

Challenges on learning and teaching of fraction operations in Grade 6 - a case study in a South African primary school

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PLAGIARISM DECLARATION

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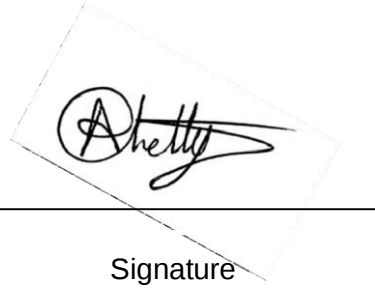
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ABSTRACT

Literature review indicates that world over, fraction operations in primary school are challenging not only for learners but also for teachers and South Africa is not an exception. There have been studies on challenges of fraction operations, but could not locate any study at a Grade 6 level in East London education district. The purpose of this study was to assess the specific strengths and challenges in the learning and teaching of fraction operations in Grade 6 in one of the schools in this district. The research paradigm for this study was post-positivism which adopted a mixed methods approach, utilising both quantitative and qualitative methods. The population consisted of three teachers and 98 Grade 6 learners in one primary school in East London. The learners were under 18 years and in order to comply with South African ethical regulations involving humans, a letter was sent out to all 98 Grade 6 learners' parents requesting their consent. Only 59 learners brought back the consent letters signed by a parent or legal guardian and hence, the sample consisted of 59 Grade 6 learners. There were two instruments: a survey questionnaire with 15 items for learners and two interview schedules, one for teachers and another for learners. The time to complete the questionnaire was 45 minutes and the time to complete the interviews was less than 20 minutes for learners and 40 minutes for teachers. As part of a pilot study, the questionnaire was given to six learners in 2017 cohort. Also, the questionnaire was given two colleagues teaching Grade 6 from two different schools. Furthermore, two learners from the 2017 cohort and two colleagues from two different schools were interviewed, respectively, to check the validity of the learners' and teachers' interview schedules. Based on the feedback, the instruments were modified for use in 2018. The learners were given the questionnaire to answer first before 10 were selected for interviews. Three teachers, one each from Grades 5, 6 and 7 were interviewed. The quantitative data were analysed to lead to a descriptive data presentation. The qualitative data were analysed thematically. It was revealed that there were similarities and differences in the manner in which teachers teach and the challenges they identified. There were similarities and differences among the responses from the learners. They found certain fraction operations challenging and their strengths were similar as well. The common outline of the learners' strengths were being able to do like-fractions while, their challenges were on doing unlike-fractions. The teachers too concurred with the learners' views. The learners got confused because they did not

understand fractions as part of a whole. The teachers indicated knowledge of more than one teaching method. The teachers also pointed out how times tables is an essential part of being able to do operations of fractions. This study could be beneficial for teachers, learners, parents and curriculum developers. The curriculum developers need to revisit the time frame, the transition between the grades, and how often the concept of fractions appears throughout every term.

Key Words

Learners' challenges on fraction operations, Like and unlike fractions, Fractions as a part of a whole, Lowest Common Denominator.



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LIST OF ACRONYMS

NAEP – National Assessment of Educational Progress

UNESCO – United National Educational, Scientific and Cultural Organisation

SEACMEQ – Southern and East African Consortium for Monitoring Educational Quality

MLATI – Mathematics Learning and Teaching Initiative

HoDs – Head of Departments

CAPS – Curriculum and Assessment Policy Statements

LCD – Lowest Common Denominator

CPTD – Continuing Professional Teacher Development

OECD – Organisation for Economic Cooperation and Development

CPD – Continual Professional Development



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CHAPTER ONE

INTRODUCTION AND BACKGROUND

1. Introduction

This chapter deals with introduction and background to the study which is followed by the problem statement, research questions, the purpose of the study, the objectives and significance of the study. Then it addresses operational terms, limitations and delimitations, followed by the theoretical framework, the relevance of the theoretical framework to this study, chapter outline and a summary of chapter one.

1.1 Background to the study

Challenges in the learning and teaching of operations involving fractions are a worldwide concern. According to the National Assessment of Educational Progress (NAEP), understanding fractions was a challenging area for North American students (NAEP, 2005). United Nations Educational, Scientific and Cultural Organisation's (UNESCO) Institute for Statistics (2012) posits that the proportionality and the attendant topics in the area of fractions represent some of the most abstract and challenging areas in primary school mathematics.

In a study in Ontario, Canada, Bruce, Chang, Flynn and Yearley (2013) sought to find out what difficulties learners experienced with the concept of fractions and how teachers were teaching the concept to the learners. They used a sample from Grades 4 to 7 in three schools. In the practical task, learners were asked to place the fraction $\frac{1}{2}$ on a number line 0-4, learners placed the half fraction at the half way point where the whole number 2 should have been placed. It was discovered that the learners struggled to recognise the place value of a whole number and fractions. The findings of the research highlighted the following concerns on teaching the concept of addition and subtraction of fractions: a) developing lessons which expose and work with the children's intuitions about what they already understand about fractions, b) the language used in fractions needs to be made suitable to learners to enhance greater understanding of fractions and, c) ensuring smooth transition between meanings of fraction and operations with fractions. Creating fraction problems which highlight

underpinning fraction concepts rather than focusing on the out-of-context procedures and algorithms is also relevant.

A study in France and Italy by the European Commission (2011) found that learners only learnt fractions at a basic level in their primary schools. The teachers in this study reported that learners spent half the time on fractions compared to the time they spent on practising adding, subtracting, multiplying and dividing whole numbers. However, on progression to secondary school, the time spent on fraction equations increased to at least 48%.

Bruce et al. (2013) built on Empson and Levi's (2011) theory regarding how difficulties of teaching and learning fractions start early in the primary school years. The learners' understanding of fractions were identified as: difficulty in understanding and representing fraction relationships, the confusion of the role of the numerator and denominator and the relations between them. It gets worse as the children go through secondary and tertiary schooling and this affects many career options for the individuals such as those in a wide-range of medicine and health care, construction and computer programming, science fields, technology and engineering, just to mention a few. The results of the research showed that there were gaps in the learners' knowledge and misconceptions were revealed through their drawing of fractions to represent portions, for example, learners represent the numerator and denominator as separate numbers and not as part of one whole.

A study from Tanzania reported by Ndalichako (2013) views fractions as one of the most complex concepts that children encounter in their primary education years. The main objective of the study was to identify possible reasons as to why the learners got wrong answers in the Mathematics examination. The collected data revealed that out of 865 221 learners who took part in the Primary School Level Examinations in 2012, only 162 078 passed. The study concluded that learners were confused about the different fraction operations as they treated numerators and denominators as separate entities. Ndalichako (2013) stated that there was a lack of understanding of the appropriate procedures to apply in solving a problem. For instance, children would add the numerators and the denominators showing that they did not understand the concept when adding or subtracting fractions. The study also drew on the aspects of Teachers' Knowledge of the concept and how they were teaching the learners.

A study done in Zimbabwe by Lillias and Chagwiza (2012) investigated the reasons behind the methods used by teachers to teach fractions. The results from the study showed the traditional methods used by teachers with basic Mathematics knowledge were used to teach fractions. Teachers drilled learners through rules and algorithms. Instructional approaches used by teachers promoted procedural understanding of fractions and they believed that these methods would enhance learners' problem solving capabilities. Lillias and Chagwiza (2012) cited Carpenter, Fennema and Romerg (1993) on the negative outcome of using traditional methods on procedural processes which limited learners' ability to fully understand the actual concept as they relied on the rules they were taught. The traditional method started to become overwhelming as there were many rules for the various operations. Learners only focused on the rules and not on the actual process. It led to the confusion and misunderstanding of different operations.

Several researchers aver that primary maths education crisis is due to the low levels of the primary Teachers' Knowledge (Fleisch, 2008; Carnoy, Chishlom, and Chilisa, 2012; Taylor and Taylor, 2013). A study conducted in Cape Town, South Africa revealed that many teachers who teach mathematics do not have adequate content knowledge and this is one of the causes for poor performance by learners. A study by Venkat and Spaul (2015) investigated the knowledge of 401 Grade 6 teachers who taught Mathematics to see whether or not they had mastered the basics of the content knowledge and the results did not reveal optimum knowledge. Other researchers such as, Taylor and Vinjevold (1999), Carnoy, Chisholm and Chilisa (2012), and, Taylor and Taylor (2013) concluded that ongoing low learners' performances led to an increase in the interest to understand the teachers' characteristics, pedagogical practices and the lack of teachers' content knowledge as possible causes for learners' weak performance. The results of tests carried out by Southern and East African Consortium for Monitoring Educational Quality, SEACMEQ (2007) revealed that only 21% of the Mathematics teachers proved to have content knowledge which was acceptable while 79% were shockingly below the Grade 6 levels of understanding.

Pienaar (2014) cited Hecht and Vagi (2010) stated that one of the most persistent problems for children with mathematical difficulties was in solving problems that involved fractions. Pienaar (2014) argued that learners with difficulty in understanding

the meaning of fractions were likely to develop difficulty in the procedural processes in the learning area. This argument was illustrated with a practical example as to why it is vital to make learners understand that certain rules are to be applied to calculations involving fractions. For instance, in finding $\frac{1}{2}$ of $\frac{3}{4}$ the rule of multiplication of two numerators and the denominators by each other is used to get the answer $\frac{3}{8}$. In order to focus on the learners' need to understand the importance of using the rule, Pienaar (2014) suggested learners preparing a sketch of the fraction $\frac{3}{4}$ showing three coloured blocks. Pienaar (2014) concluded that applying rules without comprehension makes fraction topics too abstract and difficult for many learners.

