UNIVERSITY OF FORT HARE

STS 502

DEGREE EXAMINATIONS

June 2017

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Time: 3 HOURS

Subject: APPLIED TIME SERIES ANALYSIS

Marks: 100

This paper consists of 10 pages including cover page

Internal Examiner

Mr. C.S Marange

Dr. J Ndege

External Examiner

Prof D. J. de Waal

Instructions

1. This paper consists of 10 pages (including the cover page).
2. This paper is made up of 2 sections, Section **A** and Section **B.**
3. Answer **ALL** Questions in Section **A** (**compulsory Questions**) and either Question **5** or **6** in Section **B**.
4. Calculators may be used.
5. Statistical tables are provided.

SECTION A

ANSWER ALL QUESTIONS FROM THIS SECTION

QUESTION 1 [20 Marks]

a) Define weak stationarity.

[1 Mark]

b) Give an example of a non-stationary process. (Make sure to state the property of the process that varies through time)

[2 Marks]

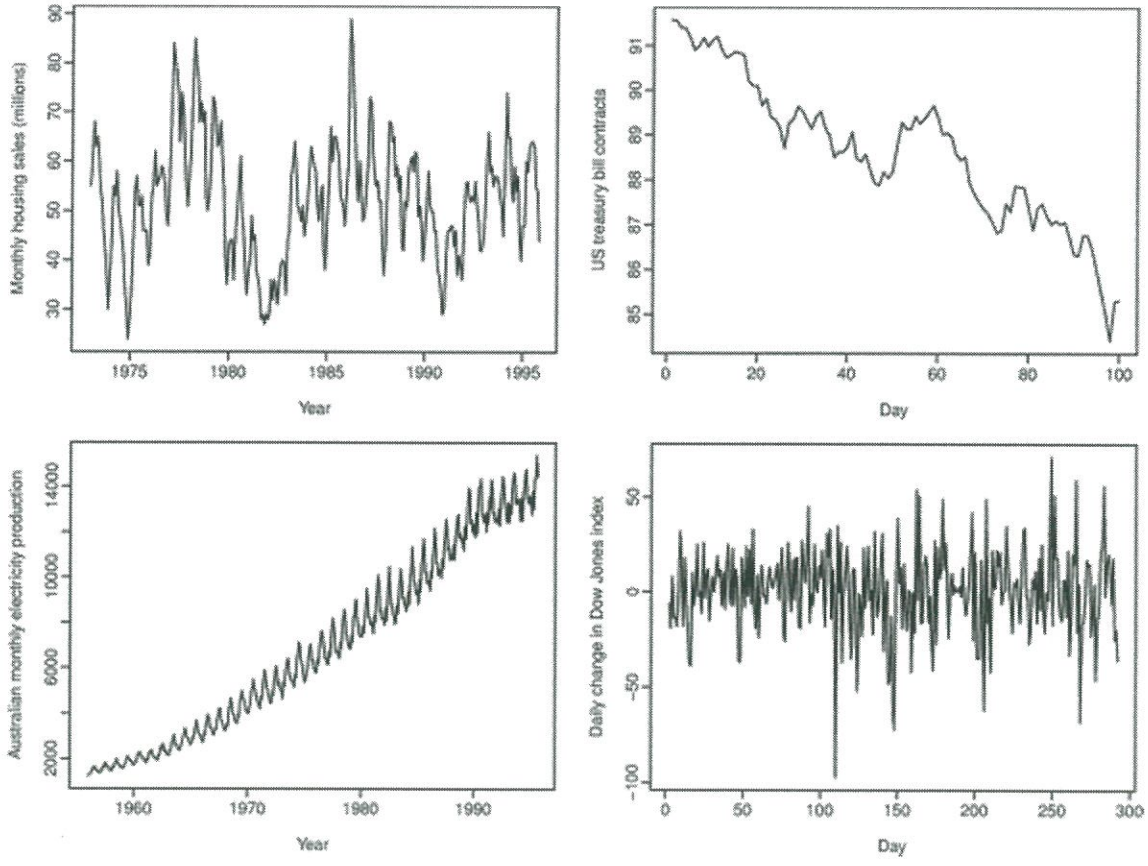
c) How are the empirical autocorrelation function and theoretical autocorrelation function used to identify a time series model?

[3 Marks]

d) Name and describe any three components of a time series. For each component, give a real life example that can lead to such component/s.

[6 Marks]

e) The following four examples show different combinations of the Time Series components.



State the components that exist in each of the examples given above.

i. The monthly housing sales (top left)
ii. The US Treasury bill contracts (top right)
iii. The Australian monthly electricity production (bottom left)
iv. The daily change in the Dow Jones index (bottom right)

[4 Marks]

f) Explain briefly any four differences between Time series and Regression analysis.

