copying numbers. 60 (75%) learners agreed, 15 (19%) learners strongly agreed, 3 (4%) disagreed and 2 (3%) learners strongly disagreed that they had a problem in copying the numbers. The majority of the respondents who supported statement number three and agreed were 60 (75%) learners. This proves that they lack concentration or that they confuse the numbers.

Mathematical signs confuse me

Learners were expected to state in statement number four whether they had a problem in confusing mathematical signs. 52 (65%) learners agreed, 11 (14%) learners strongly agreed, 8 (10%) learners disagreed and 9 (11%) learners strongly disagreed that they had a problem with mathematical signs. 52 (65%) respondents who agreed were the majority who agreed that they confused the mathematical signs, for example 12 (15%) becomes 21, 31 becomes 13, 15 becomes 51 and many more.

I always finish my tasks

In response to statement number five learners were expected to state whether they always finished their tasks. 7 (9%) learners agreed, 22 (28%) learners strongly agreed, 15 learners disagreed and 36 (45%) learners strongly disagreed that they always finished their tasks. The respondents who strongly disagreed that they finished their tasks were 36(45%) learners and they were the majority which proves that they need assistance.

I learn best when using different kinds of colours

In response to this statement number six learners were expected to state whether they learned best using different kinds of colours. 45 (56%) learners agreed, 30 (38%) learners strongly agreed, 10 (13%) learners disagreed and 5 (7%) learners strongly disagreed. The majority of the respondents who supported this statement were mostly those who agreed, supporting the statement that they learned best when using different kinds of colours.

I have problems understanding mathematical language

Learners in statement number seven were expected to state whether they had a problem with understanding mathematical language. 35 (43%) learners agreed, 5 (6%) learners strongly agreed, 20 (25%) learners disagreed and 20 (25%) learners strongly disagreed. The majority of the respondents which were 35 (43%) learners agreed that they needed help, because they did not understand the mathematical language whereby they were given symbols to write and in word sums they experienced difficulties as they were unable to read or spell.

4.3.2 Table 7. Statements and scores of the respondents

Statements	A	%	S A	%	D	%	S D	%	Totals	%
Statement	50	63	18	23	7	9	5	6	80	100
Statement	25	31	32	40	11	14	12	15	80	
Statement	60	75	15	19	3	4	2	3	80	100
Statement	52	65	11	14	8	10	9	11	80	100
Statement	36	45	22	28	15	19	7	9	80	100
Statement	45	56	30	38	2	3	0	0	79	98
statements	35	43	5	6	20	25	20	25	80	100

4.3.3 SUMMARY OF RESEARCH QUESTION 2

This study had as its aim to investigate the numerical errors that learners encounter in the foundation phase. Accordingly the respondents were given 7 statements to which they had to respond. On the statements the majority agreed to all the statements that they made numerical errors. Regarding mathematical language, they always made mistakes, confused the mathematical signs, but they learned best using colours, even though they did not finish their tasks, mainly because they had problems with reading the numbers that contain more than 1 digit and furthermore, they found it difficult to copy the numbers.

4.4 ANALYSIS OF RESEARCH QUESTION 3

This study intended to assist teachers in controlling numerical errors. In this section teachers were given 7 statements (section B) in a questionnaire (cf appendix 2) about the implementable strategies of numerical error analysis in the foundation phase. The majority of the respondents agreed in these statements that they needed assistance in order to control numerical errors in the foundation phase.

4.4.1 Which implementable strategies are present in numerical error analysis?

This section comprises illustrations and discussions about the biographical data and background information of the teachers by using percentages. The biographical data of teachers included questions regarding gender, age, marital status, professional qualifications, years of teaching experience, the area where the school was located, race and the kind of a school where they taught. Quantitative and qualitative analyses were used to find the strategies to measure shared variables believed to attribute to numerical errors in foundation phase mathematics.

There were sixteen (16) participants, of which 12 were female teachers and 4 male teachers in the foundation phase, one from each of the 16 participating schools. All the participants agreed to participate in the research.

The same respondents participated in section A (cf appendix 5). All the participants agreed to participate in this research section.

4.4.1Table 8: Biographical /Personal Analyses

Educator participants	Gender	Age	Marital status	Professional qualification	Years of teaching experience	Area where the school is	Race	Kind of school

						located		
Educator	female	35	Married	H.D. E	5	Rural area	African	Public school
Educator	female	37	Single	P. T. D	8	Big city	African	Public school
Educator	male	33	Single	H. D. E	4	Big city	English	Model c school
Educator	female	38	Married	P.T D	10	Rural area	African	Public school
Educator	female	40	Married	S.T.D	8	Rural area	African	Public school
Educator	female	41	Married	H.D.E	12	Big city	English	Model c school
Educator	female	41	Single	P.T.D	15	Rural area	African	Public school
Educator	female	43	Divorced	P T.D	15	urban	African	Public school
Educator	male	43	Widow	S.T.D	16	urban	African	Public school
Educator	male	45	Married	P.T.D	6	Small city	African	Public school
Educator	male	50	Married	P.T.C	3	rural	African	Public school
Educator	female	50	Divorced	P.T.C	17	Small city	African	Public school
Educator	female	53	Divorced	P.T.C	19	urban	African	Public school
Educator	female	53	Married	P.T.D	19	urban	African	Public school
Educator	female	55	Widow	P.T.C	9	rural	African	Public school
Educator	female	56	Married	P. T.C	7	rural	African	Public school

Section B of this questionnaire (cf appendix 5) has 7 statements, investigating whether teachers were aware of strategies for numerical errors that learners encounter in the foundation phase.

4.4.2 ANALYSIS OF STATEMENTS THAT ARE EXPOSED TO IMPLEMENTABLE STRATEGIES.

Section B of this questionnaire (cf appendix 5) was concerned with the teachers' input about which strategies to be implemented to improve the numerical error analysis. The respondents were expected to respond true, false and uncertain in statements one, two and three. In statement four the respondents were expected to say twice a year, once a year or once in five years. In statement five the respondents were expected to respond agreed, disagreed and uncertain. In statement six respondents were expected to say yes, no and not sure. Only 16 teachers were used as respondents to this section.

In the sixteen (16) schools, as illustrated in table 4 chapter 4 it was noticed that most of the foundation phase educators were females. There were only four 4 (25%) male teachers in all the sixteen foundation phase schools researched, the majority 12 (75%). were females. All the teachers that received questionnaires indicated that they had several years of teaching experience. Their ages ranged from 35 to 56 as illustrated (cf Chapter 4, table 4).

The teachers who participated had different marital statuses, three (3) teachers were single, eight (8) teachers were married, three (3) teachers were divorced and two (2) teachers were widows. All sixteen (16) teachers had professional qualifications. Twelve (12) teachers were qualified to teach in the foundation phase with either a Primary Teacher's Diploma or a Primary Teacher's Certificate. Four (4) teachers were qualified to teach the higher classes with either a Higher Diploma in Education or a Secondary Teacher's Diploma.