Lukhele, Murray and Olivier (1999) conducted a study in Port Elizabeth, South Africa, where learners were tested on their understanding of basic addition of fractions. The question asked was $\frac{7}{8} + \frac{7}{8}$, and these were two of the answers received: $\frac{14}{16}$ and 30.

Learners used algorithms which they knew for adding whole numbers. Learners added the numerators and then the denominators to get $\frac{14}{16}$ and then others added their fraction answer together to get 30. This showed learners' lack of understanding when fractions were added as they used the whole number concept. Word problems were also used in a test. An example was: a girl had three half full packets and was asked on the total she had. The child drew pictures and then correctly carried out the operation as $(\frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 3)$. However, it could have been changed into a mixed fraction which the learner did not do even in the other examples. The same learner who did not perform well in the pre-test certainly improved after being taught through the Mathematics Learning and Teaching Initiative (MLATI) programme.

1.2 Problem statement

Baseline tests are carried out every year in Grade 6 and clearly state the learners score low on the fractions component. This prompted the present researcher to find out what challenges the Grade 6 learners face that constantly lead to low learner performance in the solving of fractions operations. Cai, Perry, Wong and Wang (2009) and Empson and Levi (2011) describe how teachers view Mathematics as being complex, challenging, exciting and logical procedures. Those researchers as well as others cited by European commission (2011), Bruce, Chang Flynn and Yearley (2013), Ndalichako

(2013) and Pienaar (2014) also explain it as based on individual differences by stating that individuals distinguish different concepts based on their abstract knowledge systems and that the learners treat numerators and denominators as separate entities. Furthermore, it is of interest to note that several researchers also pointed out the weak basic Mathematics Content Knowledge and pedagogical content knowledge of teachers (Taylor & Vinjevold, 1999; Hoadley, 2007; Fleisch, 2008; Taylor, 2011; Carnoy, Chisholm, Addy, Arends, Baloyi & Irving, 2011; Askew, Venkat & Matthews, 2012).

Understanding the different methods which teachers use during teaching of addition, subtraction and multiplication of fractions and their effectiveness is pertinent in this context. There are studies which have been conducted at university level (Coetzee & Mammen, 2017) as well as in high school (Rabaza, 2015; Awuah, 2016). The University of Fort Hare has produced two research outputs: a) Pedagogical experiences of educators implementing Mathematical literacy in three FET colleges (Gerber, 2011) and; b) Analysis of Grade 10 Mathematical Literacy students' errors in financial Mathematics (Khato, 2014). However, studies on Grade 6 in the East London district could not be located despite repeated literature search efforts.



1.3 Research questions

- Main Research question;

How can the Grade 6 learners' as well as the teachers' specific strengths be enhanced and the challenges in the learning and teaching of fraction operations be addressed?

- Research Sub-questions;

1. What are the Grade 6 learners' specific strengths and challenges in learning the fraction operations of addition, subtraction and multiplication?
2. What are the Grade 6 teachers' specific strengths and challenges in teaching the fraction operations of addition, subtraction and multiplication?
3. What strategies are appropriate to enhance the specific strengths and to address the specific challenges in learning and teaching fraction operations of addition, subtraction and multiplication in Grade 6?

1.4 The objectives of the study

1. To assess Grade 6 learners' specific strengths and challenges in learning the fraction operations of addition, subtraction and multiplication.
2. To evaluate Grade 6 teachers' specific strengths and challenges in teaching the fraction operations of addition, subtraction and multiplication.
3. To identify constructive strategies which can enhance the specific strengths and address the specific challenges in the learning and teaching of the fraction operations of addition, subtraction and multiplication.

1.5 Purpose of the study

The purpose of the study is to assess the specific strengths and challenges in the learning and teaching of fraction operations in Grade 6.

1.6 Significance of the study

This research sought to find out the answers to the research questions and possible solutions or alternative methods which could benefit the teaching and learning of the operations of addition, subtraction and multiplication of fractions. Through identifying the specific strengths and challenges learners encounter as well as the specific strengths and challenges teachers encounter when teaching the concept to learners, possible and constructive solutions can evolve. The results could help teachers and learners to see if they experience similar strengths and challenges and to find ways to solve such challenges. The research could be useful for learners, parents, teachers, Heads of Departments (HoDs) and subject advisors to improve the teaching/learning interface in fraction operations. The result of this study is also expected to positively influence intermediate phase Mathematics curriculum developers as well as pre-service and in-service course developers and implementers. This could also be helpful for Mathematics lecturers in pre-service teacher education to highlight teachers' and learners' difficulties regarding fraction operations in Grade 6.



1.7 Definitions of Operational Terms

Algorithms – Is a finite sequence of unambiguous instructions to perform a specific task (Brogan, n.d). A set of rules which are to be followed in sequence when doing calculations (Soicher & Vivaldi, 2004). The steps or rules which learners were taught to use when doing calculations of addition, subtraction and multiplication of fractions.

Numeracy – The ability to understand and work with numbers, and understanding of scientific methods (Marriem-Webster, 1828; O'donoghue, 2002).

Pedagogical – The specialised knowledge of teachers for creating an effective teaching and learning environments for all students (Guerriero, 2009; Voss, 2014).

Learners – A person who is learning a subject or skill. Learners bring unique learning styles, preconceptions, attitudes, and needs to the learning environment; they are individuals (Cobuild, n.d; Oblinger, 2005).

Teachers – A person who teaches, especially in a school. A teacher is a person who is capable of imparting knowledge and shaping the youth (Zombwe, Missokia, Saiguran & Mihayo, 2008-2013).



Fraction – A fraction is a numerical representation for part of a whole (EAP,2011, Fraction Facts LSC-O; Bruce, Chang, Flynn & Yearley, 2013, p. 17).

Threshold concepts – Threshold concepts are the possible characteristics of strong teaching and learning (Meyer and Land, 2005). Nonetheless, Cousin (2006) defined threshold concepts as a valuable tool in facilitating students' understanding of their subjects.

1.8 Limitations and Delimitations

This research will not focus on division of fractions, conversions from decimals to fractions and vice versa. Division will feature if learners do multiplication sums where they have to divide denominators by whole numbers should this method be chosen. The study was on one East London School, one Mathematics teacher from Grades 5-7 and only Grade 6 learners. This research also briefly discussed the outline of the concept fractions and how much time and exercises there were in each term for the Grade based on the prescribed CAPS aligned Platinum series textbook as well as to

analysed whether the teachers thought it was adequate or inadequate. The information gathered would hopefully provide insights on how teachers teach the concept fractions. The reason for only choosing one school, with three teachers and only Grade 6 learners was due to limited funding and tight time frame issues. The following was done to overcome the limitations; the questionnaire was given to six learners in 2017 cohort. Also, the questionnaire was given to two colleagues teaching Grade 6 from two different schools. Furthermore, two learners from the 2017 cohort and two colleagues from two different schools were interviewed, respectively, to check the validity of the learners' and teachers' interview schedules.

1.9 Theoretical framework

The theoretical frame work is based on the cognitive learning theory of Grider (1993) who explored the depths of the mind from the perspective of process. This theory explored the way one perceived, organised, stored, and retrieved information. Learning is viewed as only one of the various processes comprised by the human mind. Hence the importance of how learners perceive fractions as being difficult and cannot seem to organise the information in a way that they would understand, this leads to them not storing the information and are unable to retrieve it. On the other hand, Mayer (1989) explains that some approaches deal with the detailed analyses of information-processing skills, whereas other psychologists focus on mental models or cognitive growth and development. Mayer (1989) explained how information was retrieved from memory - the schemata only selects information which corresponds to the current activity. Therefore, one's learning and application of knowledge depends on one's schematic framework. Mayer (1989) stated that there were several processing stages which took place when learning occurred, including the experience of sensory input in which the raw data is momentarily captured in a sensory buffer.

a) Learner

Information can disappear quickly unless attention transfers it to short-term memory which can hold up to seven items. Then there is the working memory which is relevant to the problem as this memory stores specific mental operations such as addition. This questions why learners are not applying their working memory when working with fraction operations. Long-term memory is information which is organised, meaningful

and permanent. Why are the learners not able to retain the concepts of fraction operations? Do they not understand the meaning of why there are different rules when working with the different operations in fractions? How are they not able to organise the information for permanent retention? Long-term memory is divided into two categories; semantic memory where information retained is from the environment, such as your address or directions. Episodic memory revolves around events experienced by the individual and which have made an impact in their lives. Calculations of fractions need to become part of learners' episodic memory in order for them to master the concept.

This theory is supplemented and complemented by those of other cognitive constructivist theorists such as Piaget where individual learners are thought to construct knowledge.

b) Teacher

The pedagogical implication is that the teacher has to structure information so that the children can work from the familiar to the unfamiliar and so building knowledge from 'simple to complex', 'known to unknown' and 'concrete to abstract'. Hence, prior knowledge and educational scaffolding become pertinent.

However, for teachers, the pedagogical content knowledge is relevant, i.e., the teacher must have both deep Mathematics content knowledge of the fraction operations and the pedagogical knowledge on instructional methodologies. When these two interact, the pedagogical content knowledge evolves. That is when the teacher can plan to implement the teaching of fraction operations in order to lead learners away from possible errors and guide them to understanding. As such, the teacher's role in facilitating the episodic memory, and encouraging educational scaffolding and various learning social interactions can happen.

c) Social interaction

Furthermore, Bandura's (2001) social cognitive theory is also relevant since it focuses on personal factors, behaviours and the environment because it encompasses attention, memory, and motivation. Bandura also stated a persisting change in human performance is a result of the learner's interaction with the environment. The more involved learners are in the process of their own learning the more they will retain.