[4 marks]

QUESTION 2 [20 Marks]

a) The autocorrelation function (ACF) between X_t and X_{t+k}, is defined as:

$$\rho(k) = \frac{Cov(X_t; X_{t+k})}{\sqrt{Var(X_t)}\sqrt{Var(X_{t+k})}} = \frac{\gamma(k)}{\gamma(0)}$$

Use it to prove that $|\rho(k)| \leq 1$

[5 Marks]

b) Consider the two time series models in a first-order MA processes given below;

$$A: X_t = Z_t + \theta Z_{t-1}$$

$$B: X_t = Z_t + \frac{1}{\theta} Z_{t-1}$$

use the models to explain the invertibility condition.

[5 Marks]

c) Let $Z_t \sim N(0; \sigma_z^2)$ be our white noise. X_t is said to be an AR process of order p given by;

$$X_t = \alpha_1 X_{t-1} + \alpha_2 X_{t-2} .... + \alpha_p X_{t-p} + Z_t$$

Derive the general ACF using the above expression.

[5 Marks]

d) An observed autoregressive time series process of order 2 is given by the equation;

$$X_t = \frac{1}{12}X_{t-1} + \frac{1}{12}X_{t-2} + Z_t$$

Determine its autocorrelation function?

[5 Marks]

QUESTION 3 [20 Marks]

a) Given the following ARMA model;

$$X_t = 0.5X_{t-1} + Z_t - 0.3Z_{t-1}$$

i. Find the ψ-weights

[2 Marks]

ii. Find the π-weights

[2 Marks]

b) Show that the ACF of the ARMA (1, 1) $X_t = \alpha X_{t-1} + Z_t + \beta Z_{t-1}$ where $|\alpha| < 1$ and $|\beta| < 1$ is given by;

$$\rho(1) = \frac{(1+\alpha\beta)(\alpha+\beta)}{1+\beta^2+2\alpha\beta} \text{ and } \rho(k) = \alpha\rho(k-1) \quad \forall k = \pm 2; \pm 3....$$

[6 Marks]

c) Given a model;

$$X_t = 1.1X_{t-1} - 0.2X_{t-2} + Z_t + 0.7Z_{t-1},$$

where $Z_i \approx iiN\left(0; \sigma_z^2\right)$,

Rewrite the model in backward shift operator notation.

[3 marks]

d) Define lead time as it is used in forecasting in time series analysis.

[2 Marks]

e) For an ARMA (p, q) model, we can write;

$$X_t = \sum_{i-1}^{\infty} \psi_i \beta^i Z_t$$

Since $X_{t+h} = Z_{t+h} + \psi_1 Z_{t+h-1} + \psi_2 Z_{t+h-2} + ...$ give the variance of the h- step forecast error.

[5 marks]

QUESTION 4 [20 Marks]

a) Buffalo District Municipality wants to understand the pattern of the number of beneficiaries of the child support grant. They collected the following quarterly data on the number of beneficiaries of the child support grant over a period of 4 years: (Data provided in Appendix A)

i. Obtain the quarterly seasonally-adjusted indices. Interpret the values.

[5 Marks]

ii. Compute the least square estimates for the trend line. Use them to obtain estimates for the four quarters of 2017.

[10 marks]

b) ARIMA models include a parameter, d, that controls the number of times a time series is differenced before being modeled by an ARMA process.

i. Why is differencing a time series sometimes necessary?

[2 marks]

ii. How could you choose the amount of differencing required for a particular time series?

[2 marks]

SECTION B

ANSWER ONE QUESTION FROM THIS SECTION

QUESTION 5 [20 Marks]

a) Why is diagnostic checking necessary in the Box-Jenkins modelling procedure?

[2 Marks]

b) In diagnostic checking, name any 3 checks that we test for, from the residuals and describe how you would test, and state clearly what one should expect in order to say that the fitted model is an adequate model.

[6 Marks]

c) For a certain ARIMA model, an analysis was performed and the output produced is shown in Appendix B to F.

i. Interpret each of the graphs in the appendices (B to F).

[10 Marks]

ii. From the plots, what conclusions can you make about this ARIMA model?

[2 Marks]

QUESTION 6 [20 Marks]

a)

i. Explain the term "spectral analysis" as used in time series.

[2 Marks]

ii. Consider the spectral distribution function given by the model

$$X_t = \mathrm{R}\cos(\omega \mathrm{t}+\theta)+\mathrm{Z}_t$$

Explain the features of this model.

[4 Marks]

iii. Determine the spectral representation (of the autocorrelation function) of the model.

[5Marks]

b) For the AR(1) model

$$y_t - \mu = (1 - \phi B)^{-1} e_t$$

Use the covariance-generating function:

$$g_y(z) = [\sigma^2 / (1 - \phi_1 z)(1 - \phi_1 z^{-1})].$$

to calculate the spectrum of the model.