Most of the schools used in this research were located in rural areas which accounted for seven (7) schools in the foundation phase. Four (4) schools participated in the urban areas, three (3) in the big city and two (2) participated in the small city. The majority of the schools were African schools which use their African language as medium of instruction, which accounted for thirteen (13) and three (3) schools were English schools which use English as their medium of instruction. Only one (1) of the urban area schools used in this research uses English as their medium of instruction. In this research only two different kinds of schools were used, as the researcher tried to identify which kind of schools has numerical errors. There were fourteen Public schools and two Model C schools.

Section B of this questionnaire (cf appendix 5) is concerned with the implementable strategies that teachers will use to improve the errors in foundation phase mathematics. There were 7 statements to which respondents were expected to respond. Those were whether the statements were true, false or uncertain, twice a year, once a year and once in five years, they were also expected to respond yes, no and not sure and agree, disagree and uncertain.

The new ANA document will provide a great improvement to the learners

In response to statement number one where sixteen teachers were expected to answer true, false and uncertain, nine teachers ticked true, five teachers ticked false and two teachers ticked uncertain to this question. The respondents who supported “yes” were 12 females and 4 males. . Between the ages 30-39 there were 6, between 40-49 there were 6 and 4 ranged from 50 to 59 years old. 3 teachers were single, 8 teachers were married, 3 teachers were divorced and 2 teachers were widows.

Only 7 teachers were from the rural areas, 4 teachers were from the suburban area, 3 teachers were from the big city and 2 teachers were from the small city. 5 teachers had Primary. Teacher’s Certificates, 6 teachers had Secondary Teacher’s Diplomas, 3 teachers had Higher. Diplomas in Education and 2 teachers had Secondary .Teacher’s Diplomas. Two (2) teachers has 0-4 years of experience, 6 teachers had 5-9 years of experience, there were 4 teachers with 10-15 years of experience and 4 teachers with 16-19 years of experience. Therefore 14 schools were African-speaking schools using isiXhosa as their medium of instruction and 2 were English-speaking schools using English as their medium of instruction. There were no Afrikaans schools. There were 14 Public schools and 2 Model C schools. (Independent schools)

The need for Government to involve teachers at ground roots level prior to policy changes

In statement number 2 ten teachers answered true to this statement and six teachers ticked uncertain, no teacher ticked false to this statement. 12 teachers were females and 4 teachers were males, 6 teachers were between the ages 30-39, 6 between the ages 40-49, and 4 teachers were between the ages 50-59. 8 teachers were married, 4 single, 2 teachers were divorced and 2 were widows. 3 teachers had Higher. Diplomas in Education, only 2 teachers had Secondary Teacher's Diplomas, 6 teachers had Primary .Teacher's .Diplomas and 5 teachers had Primary Teacher's Certificates. This study was interested to learn whether teachers wanted to be involved in the areas in which numerical error analysis strategies could be included before the government changed the policies

14 schools were Public schools, 2 schools were Model C schools and there were no Afrikaans schools. 2 teachers were from the small city, 3 teachers from the big city, 4 teachers from the urban areas and 7 teachers from the rural areas. 2 teachers had 0-4 years of experience, 6 had 5-9 years of experience. There were 4 with 10-15 years of experience and 4 teachers with 16-19 years of experience. 14 were African-speaking teachers and 2 teachers spoke English. Ten (10) teachers were the majority in this statement, supported that government should involve them in policy changes in order that the strategies for numerical errors could be considered when policies are changed. The majority of the teachers who supported this statement were females, from the rural area schools, with teaching experience of 6 years and they were African-speaking teachers.

Numerical error analysis is becoming more difficult to manage and understand in schools

In statement number three, 10 teachers responded true, 4 teachers responded false and 2 teachers were uncertain. 12 teachers were females and 4 teachers were males. 7 teachers were from the rural areas, 5 teachers were from the urban areas, 3 teachers were from the big city and 2 teachers were from the small city. 3 teachers had Higher Diplomas in Education, 6 teachers had Secondary Teacher’s Diplomas, 5 teachers had Primary Teacher’s Certificates and 2 teachers had Secondary. Teacher’s Diploma. Two (2) teachers had 0-4 years of experience, 6 had 5-9 years of experience, 4 had 10-15 years of experience and another 4 had 16-19 years of experience.

There were 14 African schools and 2 Model C schools.14 teachers were African-speaking teachers and 2 were English-speaking teachers. 8 teachers were married, 4 teachers were single, 2 teachers were divorced and 2 teachers were widows. 6 teachers were aged between 30-39 years, 6 between 40-49 years and 4 between 50-59 years..The majority of respondents on this statement agreed that learners made errors in mathematics, therefore by implementing strategies for numerical errors this will assist. The reason could be because they experienced difficulty during teaching mathematics in Grade 3 foundation phase.

Frequent training is important to teachers for professional development

Teachers were expected to state whether there was a need for the teachers to get frequent training twice a year, once a year or once in 5 years. Ten (10) teachers supported the statement twice a year, 4 teachers responded once a year and 2 teachers responded once in five years. There were 12 female teachers and 4 male teachers. 8 teachers were married, 4 teachers were single, 2 teachers were divorced and 2 teachers were widows.6 teachers had Primary Teacher’s Certificates, 5 teachers had Primary Teacher’s Diplomas, 2 teachers had Secondary Teacher’s Diplomas and 3 teachers had Higher Diplomas in Education. 14 teachers were Africans and 2 teachers were English-speaking.

There were 14 African schools and 2 Model C schools. 2 teachers had 0-4 years of experience, 6 had 5-9 years of experience, 4 had 10-15 years of experience and 4 had 16-19 years of experience. 6 teachers were between 30-39 years old, another 6 between 40-49 years old and 4 between 50-59 years. 7 rural area schools were represented, 4 from the suburbs, 2 from the small city and 3 from the big city.10 teachers, the majority, supported this statement and it proves they are committed to implement the strategies for numerical errors after they attended training and this can only help teachers to assist learners with numerical error problems.

Topics need to change frequently to promote and sustain interest in learners.

In statement number six respondents were expected to say whether topics needed to be changed to promote interest in learners. Twelve (12) teachers agreed, 2 teachers disagreed and 2 teachers were uncertain. Of the teachers 12 were female and 4 male. Eight (8) teachers were married, 4 teachers were single, 2 teachers were divorced and 2 teachers were widowed, 6 teachers had Primary Teacher’s Certificates, 5 teachers had Primary Teacher’s Diplomas, 2 teachers had Secondary Teacher’s Diplomas and 3 teachers had Higher Diplomas in Education. 2 teachers had 0-4 years of experience, 6 had 5-9 years of experience, 4 had 10-15 years of experience and another 4 had 16-19 years of experience.