Learners need to be open minded to what they are learning, if they have a negative attitude towards learning the concept then they will not be able to understand it. Vygotsky's socio-cultural theory expanding on the last factor on environment pointed out by Bandura is also relevant in that the child does also learn from his culture, environment, peers and teachers. Hence social interactions also become pertinent. Vygotsky expands on the socio-cultural theory where learners retain more from their environment and what they are exposed to daily. They also need to interact with each other as they learn more from one another. Vygotsky mentioned the Zone of Proximal development, it is the area of exploration for which the learner is cognitively prepared, but requires help and social interaction to fully develop. For instance, a teacher or more experienced peer is able to provide the learner with "scaffolding" to support the learner's developing understanding of knowledge domains or development of complex skills. There are a few strategies which could be integrated; collaborative learning, modelling, and scaffolding, these support the intellectual knowledge and skills of learners and facilitating intentional learning.

1.10 The relevance of the theoretical framework to this study

A combination of the above theories is relevant to this research. Such a combination helps to understand the way teachers and learners process relevant information. Learner to learner interactions, teacher-learner interactions and other social interactions help the individual learner to focus personal factors on learning. When the teacher facilitates educational scaffolding, personal knowledge construction also occurs simultaneously. As such, Grider's theory of cognition as applicable to learning and Piaget's cognitive learning theory, Bandura's social cognitive theory and Vygotsky's socio-cultural theory as applicable to both teaching and learning are all relevant in the formal learning of fraction operations.

The learner, teacher and social interaction are relevant to the study as all three play a role in the development of the learning process. Piaget's theory on the episodic memory where children retain more information through life's experiences is relevant to learning operations of fractions, for this teachers could use tangible and practical things to create a memorable lesson. Bandura's social cognitive theory and Vygotsky's theory on social interaction are also important as both are about retaining information either in the long-term or short-term memory depending on how information is being

related. Therefore, the three aspects and the theories mentioned above are relevant to the study as these are linked to the challenges and strengths experienced by learners and teachers through the teaching and learning processes.

1.11 Chapter outline

Chapter one gives the background of the study and shows that the research title is an important concern throughout the world. The research statement, research questions, the aim, objective, significance of the study, operational terms, delimitations, limitations and dissertation outline are what chapter one covers. Chapter two is the literature review which includes teacher problems, learner difficulties, curriculum outline and methods for teaching fractions. Chapter three is on the research methodology which was used in my research. The following subtopics are covered; research paradigm, approach, design, sampling procedures, data collection and analysis, reliability, validity, trustworthiness and research ethics. Chapter four is on data analysis and discussions. The methodology approaches used were qualitative and quantitative data. The qualitative data were structured into themes and sub-themes, whereas the quantitative data were collated into pie graphs and double bar graphs. Chapter five consists of the conclusions and recommendations.



1.12 Summary

Chapter one gave the introduction, background, problem statement and research questions, followed by the aim, objective and significance of the study. The operational terms, mention of the limitations and delimitations, theoretical frame work with its relevance to the study and a brief chapter outline followed.

CHAPTER TWO

LITERATURE REVIEW

2. Introduction

This chapter will include the curriculum outline, learner difficulties, teacher difficulties, the CPTD related to algebra, the Threshold concepts in line with helping Mathematics teachers and for professional development. It also discusses the difficulties learners present in fractions which teachers need to be informed about.

Xuhui (2011) focused on the mathematical knowledge for teaching algebraic routines in the context of solving quadratic equations. The study identified three components of Teachers' Mathematical Knowledge as (a) Mathematics content, (b) knowledge of pedagogical representations and, (c) knowledge of learners' conceptions (Harel & Lim, 2004). Each component was explained as follows; (a) knowledge of the mathematical content as teachers' content knowledge formed out of interrelated concepts, processes, properties, methods and sequences of reasoning; (b) knowledge of pedagogical representations as the teachers' pedagogical form of mathematical content, presented by textbooks, visual aids and electronic technologies, and through various teaching strategies; (c) knowledge of learners' conceptions as the learners' mental representation of Mathematics as a subject matter, resulting from learners' interaction with the subject, the teacher, and other learners. The summary of the argument was that the above components would evolve, and that there would be growth in teachers' Mathematics learning experiences and teaching expertise that would help in the teachers' professional planning. The above mentioned components would be consistently modified during the teachers' daily planning of the curricular content.

Nonetheless, Mundy, Floden, McCrory, Burrill, and Sandow (2015) conducted a study explaining the conceptual framework for understanding and assessing teachers' knowledge for teaching algebra. Two dimensions were identified: tasks of teaching and categories of knowledge. Siemon (2004) built onto Begle (1979), Monk (1994), Rowan, Chiang and Miller (1997) these researchers observed the efforts to link teacher knowledge to student achievement produce ambiguous and contradictory results. Siemon (2004) argued that a lack of multiplicative reasoning, which requires a clear

conceptual understanding and full knowledge of mathematical processes and the relationships among them, appears to be a major cause of difficulties with further mathematics. These researchers addressed the theoretical aspects of teachers' mathematical knowledge for teaching algebra and laid an empirical research that probed the connections between teachers' mathematical knowledge and student outcomes in algebra. They used Shulman's (1986) conception of pedagogical content knowledge which included an understanding of what makes the learning of specific topics easy or difficult. The proposition they put forth is that mathematical knowledge for teaching as teachers may know and use could be different from what is required. They consider research on the methods used to improve content knowledge of how Mathematics knowledge is taught and learnt as pertinent.

2.1 Curriculum outline

Bruce, Chang, Flynn and Yearly (2013) cited Moss and Case (1999), mentioned attributing students' difficulties in learning fractions emanated from the fact that most teachers devote too much time teaching the procedures of manipulating fractions and too little time on teaching their procedural meaning. Bruce, Chang, Flynn and Yearly (2013) also cited Empson and Levi (2007) and Moss and Case (1999) and explained how students' competence in memorising the procedures in Mathematics acts as a major obstacle when they have to conceptualise their understanding of fractions, which leads to an inability to monitor their work. Learners can only check their answers through repetition of the memorised procedure used when working out fractions. If learners are not continuously using the same methods over and over to develop a good sense of understanding, then they will not be well developed in the content dealing with fractions.

The time allocation for algebraic Mathematics is set out in the Curriculum and Assessment Policy Statement (CAPS). According to CAPS, the weighting for the content area under which fractions are placed decreases from Grade 4 to Grade 6.

Fractions is only $\frac{1}{6}$ of the learning area which falls under Numbers, Operations and

Relationships content area, and in one term there are five content areas which are to be covered. The weighting decreases from Grades 4–6 as follows: from 40% in Grade 4 to 35% in Grade 5 and, 30% in Grade 6. The researcher as a teacher always felt that the decrease in weighting has been a contributing factor towards the low achievement

results in this learning area. Furthermore, the inadequate time spent for exercises negatively affects the building of a concrete foundation for this concept and fails to prepare learners successfully for the next phase.

2.2 Learners' difficulties

According to Saxe, Taylor, McIntosh and Geahart (2005), learning and mastering fractions is still a major issue for students in both primary and middle schools as they show difficulties in understanding and use written notations for fractions.

Ndalichako (2013) stated that learners treated fractions as whole number concepts and treated denominators and numerators as separate entities. For example, if learners were asked to solve $\frac{1}{3} + \frac{1}{5} = ?$, one would find an answer of $\frac{2}{8}$. Learners' tendency to get confused with the denominators when adding or subtracting fractions is also cited by the European Commission (2011), Bruce et al. (2013) and Pienaar (2014). The authors note misconception that denominators should also be added or subtracted just as it is done with numerators.

The same scenario happens with subtraction if learners are given a sum as follows; $\frac{3}{5} - \frac{1}{2} = ?$, one is possibly going to get an answer $\frac{2}{3}$. Learners are either unaware of the terminology which is used specifically for fractions or they may be unaware of the standard rule to be applied when adding or subtracting fractions, that is, the denominators need to be the same while doing the operation.

According to Ndalichako (2013), possible reasons stated for such difficulties include the fact that there are many rules associated with the computation of fractions which are more complex than those of natural numbers. For instance, when adding fractions with different denominators, the first rule is to find the lowest common denominator (LCD). This rule alone can be quite complex for learners, if they do not know how to find the LCD. Once the LCD is found, learners need to know that the next rule is to multiply the numerator by the same number as they did to the denominator. These rules and steps which need to be followed can become confusing to learners especially if they do not understand the terminologies used in fractions and if they do not have a clear understanding of denominators and numerators.

Multiplication of fractions is slightly different from addition and subtraction since learners need to multiply the numerators and the denominators together. Learners get confused and use the rules for addition or subtraction when multiplying fractions. Seah (2004) found that many students demonstrated very limited understanding of the multiplication concepts, with their knowledge restricted to procedural rather than conceptual understanding. Usiskin (2008), vom Hofe, Kleine, Blum and Pekrun (2006) and Prediger (2008) underlined the importance of understanding multiplication of fractions which is needed.

Maharaj, Brijlall and Molebale (2007) mentioned how the South African learners learn the operations of fractions through traditional methods of drilling the appropriate use of algorithms relevant to the operations. It has been said that this method of teaching only teaches procedural learning but does not promote conceptual understanding.

2.3 Teacher difficulties

Siegler, Carpenter, Fennell, Geary, Lewis, Okamoto and Wray (2010) stated that more research on fractions is still needed, especially studies addressing the efficacy of teaching fractions. The procedural process is an important aspect to address how well teachers understand different calculation methods which can be used to teach fractions. A study by Lloyd, Remillard and Herbel-Eisenman (2009) concluded that there is a growing interest in the actual teaching of Mathematics. Their view stems from research on teachers' use of curriculum materials. One cannot only focus on the learners and solely blame them for the weak performance. Teachers also need to be investigated to see if their methods are contributing towards learners' failure in fraction operations.