[5 Marks]

c) Derive the power spectral density function of a purely random discrete process.

[4 Marks]

Total: 100

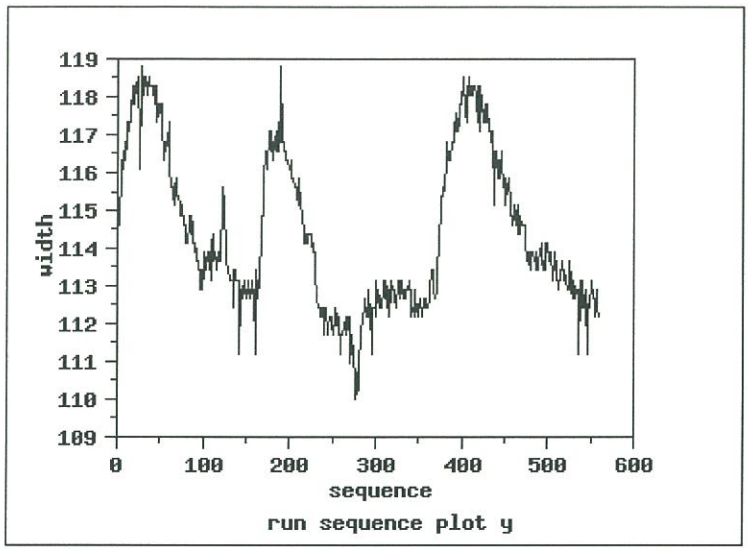
APPENDIX A

Ratio-To-Moving-Average Method (multiplicative form)

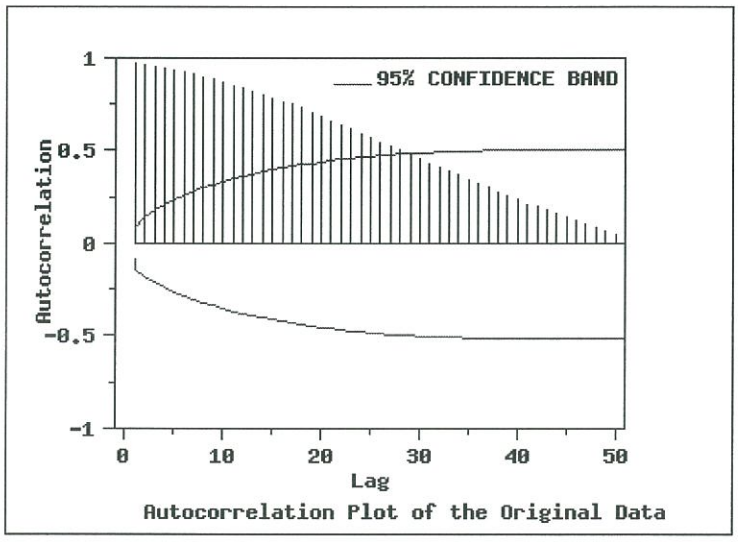
	1st Quarter	2nd Quarter	3rd Quarter	4th Quarter
2013	153	189	221	215
2014	133	177	241	228
2015	145	200	187	201
2016	111	170	243	178

Year	Quarter	t	X_t	4-Point Totals	4- Point Moving Averages	Centred Moving Averages	Seasonal Index
2013	1st	1	153				
	2nd	2	189				
				778	195		
	3rd	3	221			192	115.1
				758	190		
	4th	4	215			188	114.36
				746	187		
2014	1st	5	133			189	70.37
				766	192		
	2nd	6	177			193.13	91.65
				779	195		
	3rd	7	241			196.25	122.8
				791	198		
	4th	8	228			200.63	113.64
				814	204		
2015	1st	9	145			196.75	73.698
				760	190		
	2nd	10	200			186.63	107.17
				733	183		
	3rd	11	187			179	104.47
				699	175		
	4th	12	201			171	117.54
				669	167		
2016	1st	13	111			174.25	63.702
				725	181		
	2nd	14	170				
	3rd	15	243				
	4th	16	178				