14 teachers were Africans and 2 teachers were English-speaking. There were 14 African schools and only 2 Model C schools. 6 teachers were between 30-39 years old, 6 between 40-49 years old and 4 between 50-59 years old. 7 teachers were from the rural areas, 4 teachers were from the urban areas, 2 from the small city and 3 from the big city. The majority of teachers, 12,

agreed with the statement and supported that topics needed to be changed frequently to arouse interest in the learners when teaching. This will help the teachers to improve the culture of teaching and learning.

The curriculum should be characterized by being relaxed, welcoming, empathetic and having a low stress atmosphere.

In statement number six, 10 teachers said yes and 4 teachers said no and 2 teachers were not sure. 12 teachers were females and 4 teachers were males, 14 schools were African schools and 2 schools were Model C schools, 14 teachers were Africans and 2 teachers spoke English. Two (2) teachers had 0-4years of teaching experience, 6 had 5-9 years of teaching experience, 4 had 10-15 years of experience, and 4 had 16-19 years of teaching experience. 6 teachers were between the ages of 30-39, 6 between 40-50 and 4 between 50-59 years, 8 teachers were married, 4 teachers were single, 2 teachers were widows and 2 teachers were divorced. 6 teachers had Primary Teacher's Diplomas, 6 teachers had Primary Teacher's Certificates, 2 teachers had Secondary Teacher's Diplomas and 2 teachers had Higher Diplomas in Education. Seven (7) teachers were from the rural areas, 4 teachers were from the urban areas, 2 teachers were from the small city and 3 teachers were from the big city, 10 teachers who responded yes formed the majority in this statement, therefore if the curriculum was to be empathetic, relaxed and welcoming it would be easy to implement it.

4.5. CONCLUSION

The research questions formed the basis of the underlying numerical errors analysis in foundation phase mathematics. There were two sections A and B (cf appendices 4 and 5) in the questionnaires presented in this chapter. In the sections of the learners' questionnaire the exam test whereby learners were exposed to answer the questions, was cited. Furthermore the relationships underlying numerical errors were analyzed and conducted using descriptive statistics to establish the authenticity of the data. The descriptive statistics sought to describe the participants with the assistance of their teachers, according to their background and personal characteristics. The data was checked to establish the overall distribution of mathematical scores. The last section concerned the teachers and related to the strategies of the numerical error analysis. The data analysis involved the personal and biographical profiles of teachers in order to seek a way to improve the culture of teaching and learning in the foundation phase. Findings of chapter 4 are presented and discussed in detail in the next chapter.

CHAPTER 5

DISCUSSIONS OF THE FINDINGS

5.0 INTRODUCTION

In the previous chapter the results of the study were discussed. In this chapter the researcher drew from the literature study and the empirical research with regard to numerical errors analysis to explore in order to further interoperate the main research questions as noted below. Discussions and recommendations are highlighted and made on the basis of the findings in this investigation which have educational implications to improve the experiences in numerical errors in teaching foundation phase mathematics. This study involved 80 learners and 16 teachers who were all investigated by means of questionnaires.

The present study sought to answer the following research questions as studied in foundation phase mathematics:

- What are the underlying numerical errors that learners encounter in obtaining, processing, retaining and producing information contained in foundation phase mathematics?
- What are the relationships underlying numerical error analysis?
- What are the implementable strategies present in the numerical error analysis?

The findings proved that there are many different reasons that caused learners to make numerical errors. The following section gives a summary of the research findings as a whole.

5.1 SUMMARY OF THE MAIN FINDINGS

In this chapter a summary of the entire thesis, based on the main research questions and the relationship with the theoretical framework, is provided. The findings of the research questions in this study are presented below.

5.2 RESEARCH QUESTION 1

5.2.1 What are the underlying numerical errors that learners encounter in obtaining, retaining, processing and producing information contained in mathematical tasks in foundation phase mathematics?

The results of the descriptive analysis of the first research question in this study showed 80 learners answered the questionnaire which was based on the test examination, and the findings showed that 70% of the respondents, the majority, were able to count backwards and forwards with only a few learners making numerical errors. Only in this question learners showed that they had the ability to count and needed no assistance. In section A questions 1 to 5 of the questionnaire, for example in calculations, the calendar, the graph and problem-solving mathematics, 75% of the respondents made many errors. Some learners had problems with mathematical language and others had problems with mathematical signs and word sums.

The respondents were expected to show all the calculations by using mathematical signs, to put each number under its digit number using the place values, which are hundreds, tens and units. This was the most difficult part for them because the majority showed they needed assistance. 50% of learners confused the three digit numbers, confused also the mathematical signs and instead of subtracting the majority added, also when they were supposed to borrow a number from the tens if the number they used on top was smaller than the number at the bottom, most of them were lost. .

The respondents added instead of multiplying and when they were supposed to divide they also add, whereas division involves all the mathematical signs. Ansari and Holloway (2009) reported that tests that assess numerical magnitude processing in children should test these abilities under timed conditions. When symbolic representations of numerical magnitude are used, timed tests tapped into the ability of children to quickly access the semantic meaning that the symbols represent and to perform a comparison of the semantic representation of the two numerals. Therefore learners proved that they needed assistance to improve the underlying numerical errors. The findings of this study are consistent with studies that support the numerical error analysis of foundation phase mathematics.

The dilemma of having numerical errors, as Bornman (2010) states, is that ten to sixteen per cent of learners are perceived by their teachers as having a learning difficulty in counting. The Department of Education reported in its annual national assessment document (ANA 2011) that the country's schooling system performed well below in the areas of reading, writing and solving numerical problems. Therefore learners share the foundations of mathematics.

According to Pardesi (2008) there are common misconceptions, errors and mistakes learners make in the classroom. An error is a mistake that is repeated and which the learner thinks is correct, errors often occur when learners think they understand. Errors occur where little or no progression in terms of conceptual mathematical development across the foundation phase exists.

Goswami (2008) states that current standardized tests of numerical and mathematical processing do not routinely contain items that tap into the basic representations of numerical magnitude. Most of the available standardized tests consist of either calculations or so-called 'Mathematical Reasoning' tests. For the calculation tests, children are presented with a series of calculation problems of all four arithmetic operations and are required to solve these problems under timed or untimed conditions. For the tests of 'Mathematical Reasoning' children are asked to solve diverse items such as judging the order of items of varying sizes or solving illustrated word problems. These studies supported this research in the literature review and they were consistent with this study of the numerical errors in the foundation phase.

The first research question drew largely on the cognitive theory method to establish a context for the exploration of a test given to the learners to investigate their numerical errors and determine the content validity and reliability of the factors of numerical error analysis. The cognitive fit theory was also established to explore the mental mode inside the head of the user, which helped the researcher to identify the numerical error disorders of the learners. The findings of this study are clearly consistent with studies that support these theories which had similar results, except that the majority of learners were able to count forwards and only a few learners were unable to count.