Ball, Lubienski and Mewborn (2001) cited earlier by Lillias and Chagwiza (2012) state that teachers need to know the topics which learners find difficult and from there, find useful teaching representatives that are specific for those content areas. Therefore, it is vital that teachers get to know each and every single learner in their respective classes in order to know the different types of teaching styles they should use. A baseline test at the start of the year should be done for Mathematics in order for teachers to diagnose the areas where learners are not performing well and to focus on these areas to develop them.

Ball, Thames and Phelps (2008) explain that teachers have to choose which examples to start with and, which examples to use in order to guide students to a deeper understanding and how to balance the pros and cons of representations to illustrate a specific mathematical idea. This process needs to be well-planned as teachers need to use more than one example to show learners that there are many methods which can be applied when teaching fractions. Ball et al. (2008) further explain that every task requires an interaction between specific mathematical concepts and an understanding of pedagogical issues that affect student learning. Teachers need to be able to understand the different methods as well as how to teach the various methods to enhance understanding. As mentioned earlier, SEACMEQ (2007) attributed poor learner results to teachers' low content knowledge of fraction operations, hence teachers' knowledge needs to be assessed and improvement plans need to emanate.

Torbeyns, Schneider, Xin and Siegler (2015) establish that even amongst the most prospective teachers around the world find fractions to be difficult. Ma (1999), Clarke (2006) and, Booth and Newton (2012) together with Lortie-Forgues, Tian and Siegler (2015) mentioned the difficulties learners encounter when learning about fractions and how it is even difficult for the teachers to teach.



2.4 Continuing professional teacher development (CPTD) related to algebra

Mthethwa (2017) conducted a study on the implementation of Continuing Professional Teacher Development (CPTD) in Mpumalanga province. Mthethwa (2017) mentioned the Inadequate Teacher Performances despite them attending professional teacher development workshops. The sample consisted of 200 teachers. In this sample of teachers, there was one Mathematics and one English teacher, another 200 were relevant SMT members, a Principal and a HoD from each of the 100 schools. District managers and subject advisors were also included in the study. Four Curriculum Managers and four subject advisors were selected purposefully based on their direct dealing with Grade 6 teachers.

Mthethwa's (2017) study revealed that CPTD programmes did not address teachers' needs because they were not consulted resulting in them failing to gain confidence despite attending several CPTD sessions. The teachers who participated in the study

expressed their unhappiness with the timing of CPTD criteria and activities. They felt that time for workshops should not have been held over the weekends or public holidays as it took up teachers' personal time. The teachers felt that CPTD enrichment should be done during school hours instead.

2.4.1 Positives

There are positives which come from CPTD, which is the improvement of teachers' knowledge and capabilities. It was mentioned that those in management positions should often attend these workshops to keep themselves sharp and up-to-date. It also benefits the Grade 6 Mathematics teachers through in-service training sessions.

2.4.2 Negatives

Although government is rolling out CPTD, it is not achieving the expected outcomes. It was questioned whether or not the workshops were organised to benefit the teachers or the government which could claim that something was being done to help improve the quality of teaching. Even though these workshops and in-service training programmes are run, there has not been an improvement in the academic outcome on the specific topics learners struggle within certain learning areas. Teachers must also be trained to identify their weaknesses so that they can find solutions to help them improve their skills.

2.5 Threshold concepts and its uses for Mathematics teachers' professional development

Several researchers have reported on threshold concepts (Perkins, 1999, 2006; Meyer & Land, 2003; Meyer & Land, 2005; Cousin, 2006; Smith, 2006; Barber & Mourshed, 2007; OECD, 2009, 2010; Land, Meyer & Baillie, 2010; Raiker & Procter, 2012).

2.5.1 Threshold concepts

Threshold concepts are pertinent to the understanding of how learning takes place. The various characteristics of threshold concepts explain how the process of learning occurs, from the known to the unknown and the gaps created between learning new concepts and what already exists. It also shows how and why adding new knowledge is relevant and important. In mathematics, learners tend to create a liminal space or

gap when a new concept is compared to what they already know. Linking concepts in other subjects is important as it shows learners where different concepts can be integrated into subjects other than Mathematics.

2.5.2 Features of threshold concepts

a) Transformative

Meyer and Land (2003) explained the fundamental concept of being transformative as an opening to the new and previously inaccessible way of thinking about something. “It represents a transformed way of understanding, interpreting or viewing, without which the learner cannot progress.” Once the learners understand the concepts, their views will be transformed with new knowledge.

b) Troublesome

Threshold concepts are likely to be troublesome for the student. Perkins (1999, 2006) suggests that knowledge can be troublesome for students. This occurs because the learner remains ‘defended’ and does not wish to change or let go of his/her customary way of seeing things. This often causes a mental block and creates the liminal space between the known and the new knowledge that needs to be understood. A secured learner is someone who holds onto what they know and has trouble opening up his/her mind to new knowledge.

c) Irreversible

Meyer and Land (2003) stated the transformative threshold concept is also likely to be irreversible. Once learners have learnt a specific concept in a certain way, it is difficult for them to unlearn that one way and open up to learning new ways. They rely on the ‘old lens’, what they are familiar with, instead of the ‘new lens’. Only when the learners become comfortable with the new concepts will they be open to using it.

d) Integrative

Meyer and Land (2003) explained once concepts are learned, learners are likely to bring together different aspects of the subject that they previously did not know and they can relate the newly-learnt concepts to what they already knew. Several concepts appear in more than one topic throughout the syllabus in Mathematics. Through

integration of various concepts, learners are able to grasp the concepts they need more practice in so that they eventually understand the new information.

This type of threshold often occurs in mathematical concepts where it is integrated into different topics learners are expected to cover in their syllabus. Through integration learners are able to find more examples of how the concept they do not fully understand can be used in more than one topic. It will also show how most concepts are interrelated. It will also show learners the importance of understanding the basics in Mathematics and learning other ways to do calculations.

e) Bounded

Smith (2006) defined this threshold as follows: “to describe a particular conceptual space, serving a specific and limited purpose.” Meyer and Land (2005) built on to this by explaining the term bounded as any conceptual space having fatal frontiers, bordering with thresholds into new conceptual areas. This happens when there is limited willingness to learn new concepts and this presents troublesome conceptual areas.



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f) Discursive

Meyer and Land (2005) suggest that the crossing of a threshold will be beneficial as it will enhance and extend the use of language. This is not only beneficial for language but also in Mathematics to cross threshold concepts and help learners close the liminal spaces created between prior knowledge and new knowledge. The more learners see the concepts being taught in various subjects, the more important understanding a concept becomes. It is hard to imagine any shift in perspective that is not altogether accompanied by an extension of the student's use of language, in this case, in the learners' understanding of fraction operations.

Through this elaboration of discourse, new thinking is brought into being, expressed, reflected upon and communicated. If the learners are exposed to using fractions in a variety of different situations other than in Mathematics, it will highlight the importance of acquiring the understanding of new knowledge or concepts. In this way, they can link what they know and expand on it through its use in other contents. This could lead

to specific discipline, community practice and even self-generated discipline and respect for the concepts.

g) Reconstitutive

This threshold, according to Smith (2006), is linked to the transformative and discursive thresholds which imply how the understanding of new concepts is subjectively chosen by learners. Such rebuilding is mainly recognised by others, and also takes place over time. Cousin (2006) explains how there is no easy method to combine prior knowledge with the new or more difficult knowledge learners are required to know. "In short, there is no simple passage in learning from 'easy' to 'difficult'; mastery of a threshold concept often involves messy journeys back, forth and across conceptual terrain" (Cousin, 2006).

h) Liminality

Land, Meyer and Baillie (2010) state that understanding the concept of this threshold could be difficult and leads learners to be in a state of 'liminality', a suspended state of partial understanding, or 'stuck place' in which understanding lacks authenticity. This space is created when learners struggle to learn new concepts which they can build onto if they retain and understand the new knowledge in term of what they already know. This concept was explained in the transformative threshold. Further complications might be the operation of an 'underlying situation' which requires the learner to comprehend the ways of thinking and practising essential information.

There are three phases to the liminal state: (a) pre-liminal-gaps which have existed before the new knowledge is taught and learners struggle with the knowledge encountering troublesome blockers; (b) liminal - the space or gap that is created when learners are not open to building onto prior knowledge and (c) post-liminal - the state after new concepts have been taught where transformation occurs and learners allow conceptual crossing and discourse changes.

2.5.3 Studies based on threshold concepts

Raiker and Procter (2012) conducted a study which introduced the threshold concepts to Mathematics teachers over four European countries. Their partnership revealed a continuing nation-wide concern of Mathematics teachers about low pupil confidence

and attainment in two key areas in Mathematics of algebra and ratio. The Organisation for Economic Cooperation and Development (Barber & Mourshed, 2007; OECD, 2009, 2010) report that the biggest influences on the process of learning depend on the quality and effectiveness of teachers and teaching. The report is based on the evidence collected from all over the world. It is for this reason that teachers need to continually develop their professional knowledge through the various professional development programmes and workshops offered. The project where Raiker and Procter (2012) were involved was designed to explore the notion of enhancing the quality of CPD through teachers coming together.

Perkins (1999) and Meyer and Land (2003) explained how threshold concepts can be troublesome for learners with the difficulty to link their prior knowledge with new knowledge that is being taught. The diagram below explains the gap between the known and unknown knowledge of ratio.