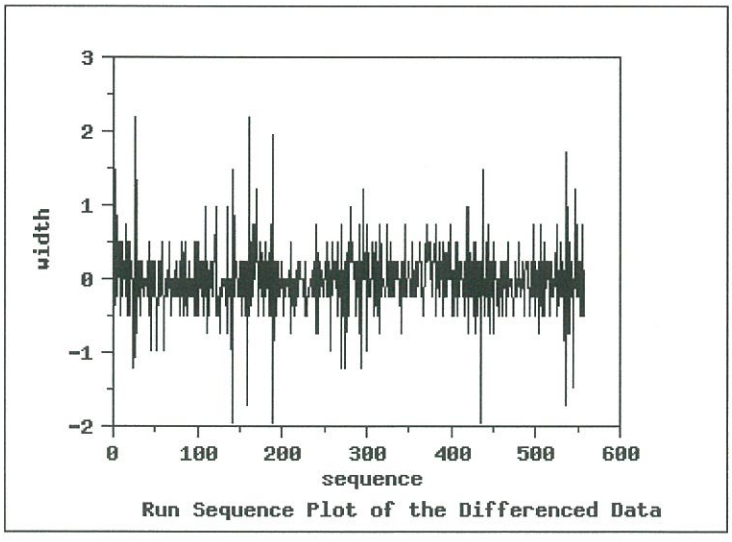
APPENDIX B



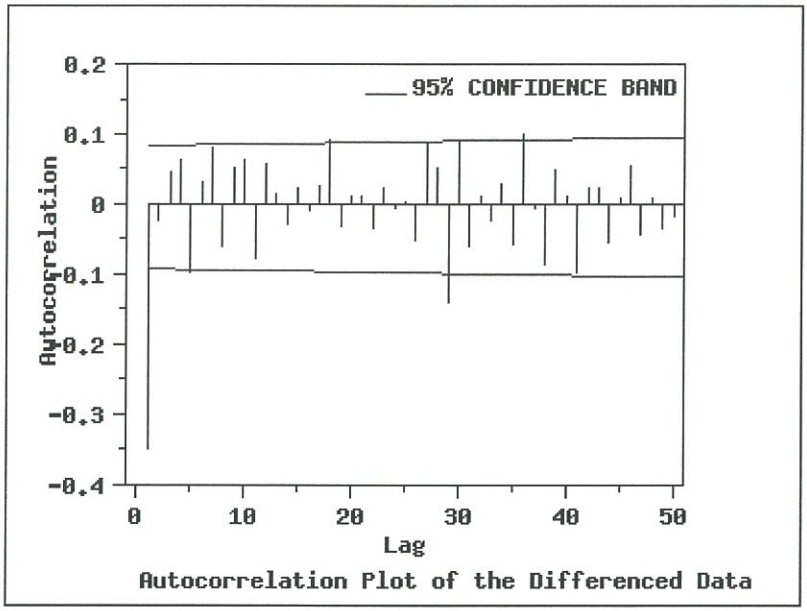
APPENDIX C



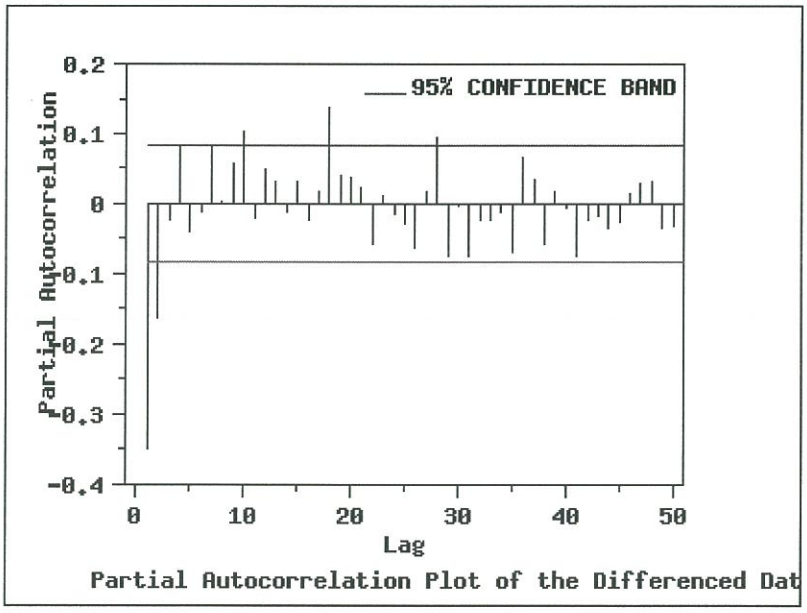
APPENDIX D



APPENDIX E



APPENDIX F



SWK 214 SUPP

QUESTION1

Human Diversity; Cultural Competency; Oppression; Populations –at-Risk; Priviledge; Power; Acclaim;Group Membership and values;Societal Perspectives on Group Differences; Empowerment; Strength Perspective

QUESTION2

System; subsystem; boundary; equifinality; Input; output; Feeback; Positive Feedback; Entropy; Negative entropy

QUESTION3

Enabler; Mediator; Resolving conflicts; Integrator/Coordinator; Manager; Educator; Analyst; Broker; Facilitator; Advocate; Initiator; Negotiator

QUESTION4

Biological Theories; Behavioural Theories; Interactionist theories

QUESTION5

- Trust vs Mistrust
- Autonomy Vs Shame & Doubt
- Initiavive Vs Guilt
- industry vs inferiorirty
- Identity vs Role confusion
- Intimacy vs Isolation
- Generativity vs Stagnation
- Integrity vs despair