5.3 RESEARCH QUESTION 2

5.3.1. What are the relationships underlying numerical errors?

The method used in this research question was a 4 Lickert scale, which required that learners either Strongly Agreed, Agreed, Strongly Disagreed or Disagreed on a questionnaire in order to collect data from the learners. Statements were used to investigate the relationships underlying numerical errors. Learners were expected to use a tick if they Agreed, Strongly Agreed, Disagreed or Strongly Disagreed.

The majority of the 80 respondents answered the 7 statements. Only one respondent decided not to answer the statements. 60% of the respondents formed the majority who showed by answering the questionnaires that they made errors, which could have a variety of causes, but the statements showed they need assistance.

I always make mistakes when counting

85% of the respondents gave a positive response that they agreed and therefore they need help to overcome the mistakes they persistently make in mathematics.

According to Radatz (2005) errors are the function of other variables in the educational process. Errors in the learning of mathematics are the result of a very complex process. A sharp separation of the possible causes of a given error is often quite difficult because there is a close interaction among causes. The findings should help to improve avoidance of the numerical errors in the foundation phase.

I have a problem in reading the numbers which contain more than one digit.

In chapter 4, section A of the above statement the majority of the learners (50%) confirmed that they strongly agreed they had a problem with the place value, because in order for a child to be able to differentiate the value of numbers he or she must be able to know the place value, that is hundreds, tens and units and this is a crucial stage or level in mathematics. For the child to be able to count in addition, subtraction, multiplication and division he or she must be able to know the place value to get the correct answer. This study sought to find out which errors exist when learners calculate and use the places value of numbers in mathematics in the foundation phase.

Pipping (2005) suggests that errors of interference, in which different operations or concepts interfere with each other, will be of great value in exposing numerical errors. The following examples in section B of chapter 4 were consistent with the studies that support the interference between the algorithms for addition and subtraction. As learners experience difficulties in numerical errors in foundation phase mathematics, this study aimed to improve the methods and strategies concerned with numerical errors that learners encounter in foundation phase mathematics.

The majority of the respondents agreed that they had problems when copying the numbers, and this statement informs teachers that they need to supervise the learners to avoid many errors while doing their work in class and even the parents need to thoroughly supervise their homework.

Quinn (2011: 125) states that “the neuroscience studies confirm that incorrect internalizations quickly become difficult to correct. It is therefore vital that they be found and fixed as soon as possible. To accomplish this, every assignment must be checked for correctness and learners should be required to locate and correct errors in their work record. Always having to find error provides consistent reinforcement for accuracy and good work habits.”

According to Blakemore (2005) the areas in the brain that “do mathematics” are fragmented into specialized systems. The inputs and outputs of these areas must be integrated in order for people to estimate and calculate correctly. Lack of integration is one potential explanation for dyscalculia, the difficulty that some children have in mathematical performance. The results of this study are consistent with these researchers as they support the numerical errors that are present and the strategies that can be used to help the learners in the foundation phase.

Mathematical signs confuse me

65% of the respondents supported the statement that the mathematical signs confused them. For example 12 becomes 21, 51 becomes15 and many more, which means learners still need to understand the number operations in mathematics. This is very important because if a child could not handle number operations, according to the CAPS document, that child cannot proceed to the next class.

Bornman (2010) reports that dyscalculia means to “count badly” which describes the essence of impairment. Learners with dyscalculia may have difficulty in understanding simple number concepts, confuse similar looking numbers (e. g. 6 and 9, 3 and 8), lack an intuitive grasp and have problems learning numbers and mathematical procedures. Particular difficulties are seen when they have to read numbers that contain more than one digit number and numbers that contain a zero, for example 1007 or 1087. Furthermore, they may be confused when reading some numbers, for example 12 may become 21 although at times they might experience no difficulty with this.

Errors of interference arise when a previously learned skill or algorithm, because of its similar processes, makes the learning of a new skill or algorithm difficult.

I have problems when copying the numbers

The majority of the respondents agreed that they have problems when copying the numbers, and this statement informs teachers they need to supervise the learners to avoid many errors when they are busy doing their work in class and even the parents need to thoroughly supervise their homework.

I always finish my tasks

Concerning this statement respondent gave a positive response that they always finished their tasks. Only 15 learners out of 80 disagreed with this statement, therefore teachers need to find a strategy to help them to finish their tasks by setting a time and to try to encourage them.

Deheane (2007) suggests the areas in the brain that do mathematics are fragmented into specialized systems where both sides manipulate Arabic numbers and numeric quantities. The inputs and outputs of these areas must be integrated in order for learners to calculate correctly. Lack of integration is one potential explanation for dyscalculia, a difficulty that some learners have in mathematical performance.

56% of the learners supported the statement that they learned best when using different kinds of colours.

According to Caldwell (2008) human beings are geared to think mathematically and all children are born with a mathematical mind. Children can learn emergent numeracy skills in the early grades through play, by manipulating concrete objects and in this they can get a sense of colour, size and texture.

The choice to undertake a descriptive analysis in the second research question was important, firstly to verify the validity and authenticity of the statistics as reported in table 4.5.1. The quantitative research method was used to frame the methodological structure and to expose the errors that are the cause of learner performance. The fit theory was used to ensure that the results fit the existing knowledge, and to ensure that the results of this study are well grounded scientifically and in existing knowledge theories. The reason for this study to be conducted was to investigate the numerical errors in the foundation phase and the strategies that can be used to improve mathematics.

5.4 RESEARCH QUESTION 3

5.4.1 What are the implementable strategies present in numerical error analysis?

Section A question 2 of the questionnaire (cf. appendix 2) provides biographical data and illustrations of the teachers' age, gender, marital status, teaching experience, professional qualifications, race, kind of school and the area where the school was located. There were only 16 teachers who participated in this research, of which 12 were females and 4 males, with their ages ranging from 35 to 56 years.

The majority of teachers were females 12 (75%) and their ages ranged from 35 to 56. 8 teachers were married, 3 were single, 2 were widows and 3 were divorced. 12 teachers had Primary Teacher's Certificates and Primary Teacher's Diplomas, 4 teachers had Higher Diplomas in Education and Secondary Teacher's Diplomas. Seven (7) schools were from the rural areas, 4 from the urban area, 3 from the big city and 2 from the small city. 13 schools used an African language as medium of instruction, only 1 African school and 2 other schools used English as their medium of instruction. 14 schools were African schools and 2 schools were Model C schools. This information is illustrated in table 4.6.3

In section B teachers were provided with statements on strategies that can be implemented to improve the numerical error analysis. The respondents were expected to answer whether true, false, or uncertain; once a year, twice a year, once in five years; agree, disagree or uncertain; yes, no or not sure. This section had six statements as illustrated in table 4.6. The majority of the respondents agreed that they needed assistance to help learners avoid the numerical errors in foundation phase mathematics.