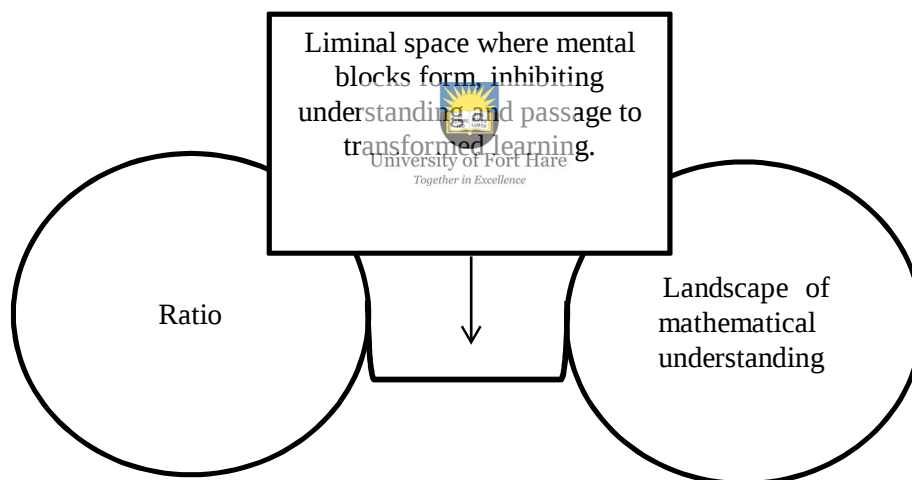


Fig. 2.6.1 (Source: Raiker, 2010; Hodgen, Brown, Küchemann & Coe, 2011)

The gap between ratio and the understanding of the topic is called the liminal space, the time where learners do not understand the new knowledge of a known topic they are taught. This is the same with fraction operations as there is a liminal space created when learners learn different forms of fractions - they can add or subtract fractions with the same denominators, but the minute the denominators are not the same, the learners get confused.

A pre-test for learners was done in the four participating classes in each of the four European countries. The questions were based on ratio and associated concepts to ensure standardisation. The teachers were introduced to the project and resources. All lesson plans by teachers were used as data and all lessons taught were recorded. Two separate cameras were used, one focused on the teacher and the other focused on the group of learners. The teachers and the researchers identified learners who struggled to learn the concept of ratio based on the information recorded. Ways to improve the lessons and resources were discussed afterwards between the research team and the teachers in order to find supportive methods which can aid the learning process. The aim of the process was to identify the threshold concepts and liminal space. Knowledge maps were used to support and address the difficulties, and misconceptions and misunderstandings were identified in the learning process. A post-test was done by the research team with reference to the pre-tests, where learners completed knowledge maps and online explanations.

There were two sets of interviews: (a) task-based interviews for the learners who experienced difficulties and who had support from the teachers and, (b) semi-structured interviews for the teachers who participated in the study. The teachers were shown the video of their own lessons and the strategy of using the knowledge map which they had used to support their learners who had difficulty in understanding the concept.

Northcote (2012, 2013) conducted a study on the threshold concepts and attitudes in Mathematics education through listening to pre-service students' past, present and projected stories. Two groups of these teachers enrolled in a first year Mathematics education course. The aim of the course was to enable them to develop an understanding of the Mathematical learning and the understanding and attitudes experienced by young children aged between 4-8 years. These pre-service teachers were expected to demonstrate knowledge and understanding of Mathematical concepts taught to these age groups. A narrative account was recorded on how the preservice teachers experienced teaching Mathematics in the past, how they experienced it in the present and how they expect to experience it in the future.

These pre-service teachers were also involved in a range of hands-on teaching of Mathematics through course-based activities and it was demonstrated for them. The

teachers were also guided through the process of creating engaging, purposeful and enjoyable Mathematics activities for the young children. This method of training reduced these teachers' anxiety and fears and increased their confidence levels for teaching Mathematics.

2.6 Teachers awareness about the difficulties learners present in fractions

Torbeyns, Schneider, Xin and Siegler (2015) stated how teachers should be informed of their students' difficulties with fractions so that they could adjust their teaching to their students' current skill levels. Therefore, it is important that teachers identify their learners' problem areas when it comes to fractions so that they can plan lessons to cater for those areas of concern. If teachers do not know where the problem areas lie, they will not be able to teach or build onto a lesson. If their learners are not where they should be, an evaluation should be done in order to find out where the problem areas are and teachers should find suitable strategies to assist and guide the learners to close the liminal space. If teachers do not know multiple ways of teaching the concept of fractions, then they need to attend development workshops as implemented by CPTD to benefit themselves, the school and most importantly, the learners.



Siegler, Duncan, Davis-Kean, Duckworth, Claessens, Engel, Susperruguy and Chen (2012a), Booth, Newton and Twiss-Garrity (2014) and Siegler and Lortie-Forgues (2015) revealed through research that the understanding of fractions leads to one's success in Mathematics and it even influences one's performance in advanced courses in Mathematics. Problems with conceptual understanding of fractions are common amongst students and may last into adulthood (DoBE, 2012). Should the conceptual problems get carried into adult world, education does not only create a gap for those adults but also for future generations who might have them as parents or even educators one day. Sadly, children are pushed through the phase if they have already repeated, regardless of whether the learner understands the concepts of fractions or not. Pushing children through a phase creates big gaps that are difficult to close. The older and the further in the grades the learners are, the wider the gaps become. Fractions are a universal concern as learners in primary, secondary and even tertiary level struggle to grasp the concept. Gould, Outhred & Mitchelmore (2006) and NAEP (2005) have noted the teaching and learning of fractions to be a concern in all levels of schooling.

2.7 Summary

Chapter two consists of the literature review and includes the curriculum outline together with the learner and teacher difficulties. It also includes the positives and negatives of the CPTD system implemented, the discussion of threshold concepts and its features and the relevant connection it has within Mathematics and specifically fraction operations. The last sub-heading is on the point of teachers needing to know the difficulties learners present in fractions in order to find ways to alleviate long term gaps or complications. Each sub-heading gives detailed information and how it is relevant to this study.



CHAPTER THREE

RESEARCH METHODOLOGY

3. Introduction

In this chapter the following will be explained; research paradigm, methodology approach, population and sample size, the research design, study site, sample, instruments which will be used and why (questionnaire and interviews). The ethical producers, reliability, validity, data trustworthiness, the data collection and analysis will also be mentioned.

3.1 Research paradigm

The paradigm which was most suitable for my dissertation was a post-positivist paradigm. According to Kaboub (2004), positivism came into being as a truth-seeking paradigm and later came interpretivism. As this study focused on the causes that lead to learners' poor achievement in fractions in Grade 6, the post-positivist paradigm was relevant to my study as it gave an in-depth understanding of the data revealed by both the quantitative and qualitative approaches. Crossan (2003) explained post-positivistic approach as being used to establish the warranted justification for the phenomenon rather than absolute truth, and that such research produces findings with a focus on meaning and understanding the situation.

3.2 Methodology Approach

According to Creswell (2008), a mixed method approach that allows a combination of quantitative and qualitative approaches provides a better understanding of the research problem than a single approach. Quantitative approach enables a broader picture of the causes of the problem and responses from questionnaire surveys are useful for this approach. Sukamolson (2007) and Assalahi (2015) explain quantitative approach as collecting data by experimental study and this is used for testing hypotheses. Bryman (2004) explains qualitative research as being concerned with the theories generated from conducting research by observing people's behaviour, norms, values and culture. The reason for choosing mixed method in this study was due to the nature of my study. I wanted to improve my understanding of the problems

teachers face when teaching addition, subtraction and multiplication of fractions as well as the difficulties learners encounter while learning the fraction operations. The importance of both approaches was relevant to my study as I have used techniques from both approaches when the research was conducted. This could only be possible through a mixed methods approach.

3.3 Research Design

Yin (2014) explains that case study research constitutes an all-encompassing method that covers the logic of design, data collection technique and specific approaches to data analysis and it can be used in a mixed method approach. This was a case study. The case was Grade 6 teachers and learners in one school.

3.4 Study site, Population and sample size

3.4.1 Study site

The study was done at one school in the East London District in South Africa's Eastern Cape Province. The population was the  Grade 6 learners in the school and three teachers. The researcher was employed at the study site. The researcher took steps to avoid insider conflict while gathering qualitative data. The details are given in Sub-section 3.9.

3.4.2 Population and sample size

Latham (2007) defines sampling as taking a representative selection of the population and using the collected data to answer the research questions. All members of the population were encouraged to participate in the research or survey subject to ethical compliance such as signed informed consent and guarantee of participant anonymity. Details are given in Sub-section 3.9. Since all learners were under 18 years, parents' or guardians' consents were required. Parents or guardians of only 59 learners gave consent and the sample consisted of those 59 learners. All the members of relevant teacher population consented, that is, three Mathematics teachers, one each from Grades 5, 6 and 7. Only 10 learners were interviewed: two who got all sections wrong, two who made errors in addition and subtraction, two who made errors in addition and multiplication, two who made errors in subtraction and multiplication and two who got

one question wrong in each section. The selection was randomly based on survey results and grouped to the five categories.

3.5 Instruments

There were 3 sets of instruments: two instruments were for the learners (an open-ended questionnaire - Appendix 5 and an interview protocol - Appendix 6). However, there was only one instrument for teachers, an interview protocol - Appendix 7. The learners' individual interview responses are in appendix 8-17. The teachers' individual interview responses are in Appendices 18-20.

Appendix 5

There were three sections and each consisted of 5 questions. Section A dealt with addition of fractions, section B with subtraction of fractions and section C with multiplication of fractions. Lines were provided underneath each question for learners to show how they worked out each question. Qualitative data were also collected through open-ended questions on the fraction operations in the learners' questionnaire.



Appendix 6

There were 16 semi-structured interview questions. All questions were related to how the learners would rate the different operations of fractions and also the questions in each section. The questions asked focused on finding out not only the strengths and weaknesses of the learners but also the suggestions for strategies they had for their teachers.

Appendix 7

The teacher interview questions asked for biographical information and questions on the teaching of the different operations. There were 19 semi-structured interview questions. The teachers were asked about their strengths and weaknesses and also to identify the strengths and weakness of the learners in their class.