The new ANA document will provide a great improvement to the learners

The majority of teachers who supported this statement were 12 (75%) females, of which 8 (50%) were married and their ages ranged from 30 to 49 years. 7 (44%) of the teachers were from the rural areas, and the majority taught in 14 (88%) African schools 6 (38%) had Primary Teacher's .Certificate qualifications, and 14 (87%) spoke an African language and most of those who supported this had 0-4 years of experience. This proves that teachers who supported this statement had a few years of teaching experience and the new ANA document would assist them when implementing strategies of the numerical error analysis. The Department of Basic Education (2010) reported in its annual national assessment document (ANA, 2011) that the schooling system performed well below in the areas of reading, writing and solving numerical problems. The distribution of scores by ANA emphasizes the greatest challenge that the country is facing is in numeracy and literacy. The female teachers supported this statement that the ANA document would be able to assist them to improve avoidance of the numerical errors in the foundation phase.

The numerical error analysis is becoming more difficult in schools

Again female teachers were the majority to respond yes in this statement. There were 10 (63%) who were all married (88%) and taught in African schools, the majority, 87%, spoke an African language and they had between 5-9 years of teaching experience. Their qualifications included Primary Teacher's Diplomas (38%) and their ages ranged from 40-49 years. 44% were from the rural areas. The respondents who supported this statement had teaching experience of 5-9 years, which means in their teaching of mathematics numerical errors are an increasing problem which makes teaching and learning more difficult.

Cranefield (2005) investigated Grades 1 to 3 in three schools in South Africa which included and involved 222, 257 and 240 learners in Grades 1, 2 and 3 respectively. The children's level of understanding was assessed by the use of four tests. An analysis performance, misconceptions and errors made by learners in each grade was achieved through an in-depth analysis of 48 learners. The results suggested that the majority of learners were unable to solve straight calculations, employ the strategy counting all and counting on, while none engaged in formal or innovative methods. There is no progression in terms of conceptual mathematical development across the foundation phase. In the findings of this study with regard to numerical errors female teachers agreed that they needed assistance to improve the quality of teaching and learning.

Frequent training is important to teachers for professional development.

Only 63% of the respondents supported the statement that training should occur at least twice a year. 87% of those came from the rural areas, and their ages ranged from 30 to 39 and from 40 to 49 years old. 75% of those were married and they had teaching experience of between 5-9 years. 87% were from African schools and they spoke an African language. The majority, 75%, were females. Teachers do show an interest in training on how to implement strategies of the numerical error analysis, as the problem with numerical errors becomes more difficult in schools. Singarpore's Ministry of Education reported (2006) that the concept of helping learners to monitor their own capacity to learn and to engage in mathematical thinking has received little attention thus far.

Topics need to change frequently to promote and sustain interest in learners

Teachers from the rural areas were the largest number (87%) who supported this statement. They were females, the majority (75%) spoke an African language and their schools were African schools. The ages of the respondents were between 40-49 and 50-59 years. The majority of the respondents (38%) had Primary Teacher's Diplomas and they had between 10 and 15 years of teaching experience. This suggests that according to their teaching experience teachers supported the statement that there was a need for the topics to be changed in order for the strategies of error analysis to be included to arouse the interest of learners.

Ginsburg (2000) suggested that the diagnostic and causal aspects of errors could give specific help to mathematics teachers by allowing them to integrate their knowledge of curriculum content with their knowledge of individual differences in children. The majority of educators in this questionnaire (cf. appendix 2 section B) were positive because it showed that they were willing to implement the strategies of numerical error analysis in mathematics with the help of the Department of Education.

The curriculum should be characterized by being relaxed, welcoming, empathetic and having a low stress atmosphere

The teaching and learning process should be relaxed for the learners to be ready to learn. Only 87% of the respondents who agreed with this statement were from the rural areas, they spoke an African language and they taught in African schools (75%). The majority of teachers had Primary Teacher's Certificates and they were between 40-49 and 50-59 years old with teaching experience of between 16 and 19 years. They were all females, and 75% were married. Because of their teaching experience they supported this statement with the hope this will minimize the numerical errors the learners encounter in mathematics.

According to Caldwell (2003) learning concepts at a concrete level (the use of five senses) must happen first before reading and writing of a mathematical sum happens.

Children must be encouraged to:

- Sort out and match objects in accordance with colour, size, length
- Count freely – no limitations
- Play freely with 3D objects (that allow them to learn the geometric language) i.e. top, under, above, next to and opposite.
- Sing, count and rhyme

Government needs to involve teachers at ground roots level prior to policy changes

The teachers should be involved when the policies are being changed by government. Only 63% of the respondents supported this statement, the majority of which were female teachers. Only 87% came from the rural areas and they spoke an African language. Their ages ranged from 40 to 49 years old (75%) and they taught in African schools. The respondents had Primary Teacher's Diplomas and they had between 5 and 9 years of teaching experience. The majority of the respondents who supported this statement were from the rural areas and the schools were also in the rural areas. They formed the majority of those affected by numerical errors.

Ginsburg (2000) suggests possible causes of learner's errors in mathematics. At first, errors in the learning of mathematics are not simply the absence of correct answers or the result of unfortunate accidents. They are the consequence of definite processes whose nature must be discovered. Secondly, it seems to be possible to analyze the nature and the underlying causes of errors in terms of the individual's information–processing mechanism. Thirdly, the analysis of errors offers a variety of points of departure for research into the processes by which children learn mathematics. Fourthly, the prevailing quantitative aspects of tests and similar methods for measuring achievement cannot provide sufficient criteria for efficient instructional procedures.

Consideration of the diagnostic and causal aspects of errors could give specific help to mathematics teachers by allowing them to integrate their knowledge of curriculum content with their knowledge of individual differences in children. Finally, one should note some difficulties in the present state of research into learner errors. In many of the examples cited above, it is often quite difficult to make a sharp separation among the possible causes of a given error because there is such a close interaction among causes. The same error can arise from different sources, and the same error can arise from different problem-solving processes in schools located in the rural areas and the government needs to involve all of those when planning to change the policies as there are many problems teachers experience.

This study used fit theory to predict the value of the independent variable of the third research question that is scores of teachers over against gender, age, location, language, and the type of school. Cognitive fit theory predicts the numerical errors that affect the overall performance of learners. The findings of this study in this research question reveal that the majority of teachers agree that they need to be trained for the strategies of numerical error analysis by the Department of Education as the Department already started the ANA program in 2011.

According to Pardesi (2008) there are common misconceptions, errors and mistakes that are made by learners in the classroom. A misconception can be understood as something a learner thinks is incorrect with regard to his/her understanding of a concept, an error is a mistake that is repeated and which the learner thinks is correct, while a mistake is something that a learner knows is wrong and allows it to be corrected. The reason for this study to be conducted was to make the learners aware of the errors in mathematics and the teachers to use the strategies of numerical error analysis present in foundation phase mathematics.

5.5 CONCLUSION

Errors are usually considered as a sign of inadequacy of teaching and learning. Based on the research questions of the numerical error analysis errors should be a part of improving learning and teaching. Findings of the study show that regarding the underlying numerical errors learners encounter in the foundation phase, the majority of the respondents (75%) agreed that they needed assistance to avoid numerical errors in the foundation phase, and they also agreed that they made numerical errors. The teachers should guide them in grasping numerical errors in the foundation phase. The majority of teachers agreed on the statements contained in the questionnaire that they also needed numerical error implementable strategies to guide them in alerting learners about numerical errors in the foundation phase (cf Appendix 5).

CHAPTER 6

SUMMARY, CONCLUSION AND RECOMMENDATIONS

6.0 INTRODUCTION

Chapter 6 provides a conclusion and recommendations with regard to the main research questions.

6.1 SUMMARY OF MAIN IDEAS

Chapter 1 of this study provided the background of the study, to give the context of the study. This was followed by the research statement. The other sections included the research questions and the limitation of the study. The research methodology including questionnaires, data analysis, sampling, ethical consideration and an outline of the study were completed. This study is important since valuable information will be generated to help learners to improve on numerical errors.

Chapter 2 concentrated on the literature review. It addressed the numerical errors in the foundation phase. It was noted that learners made many errors that caused problems. The other sections included research question 1, the underlying numerical errors in the foundation phase.

The Department of Education introduced the ANA program in 2011 to improve the learner performance in literacy and mathematics as the performance of learners was well below accepted levels. Secondly, the relationships underlying numerical errors in the foundation phase were considered. It was also noted by means of the literature review that most of the empirical studies on numerical errors attempted to identify a set of factors to help the researchers. Lastly, the strategies for the underlying numerical errors will improve the culture of teaching in the foundation phase. These, the cognitive and fit theories, were the theories used as the theoretical framework for the current study.

Weber (1990) reported that, cognitive theory is concerned with the presentation and tasks of the individual characteristics. A fit theory must therefore address these components, technology as a tool and technology as representation. Technology as a tool provides the physical interface for manipulating the technology as representation. Representation implies a model of the real world tasks, for example a graphical representation of a document or a mental model of a user. These will guide the researcher in identifying the numerical disorders which create a problem to the learners.

Chapter 3 looked at the research methodology and design. This was followed by a description of the selected design used in the study which was a survey design. The sampling technique, validity and reliability and questionnaires as well as the data analysis and interpretation, and ethical procedures were addressed. The cognitive and fit theories were used in chapter 3 to show the significant relationships in numerical error analysis.

Chapter 4 presented the findings and the interpretation of the empirical investigation carried out in this study. The other section presented the analysis of the research questions. The aim was to provide answers to the research questions and the analysis of the questionnaires to discover findings of numerical errors in mathematics. This chapter commenced with an overview of the specific research questions, descriptive analysis, demographics of respondents and the strategies for error analysis. The descriptive analysis in chapter 4 was to verify the validity and the authenticity of the statistics to expose the errors that are the causes of learner performances. This chapter concluded with an analysis of the research findings based on the hypothesis posed in chapter 1.

Ansari (2011:25) states that, “one of the most significant events in early number development is children’s development of counting skills. Representation and processing of numerical magnitude play a role in children’s development of counting. The development of counting is a step towards exact verbal number representation. Thus, children appear to use areas involved in attention, working memory and cognitive control during numerical magnitude processing to a greater extent than do adults. The intraparietal sulcus (IPS) plays a critical role in the processing and representation in the brain of humans and animals”.

Chapter 5 addressed the summary of the research findings given in chapter 4, the conclusion and the recommendations of the study.

Chapter 5 addressed:

1. The underlying numerical errors that learners encounter in obtaining, retaining, processing and producing information contained in mathematical tasks.
2. The relationships underlying numerical errors.
3. The implementable strategies present in numerical error analysis.
4. Conclusion and recommendations
5. Key findings: 1 the scores of the numerical error analysis, and recommendations.
6. Key findings: 2 the relationships underlying numerical errors, and recommendations.
7. Key findings: 3 the strategies to improve on the numerical errors, and recommendations.

6.2 KEY FINDINGS

In this section, the findings emanating from this chapter are presented. The purpose of this chapter is to help inform future research and interventions to improve the underlying numerical error analysis in mathematics. The recommendations are made with a view to propose concrete interventional strategies for the improvement of numerical errors in rural, small city, city and suburban schools.

Key finding: 1 The underlying numerical error analysis.

Based on research question 1, it was noted that the results of the numerical error analysis survey showed in sections A and B of the final examination test in chapter 4 (see figure 4.4.1) suggest that learners made numerical errors. The best probable reason for the low performance is a lack of sequencing, the language use and word sums. The numerical errors that learners made include, not showing all the calculations in addition, subtraction, division and multiplication in mathematics. Learners had 75% scores that were not correct and only 10% of learners understood. The majority of them managed to get only 50% of the numbers and operations, which means they understand counting forwards and backwards, because learners proved that they were able to count on numbers and operations. Only a few learners made errors.

Regarding the ANA (2011) results, the D. B. E (Department of Basic Education) (2010) reported in its Annual National Assessment document (2011) that the country's schooling system performed well below in the areas of reading, writing and solving numerical problems. The ANA (2011) results showed the Eastern Cape had the lowest percentages.

Recommendation: It is recommended that attention be paid to the use of language and word sums in order to improve cognition processes in foundation phase mathematics.

Key finding: 2. The relationships underlying numerical errors.

The relationships underlying numerical errors were based on research question 2 in chapter 4 (see figure 4.3.2). Only 60% of the respondents, the majority, showed by answering the questionnaire that they made errors which could have various causes and they needed assistance, as statements for example, they confused mathematical signs, they always did not finish their tasks, they had difficulties with reading numbers containing more than one digit number, came to light.

Halberda (2008) suggests there is a relationship between individual differences in numerical magnitude which plays a role in children's development of counting. Numerical errors can tell the teacher how far towards the goal the learner has progressed and consequently what remains for him/her to learn. This is significant if this study was to investigate the numerical errors in foundation phase mathematics.

Recommendation: This study recommends that learners are to be assisted time and again when reading or copying their work, so that they could have fewer errors in foundation phase mathematics.

Key finding: 3.The implementable strategies to improve on the numerical errors.

The results of research question 3 (figure 4.4.1) in chapter 4 were based on teachers' statements about which strategies to be used to improve foundation phase mathematics. This study found that only 75% of teachers, the majority, supported these statements and they were from the rural areas, they were female and from African schools. Teachers agreed that they needed to be trained to implement strategies on how to assist learners with numerical errors. These findings are similar to those of Siegler and Thompson (2005: 89) who state that "if one considers the case of mathematics it is clear that in order to solve calculation problems, one need to activate numerical magnitude representation. This raises the possibility that tasks such as number comparison and number line estimation can be used as educational measurements to characterize both large cohorts of children as well as provide a deeper understanding of the performance of individual children".