3.5.1 Questionnaires

The quantitative data collected from the learners were analysed using the same style as that of the Annual National Assessment, whereby each question was analysed and percentage was reflected.

3.5.2 Interviews

Cohen, Manion and Morrison (2011) explained interviews as a method of gaining an understanding of the person's thoughts on the topic being researched. Cohen et al. (2007) view interviews as a tool that enables multi-sensory channels to gain information from non-verbal and verbal cues. Drew, Hardman and Hosp (2008) defined interviews as a face-to-face personal contact and interaction to collect relevant information related to the study. There are different types of interviews such as open-ended, structured and semi-structured interviews. This study used semi-structured interviews and will elaborate on that.

Doyle (2018) explains semi-structured interviews as being a non-formalised way of asking questions - more open-ended questions are asked rather than straight forward questions. Questions are asked to draw out information such as participants' qualifications, opinions and attitudes. Semi-structured questions may lead to open-ended questions that gather more specific information from the interviewee.

Doyle (2018) elaborates on semi-structured interviews as a two-way communication, thus allowing either the researcher or the candidate to ask questions which lead to comprehensive discussions. Due to the nature of the interview, the candidates may feel more comfortable, expanding on experiences which highlight good or negative traits on the topic. Saunders, Lewis and Thornhill (2009) compared semi-structured interviews to a hybrid type of interview as some questions are structured and others are open-ended.

One teacher each from Grades 5-7 was interviewed to find out how they teach fractions and where they think the problems arise. Each interview was around 40 minutes long. The data were summarised into separate paragraphs on the three operations of addition, subtraction and multiplication. The information was also separated into themes for teachers and learners separately.

The learners were given the questionnaire to answer first before 10 of them were selected for interviews. Three teachers, one each from Grades 5, 6 and 7 were interviewed. The quantitative data were analysed to lead to a descriptive data presentation. The qualitative data were thematically analysed.

3.6 Reliability, validity and data trustworthiness

- Reliability

To ensure reliability in my research I used questions from the National exam set by the Department of Education as well as questions from the CAPS aligned textbooks. These are national bench marked instruments and hence the assessment is reliable and no other tests will be applied. Cronbach alpha's methods of test-retest were also used to ensure reliability.

- Validity

To ensure validity in my research, the interview questions and questionnaire were designed fair, non-biased, straightforward and directly related to the objectives of the study where information given by participants related to what I wanted to find out. This was a piloted study - there was pre-testing of the research instruments which included the questionnaire as well as interview schedule with only three Mathematics teachers (Van Teijlingen and Hundley, 2001; Baker 2002). As part of a pilot study, the questionnaire was given to six learners in the 2017 cohort. The questionnaire was also given to two colleagues teaching Grade 6 from two different schools. Furthermore, the two learners from the 2017 cohort and the two colleagues from two different schools were interviewed, to check the validity of the learners' and teachers' interview schedules. Based on the feedback, the instruments were modified for use in 2018.

- Data trustworthiness

Gunawan (2015) further elaborated what trustworthiness is about in research based on the study done by Sandelowski (1993). In qualitative research, trustworthiness is obtained when it accurately represents the experiences of the participants where they might recognise the findings to be true. Therefore, the methodology is clear and precise so that readers are able to tell how the research is conducted. Triangulation is defined by Burns and Groove (2001) as collecting data from numerous sources to

maintain the same outcome of results. Data trustworthiness was obtained by having various participants take part and from their responses similar strengths, weakness and challenges were provided.

3.7 Data collection

The learners were under 18 years and in order to comply with South African ethical regulations involving humans, a letter was sent out to all 98 Grade 6 learners' parents requesting their consent. Only 59 learners brought back the consent letters signed by a parent or legal guardian and hence, the sample consisted of 59 Grade 6 learners. The data were collected over a period of time when all the participants were available. The learners answered the questions in one day and all 3 teachers were interviewed on the same day as it was convenient for them. The teacher interviews were around 40 minutes each. Consent from the principal, teachers and learners' parents had been received before the data were collected. The surveys and semi-structured interviews were conducted at a time and place which was most convenient for the participants. The interviews were recorded using a cellular phone with written consent from the participants beforehand. The data were collected after school hours so as not to interfere with the normal school schedule.



The learners were given 45 minutes to answer the questions, 15 minutes per section. The instructions were clearly written on the questionnaire. In addition, verbal instructions were given to the learners so they knew what they needed to do if they were confused about any of the questions. Sufficient space for learners to show their calculations was provided on the questionnaire. Two consent letters were sent home to parents of the Grade 6 learners'. The first one requested permission for the learners to participate in answering the questionnaire. The second letter was sent to parents of only 10 learners' and it sought permission for the learners to be interviewed and get these interviews recorded. Interviews with the learners were not longer than 20 minutes to avoid fatigue. The 10 learners who were interviewed were given sweets afterwards, a small gesture to thank them for taking part. Chocolates were also given to the teachers who willingly took part in the interview process.

3.8 Data Analysis

The data collected was structured into its respective categories which relate to the title of the thesis. The information was used to outline the issues encountered by both learners and teachers when it came to fractions. Possible solutions given by the teachers and the learners could be beneficial to those who read this research. The data collected from the questionnaires are presented in pie graphs as well as in double bar graphs showing the percentages for each question answered. This information was used to see where the gaps are and how best to try and close the gaps in the teaching and learning of fraction operations. The quantitative data collected demographical information about teachers' only while descriptive statistics described the data collected from both the teachers' and learners'. Percentages show the portion of the learners who exhibit challenges in fraction operations. Pie charts, bar graphs, and tables were used to display the data that were collected from the questionnaires. The tabulated information in quantitative analysis shows percentages for the questions the learners rated. The mean, median and mode were used. Quantitative data were information collected from the 5 sample groups of learners after they have answered the questionnaire based on solving expressions of fractions. The qualitative data collected from the interviews were categorised into themes which answered the sub research questions.

3.9 Research ethics

(a) Participants to remain anonymous

According to Burns and Grove (2001), anonymity is when the researcher cannot be referred back to any participant. The anonymous ethics was used to protect the identity of the persons who willingly took part in the research. This was done so that the participants cannot be linked to the information they have given, especially if the information provided might not be accepted or agreed upon by others. The teachers who participated in this research gave pseudo names to protect their identities.

(b) Respecting the views of the participants

Burns and Grove (2001) explain that the right to self-determination is based on the ethical principle of respect for a person. The researcher must respect the perspectives

of all participants and treat all with the same respect, irrespective of their contributions. It was important to accept the answers given by participants as the research is not about you and what others say should not be disregarded.

(c) The Right to full disclosure

Polit, Beck and Hungler (2001) state that full disclosure is when the researcher has fully explained the nature of the study, and the person's right to refuse participation. This allowed the participants to have freedom of choice and made them feel free not to answer the questions they did not feel comfortable answering or to withdraw completely from the research.

(d) Informed consent from the school and the participants

Fouka and Mantzorou (2011) explained that consent is to be obtained freely by participants, with full awareness of implications. The researcher must explain in detail what it is they require from their participants and share the purpose of the research. Once consent was given by participants, it did not mean that they were liable or forced to be a part of the research. As mentioned above, at any time during the interviews or processes of collecting the data the participants were free to withdraw. This was mentioned in the consent letter so as to assure the participants that they were not forced to be part of the research. Because the members of the learner sample were minors, the parents were requested to sign the 'informed consent' form.

(e) Respect for privacy

Burns and Grove (2001) defined confidentiality as the researcher's management of private information shared by the participants and which would not be shared with others without the authorisation of the participants. Levine (1976) explained privacy as the freedom an individual has to determine the time, extent and general circumstances under which private information will be shared or withheld from others. To assure privacy to the participants, their real names were not mentioned or required nor the name of the school at which they work.

(f) Permissions from relevant authorities

Ethical clearances from the University of Fort Hare and the Eastern Cape Provincial Department of Basic Education were granted. Informed consent was given by parents of 59 Grade 6 learners.

(g) Insider researcher

Insider-outsider conflict is relevant to qualitative research. The purpose is to avoid subjectivity by the insider. However, Breen (2007) refers to Bonner and Tolhurst (2002) about the advantages of being an insider researcher. This was an insider research. To gain access, I submitted a typed letter requesting permission to conduct research at my own institution. Alan (n.d.) explained that one being an insider researcher enhances a sense of trust and relational responsibility. It also makes one closer to his/her participants, and the sense of responsibility was arguably stronger than when conducting research at an institution where there were no links.

Advantages of an insider researcher

Bonner and Tolhurst (2002) highlighted three key advantages of being an insider researcher. These are as follows: (a) having a greater understanding of the learners' and teachers' culture, (b) the ability to interact naturally with the participants and, (c) being able to establish a greater relation of familiarity with the participants, making them feel at ease through the process. Breen (2007) mentioned the importance of the researcher to remain in that role so that it creates the separation needed to conduct research effectively without any barriers. The advantage of being an insider researcher can be followed by disadvantages.

Disadvantages of an insider researcher

Bonner and Tolhurst (2002) discussed the disadvantages of the advantages mentioned above. The disadvantage of (a) mentioned above is the downfall of familiarity which could lead to losing the purpose of the interviews or conducting of questionnaires if not done correctly. The disadvantage of (b) is that an easy communication with the participants could affect the manner in which the researcher conducts himself/herself, forgetting his/her role as a researcher and taking on the role

familiar to the participants. If the role of a researcher is not balanced correctly, it could negatively impact the results collected.

3.10 Summary

Chapter three contained the research methodology which included the research paradigm and methodology approach followed by population and size, research design and instruments. The following sub-headings were also discussed in this chapter: reliability, validity and data trustworthiness, data collection, analysis and research ethics.