The test in section A question 1 can be very useful as a tool for the characterization of a learner's level of preparedness, which will in turn inform the kind of information that is being taught in school. Furthermore, such tests can be used to identify children who could be at risk of developing mathematical difficulties, such as developmental dyscalculia.

Recommendations: This study recommends that teachers be trained on how to implement strategies of numerical error analysis in foundation phase mathematics. Furthermore, teachers can use tests to identify learners who could be at risk of developing mathematical difficulties in the foundation phase.

6.3 CONCLUSION

The findings on numerical errors were presented and discussed in this chapter. The overall performance of learners (cf appendix 1) revealed that the majority of the respondents agreed that they needed assistance regarding numerical errors in the foundation phase because their performance was very low, for example in sequencing, mathematical language and confusing the place values. In the relationships underlying numerical errors, the statements showed that only 60% of the respondents, the majority, made mistakes, such as confusing mathematical signs, copying numbers and many more, and in question 2 (cf appendix 4) learners were assisted by their teachers to read those statements. Teachers were given statements in section B (cf appendix 5), namely the implementable strategies to assist and guide teachers to improve on numerical errors in the foundation phase. In those statements 75% of teachers agreed that they needed guidance to implement the strategies regarding numerical errors in the foundation phase.

This study also found that errors are usually considered to be a sign of inadequacy of the teaching and learning, however errors are a necessary part of learning and teaching mathematics in the primary system. This is not incorrect knowledge, but incomplete knowledge. An error may only surface once existing knowledge is applied to new situations. Errors occur where there is little or no progression in terms of conceptual mathematical development across the foundation phase.

6.4 RECOMMENDATIONS FOR FURTHER RESEARCH

- Errors that the learners make in mathematics in the foundation phase need to be supervised by teachers at school.
- In mathematics, teachers should take care of consequences, patterns, mathematical language, word sums and the place value which is very important when calculating addition, subtraction, multiplication and division
- Mathematics in the foundation phase is where the child starts to learn how to count, therefore I recommend further research be conducted in this area of the foundation phase.
- Involving learners in group discussions and projects will assist in building their self-esteem and developing their counting and calculating skills.
- Teachers should be encouraged to carefully analyze learners' numerical errors in the foundation phase.
- This study advocates the inclusion of the strategies of numerical errors in teacher training courses. This will help to enhance the quality of teacher preparation
- These studies should have significant implications for educational practices.

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P O Box 14

McGregor

6708

27th January 2014

Professor A Bayaga

Faculty of Education

University of Fort Hare

Private Bag X1314

ALICE

5700

Professor Bayaga,

Re: PUMLA PATRICIA NDAMASE / NZUZO

Dissertation: NUMERICAL ERROR ANALYSIS IN FOUNDATION PHASE (GRADE 3) MATHEMATICS

Herewith I confirm that the above document (apart from the Appendices already used) complies with the linguistic requirements for a text of this nature..

Sincerely

D H Barry

Telephone: 023 625 1709

Fax 086 622 3667

Appendix 1

NAME OF APPLICANT

<<Approved >>

Ethics Human 2011

OFFICE USE ONLY

Ref:	Date:

University of Fort Hare

Together in Excellence

LETTER OF PERMISSION TO THE DEPATRMENT OF EDUCATION

Enquires: P. Ndamase

Email: ndamase.pumla@gmail.com

Cell no: 0736342886

University of Fort Hare

Private Bag X1314

Alice

5700

26 October 2012

The Superintendent-General

Eastern Cape Department of Education

Private Bag X 0032

Bisho

5608

Re: request for permission to conduct research in an East London district schools.

I am a student and pursuing a Masters of Education (M. Ed) degree at the above mentioned institution and planning a research project for a full dissertation.

The title of my proposed research is: NUMERICAL ERROR ANALYSIS IN THE FOUNDATION PHASE (GRADE 3) MATHEMATICS. This is a quantitative study which will involve a sample of about 80 foundation phase learners from the 16 schools in the East London district. Learners commit numerical errors in their work. In order to reduce these errors educators and learners need to understand;

- What are the underlying numerical errors that learners encounter in obtaining, processing, retaining and produce information contained in mathematics in foundation phase?
- What are the relationships underlying numerical errors in the foundation phase?
- What implementable strategies are found in the numerical error analysis in relation to research?

The study will take a month interaction with collected data. Data collected will be conducted through questionnaires and will take a minimal use of school time. The study will not in any way harm the image of the department or the schools where research is conducted. Furthermore the anonymity of the concerned respondents, as well as confidentiality regarding information will be maintained. The data collected will be kept confidential until the research is over.

It is hoped the findings in this study will help both the educators and learners. If you would like to query anything about the study, you may contact the researcher or my supervisor at the University of Fort Hare, Faculty of Education (East London campus)

Thanking you in advance

Yours Faithfully

Ndamase Pumla (201202117)

Appendix 2

NAME OF APPLICANT | Ethics Human 2011

<<Approved >> OFFICE USE ONLY

Ref:	Date:

University of Fort Hare
Together in Excellence

LETTER TO THE SCHOOL PRINCIPAL AND THE SCHOOL GORVENING BODY (Floradale Primary school)

Enquires: P. Ndamase **Cell no:0736342886**

Email: ndamase.pumla @gmail.com

University of Fort Hare
Private Bag x 1314

Alice

5700

13 May 2013

The Headmaster

Floradale Primary School

P O Box 2073

5241

Dear Sir

RE: requestfor permission to use Grade 3 foundation phase learners in a research study. I am a Masters of Education student at the University of Fort Hare conducting a quatitative research in NUMERICAL ERROR ANALYSIS FOUNDATION PHASE (GRADE 3)MATHEMATICS, would like to formally request permission to use Grade 3 class in the research study.

I will distribute 80 questionnaires to 16 schools. Each school will have 5 learners and 1 teacher to complete the questionnaire. They will return and complte the questionnaire. The data collected will be used only for this research and the participants will not be identified.

I thank you in advanced

Yours faithfully
Ndamase Pumla **(Student no: 201202117)**

Page 1 of 1

FLORADALE PRIMARY SCHOOL

SIYAKUNQANDWA

ZIINKWENKWEZI

P.O BOX 2073

BEACON BAY

EAST LONDON

5241

Date :15 June.13

Dear Sir/Madam

ACCEPTANCE LETTER FOR THE APPLICATION TO DO RESEARCH IN GRADE 3

This is to inform you that we do accept your application to conduct research in grade 3 of the above mentioned school

Yours Faithful

N.C Ntsabo (Deputy Principal)

FLORADALE PRIMARY SCHOOL
P. O. BOX 2073, BEACON BAY
EAST LONDON, 5205

15 06 2013

PRINCIPAL'S SIGNATURE: ________________

Tel: 043- 748 1593
Email : floradaleschool@telkomsa.net

Appendix3

University of Fort Hare
Together in Excellence

ETHICAL CLEARANCE CERTIFICATE

Certificate Reference Number:	BAY03 1SNDA01
Project title:	**Numerical Error Analysis: Foundation Mathematics.**
Nature of Project:	Masters
Principal Researcher:	Pumla Patricia Ndamase
Supervisor:	Dr A Bayaga
Co-supervisor:	

On behalf of the University of Fort Hare's Research Ethics Committee (UREC) I hereby give ethical approval in respect of the undertakings contained in the above-mentioned project and research instrument(s). Should any other instruments be used, these require separate authorization. The Researcher may therefore commence with the research as from the date of this certificate, using the reference number indicated above.