CHAPTER FOUR

DATA ANALYSIS, INTERPRETATION AND DISCUSSION

4. Introduction

This chapter will be analysing, interpreting and discussing the findings of the research and answering the main- and sub- research questions. This will be categorised into the following headings and subheadings; learners' questionnaire results and discussion subdivided into quantitative data, data from learners' interview, the qualitative data which was categorised into themes. There will also be biographical information of the teachers who participated, the teachers' interviews were categorised into main and sub themes related to the research questions. The information collected will be triangulated into learners' questionnaires and interviews and then learners' and teachers' responses.

4.1 Learners' questionnaire results and discussion

4.1.1 Quantitative data

Questionnaire – Learners



Results of learners who got each question correct or incorrect out of the 59 learners.

SECTION A: ADDITION

Question 1

$$\frac{1}{4} + \frac{2}{4} = \frac{3}{4}$$

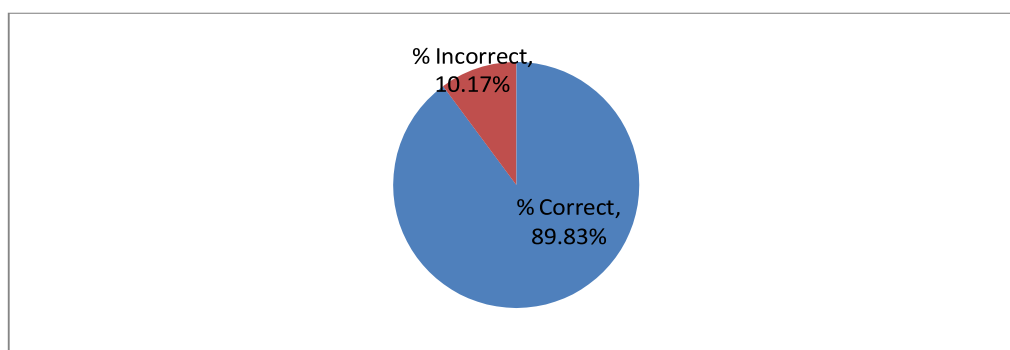


Fig. 4.1: Pie chart for correct and incorrect answers for Section A: Question 1

The correct steps:

The correct steps to be followed were:

$\frac{1}{4} + \frac{2}{4} = \frac{3}{4}$ The learners had to add the numerators together through holding the denominator as 4.

Examples of some learners' incorrect calculations:

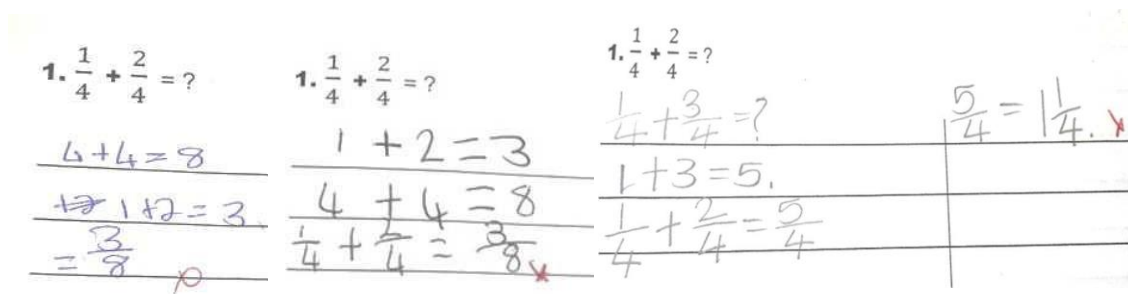


Fig. 4.1.1: Examples of some learners' incorrect calculations

For this question (number 1), only 6 out of 59 learners (about 10%) answered incorrectly. These learners added the denominators together to get a sum of 8 as denominator, or wrote an answer over an irrelevant denominator. One among them wrote the question incorrectly and this led to the incorrect addition of the sum and another one just added the numerators and wrote the answer as a whole number.

The incorrect answers given by the learners:

$\frac{3}{8}$ or $1\frac{1}{4}$ or 3

Question 2

$$\frac{3}{6} + \frac{2}{6} = \frac{5}{6}$$

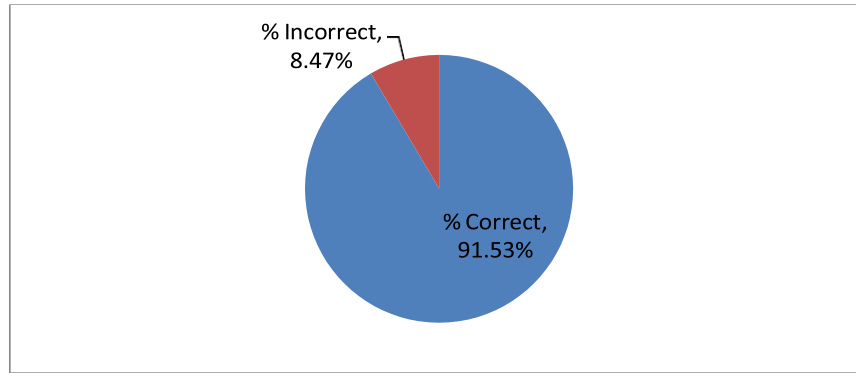


Fig. 4.2: Pie chart for correct and incorrect answers for Section A: Question 2

The correct steps:

The correct steps to be followed were:

$\frac{3}{6} + \frac{2}{6} = \frac{5}{6}$ The learners had to add the numerators together through holding the denominator as 6.

Examples of some learners' incorrect calculations:

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Fig. 4.2.1: Examples of some learners' incorrect calculations

For this question, 5 out of the 59 learners (about 8%) answered incorrectly. Again, learners added the denominators together to get an irrelevant denominator of 12, not realising that the denominators were the same and needed to remain as was. Therefore, the basic understanding of fractions was lacking. A learner incorrectly wrote down the sum and there were learners' who wrote incorrect numerator answers as seen below. The majority of the learners knew how to add the numerator only.

The incorrect answers given by the learners:

$$\frac{5}{12}, \frac{4}{6}$$

Question 3

$$4\frac{1}{4} + 1\frac{2}{3} = 5\frac{11}{12}$$

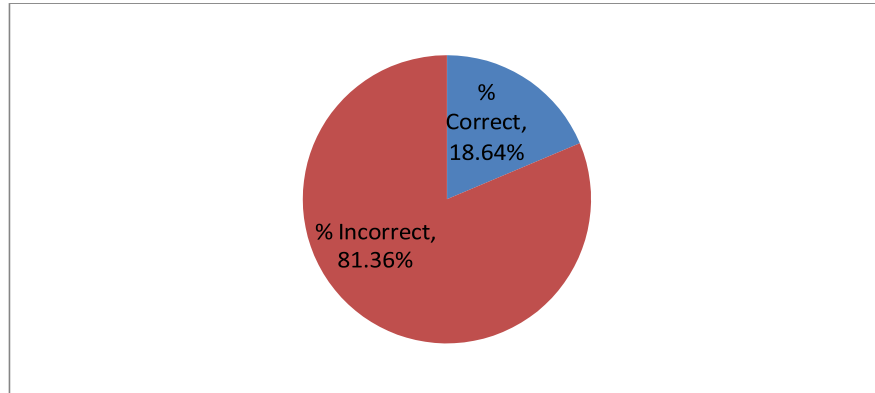


Fig. 4.3: Pie chart for correct and incorrect answers for Section A: Question 3

The correct steps:

The correct steps to be followed were:

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3. $4\frac{1}{4} + 1\frac{2}{3} = ?$

Correct Steps (Left):

$$4 + 1 = 5 \quad \text{LCD} = 12$$

$$\frac{1 \times 3}{4 \times 3} + \frac{2 \times 4}{3 \times 4} = \frac{3}{12} + \frac{8}{12} = \frac{11}{12}$$

$$5 + \frac{11}{12} = 5\frac{11}{12}$$

Incorrect Steps (Right):

$$4\frac{1}{4} + 1\frac{2}{3} = 5\frac{11}{12}$$

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Fig. 4.3.1: Correct steps

The contents on the left side of the above figure show the correct steps to calculate the answer. One could add the whole numbers first, then find the LCD for both denominators as shown in fig. 4.3.1. Once converted, add the new numerators together, write it above the denominator and then add the whole number to the fraction at the end.

The contents on the right side of the above figure are alternative correct steps which could also be used. This method converts the fractions to improper fractions. Then, calculate the LCD and add the numerators. Convert the answer in improper fraction back to a mixed fraction, and in some cases, to its simplest form.

Examples of some learners' incorrect calculations:

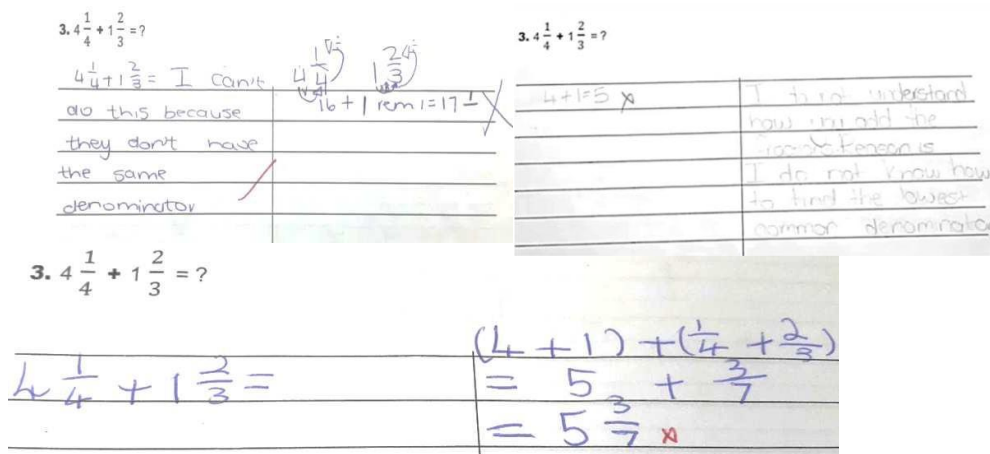


Fig. 4.3.2: Examples of some learners' incorrect calculations

For this question (number 3), 48 out of the 59 learners (about 81%) answered incorrectly. Some learners did not know or lost direction on what to do after adding the whole numbers. They appeared to be unaware that the LCD needed to be found. One among them added 1 to the numerator and 1 to the denominator to change the denominator into 4. This showed that the learner knew that the two denominators needed to be the same but did not know how to find it. There were others who explained that they did not know what to do after finding the LCD as they were unsure of what needed to happen to the numerator. One learner converted the mixed fraction into an improper fraction but after doing that, did not know that to proceed further, the LCD had to be calculated. While some learners wrote that they did not know whether to multiply or add, others wrote that they did not understand how to do addition of fractions with different denominators. Some did not even attempt to answer.