Please note that the UREC must be informed immediately of

- Any material change in the conditions or undertakings mentioned in the document
- Any material breaches of ethical undertakings or events that impact upon the ethical conduct of the research

The Principal Researcher must report to the UREC in the prescribed format, where applicable, annually, and at the end of the project, in respect of ethical compliance.

The UREC retains the right to

- Withdraw or amend this Ethical Clearance Certificate if
  - Any unethical principal or practices are revealed or suspected
  - Relevant information has been withheld or misrepresented
  - Regulatory changes of whatsoever nature so require
  - The conditions contained in the Certificate have not been adhered to
- Request access to any information or data at any time during the course or after completion of the project.

The Ethics Committee wished you well in your research.

Yours sincerely

Professor Gideon de Wet
Dean of Research

03 September 2013

Appendix 4

QUESTIONNAIRE 1

The purpose of the study is to investigate the numerical error analysis-foundational mathematics.

Enquires : P.P Ndamase Mobile : 0736342886 Tele-Fax : 0437411357

E -mail ndamase.pumla@gmail.com.

School Questionnaire:

INSTRUCTIONS

1. This questionnaire has 2 sections A, B, and you are requested to complete all of them.
a. Your support will receive a helpful consideration that will improve the performance of the quality of Basic Education.
2. Complete questionnaires and returned them to the district office by the (28/02/2013).

Section A

This section is for the learners

The purpose of this investigation the numerical error analysis foundational mathematics

1. Answer the following questions?

i. Count backwards in fives from 95 to 45

ii. Count in twenties from 510 to 1000

Fill in: 120, 140, 160, _, _, _,_,260

1000, 990, 980, - ,- ,-, -,800.

2. Fill in:

120	100	80	60	40
60	50	-	-	-

3. The car has four wheels and one from the boot. How many wheels of 12 cars?

4. May 2012

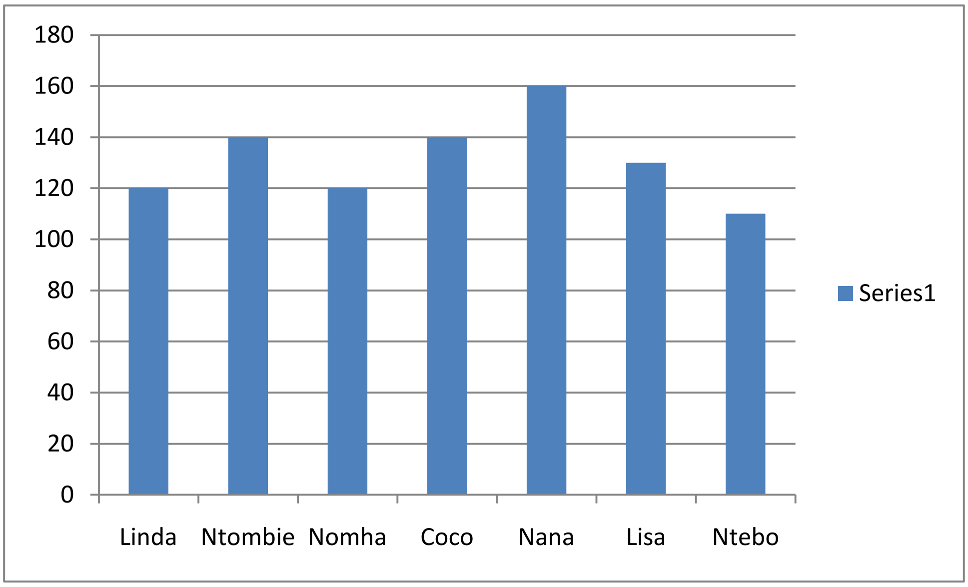
Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

I. How many days are there in May?

2. How many Sundays are there in May?

3. June starts on ---------.

5.The length of the netball players



a. The number of the netball players?

b. The tallest netball player?

c. The length of Coco is ------- cm

d. Ntebo's length is -------- cm

e. The player who has the same length as Coco is--------.

Section B

This section is for the learners

The purpose of this questionnaire is to investigate the significant relationship underlying the numerical error analysis.

Please use a tick if you, Agree (A) Strongly Agree (S A) Disagree (D) and Strongly Disagree.

	A	SA	D	SD
Do you always make mistakes when counting?				
Do you have a problem in reading the numbers which contains more than one digit number?				

Do you have problems when copying the numbers?				
Do mathematical signs confuse you?				
Do you finish your tasks?				
Do you come late to school?				
Learners learn best when using different kinds of colours				

APPENDIX 5

QUESTIONNAIRE 2

The purpose of the study is to investigate the numerical error analysis-foundational mathematics

Enquiries : P.P Ndamase Mobile : 0736342886 Tele-Fax : 043 7411357

E -mail ndamase.pumla@gmail.com.

School Questionnaire:

INSTRUCTIONS

1. This questionnaire has 2 sections A, and B you are requested to complete all of them.

a. Your support will receive a helpful consideration that will improve the performance of the quality of Basic Education.

2. Complete questionnaires and returned them to the district office by the 28/2/2013).

Section A

This section is for the teachers

The purpose of this questionnaire is to investigate the numerical error analysis

1 Personal detail (please tick the number that corresponds with your answer).

Gender	Female	Male		
Age	30-39	40-49	50-59	
Marital Status	Single	Married	Divorced	Widowed
Professional Qualification	P.T.C.	P.T.D.	S.T.D.	H.D.E.
Years of teaching Experience	0-4	5-9	10-15	16-19

The area where the school is located	Rural area	Suburb area	Small city	Big city
Race	African	English	Afrikaans	Other specify
The kind of a school	Public school	Model C School	Christian school	Other specify

Section B

1 = true 2= false 3 = uncertain

The new Anna document will provide a great improvements to the learners	1	2	3
The government needs to involve teachers at ground root level prior to policy changes	1	2	3
Numerical error analysis is becoming more difficult in schools	1	2	3
Frequent trainings are important to teachers for professional development	Twice a year	Once a year	Once in five years
Topics need to change frequently to promote and sustain interest to learners	Agree	Disagree	Uncertain

The curriculum should be characterized by being relaxed, welcoming empathetic and have low stress atmosphere	Yes	Not	Not sure