The incorrect answers given by the learners:

$$5 \frac{3}{7}; 6; 5 \frac{3}{4}; 5 \frac{9}{12}; 5 \frac{7}{12}; 5 \frac{1}{4}; 3 \frac{4}{7}; 4 \frac{3}{4}; 5 \frac{3}{12}; 5 \frac{3}{3}; 3 \frac{3}{5}; 5 \frac{2}{4}; \frac{22}{3}$$

Question 4

$$3\frac{4}{7} + 2\frac{3}{4} = 6\frac{9}{28}$$

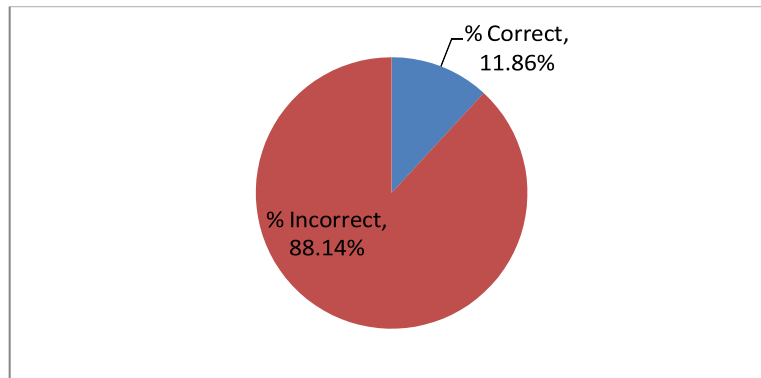


Fig. 4.4: Pie chart for correct and incorrect answers for Section A: Question 4

The correct steps:

The correct steps to be followed were:

4. $3\frac{4}{7} + 2\frac{3}{4} = ?$

Left side (Correct steps):

$$3 + 2 = 5 \quad \text{LCD} = 28$$

$$\frac{4 \times 4}{7 \times 4} + \frac{3 \times 7}{4 \times 7} = \frac{16}{28} + \frac{21}{28} = \frac{37}{28}$$

$$5 + 1 = 6\frac{9}{28}$$

Right side (Alternative correct steps):

$$3\frac{4}{7} + 2\frac{3}{4} = \frac{25}{7} + \frac{11}{4} = \frac{100}{28} + \frac{77}{28} = \frac{177}{28} = 6\frac{9}{28}$$

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Fig. 4.4.1: Correct steps

The contents on the left side of the above figure show the correct steps to calculate the answer. One could add the whole numbers first, then find the LCD for both denominators as shown above. Once converted, add the new numerators together, write it above the denominator and then add the whole number to the fraction at the end.

The contents on the right side of the above figure are alternative correct steps which could also be used. This method converts the fractions to improper fractions. Then,

calculate the LCD and add the numerators. Convert the answer in improper fraction back to a mixed fraction, and in some cases, to its simplest form.

Examples of some learners' incorrect calculations:

The image shows two examples of student work for the problem $4. 3\frac{4}{7} + 2\frac{3}{4} = ?$.

Example 1 (Top): The student shows several incorrect steps:

- Initial problem: $4. 3\frac{4}{7} + 2\frac{3}{4} = ?$
- Step 1: $3\frac{4}{7} + 2\frac{3}{4} =$
- Step 2: $3 + 2 = 5$
- Step 3: $\frac{4}{7} + \frac{3}{4} = \frac{16}{28} + \frac{21}{28} = \frac{37}{28}$
- Step 4: $5 + \frac{37}{28} = 5\frac{37}{28}$
- Step 5: $5\frac{37}{28} = 6\frac{9}{28}$
- Step 6: $6\frac{9}{28}$

Example 2 (Bottom): The student shows a different incorrect method:

- Initial problem: $4. 3\frac{4}{7} + 2\frac{3}{4} = ?$
- Step 1: $3\frac{4}{7} + 2\frac{3}{4}$
- Step 2: $(3+2) = 5$
- Step 3: $\frac{4}{7} + \frac{3}{4} =$
- Step 4: $\frac{4}{7} + \frac{3}{4} =$
- Step 5: $\frac{4}{7} + \frac{3}{4} =$
- Step 6: $\frac{4}{7} + \frac{3}{4} =$
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- Step 100: $\frac{4}{7} + \frac{3}{4} =$

Fig. 4.4.2: Examples of some learners' incorrect calculations

For this question (number 4), 52 out of the 59 learners (about 88%) answered incorrectly. Mistakes made were similar to the previous question (3 above) where learners added the whole numbers and then did not know how to proceed further with addition of unlike-fractions. There were also learners who wrote that they did not understand how to make the denominators the same. Some learners tried to find the LCD but the inability to recall the multiplication table appeared to be a challenge for them. Somewhere along their calculations, they appear to have lost direction and did not proceed further. These last-mentioned learners did not know that they could have multiplied the denominators together to find the LCD. There were another set of learners who failed to convert the improper fraction back into a mixed fraction and also forgot to add the whole number to it.

The learners who answered correctly showed a step-by-step progression. They added the whole numbers first, found the LCD and multiplied both the numerator and denominator by the same number. Once the denominators and numerators were changed, they added the fractions correctly.

The incorrect answers given by the learners:

$$6\frac{3}{4}; 7\frac{7}{11}; 6\frac{2}{8}; 7\frac{1}{4}; 5\frac{7}{7}; \frac{1}{2}; 6\frac{7}{2}; 5\frac{7}{4}; 5\frac{7}{4}; \frac{11}{11}; 5\frac{7}{32}; 5\frac{7}{9}; 5\frac{37}{28}; 5\frac{7}{28}; 5\frac{4}{7}; 5\frac{1}{7}; 4\frac{14}{7}; \frac{1}{28}; 5\frac{8}{1}; 5\frac{32}{54}$$

Question 5

$$4\frac{2}{3} + 3\frac{3}{5} + 2\frac{3}{15} = 10\frac{7}{15}$$

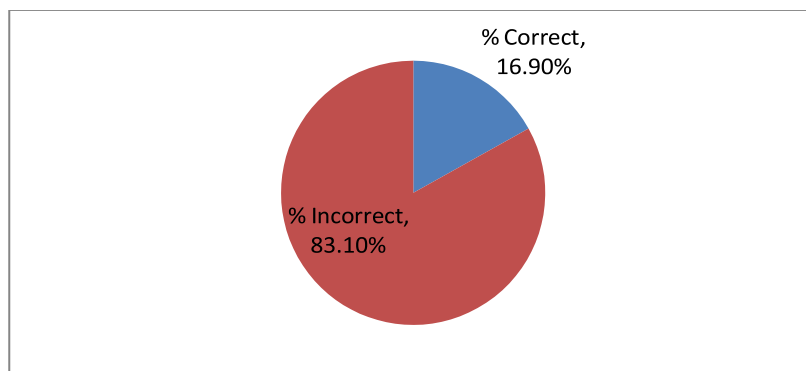


Fig. 4.5: Pie chart for correct and incorrect answers for Section A: Question 5



The correct steps:

The correct steps to be followed were:

5. $4\frac{2}{3} + 3\frac{3}{5} + 2\frac{3}{15} = ?$

Method 1 (Left):

$$4 + 3 + 2 = 9 \quad \text{LCD} = 15$$

$$\frac{2 \times 5}{3 \times 5} + \frac{3 \times 3}{5 \times 3} + \frac{3}{15}$$

$$\frac{10 + 9 + 3}{15}$$

$$= \frac{22}{15} - \frac{15}{15} (1)$$

$$9 + 1 = 10\frac{7}{15}$$

Method 2 (Right):

$$4\frac{2}{3} + 3\frac{3}{5} + 2\frac{3}{15}$$

$$\frac{14 \times 5}{3 \times 5} + \frac{18 \times 3}{5 \times 3} + \frac{33}{15}$$

$$\frac{70 + 54 + 33}{15}$$

$$\frac{157}{15} = 10\frac{7}{15}$$

Fig. 4.5.1: Correct steps

The contents on the left side of the above figure show the correct steps to calculate the answer. One could add the whole numbers first, then find the LCD for both

denominators as shown above. Once converted, add the new numerators together, write it above the denominator and then add the whole number to the fraction at the end.

The contents on the right side of the above figure are alternative correct steps which could also be used. This method converts the fractions to improper fractions. Then, calculate the LCD and add the numerators. Convert the answer in improper fraction back to a mixed fraction, and in some cases, to its simplest form.

Examples of some learners' incorrect calculations:

The figure displays three examples of student work for the problem: $5. 4\frac{2}{3} + 3\frac{3}{5} + 2\frac{3}{15} = ?$

- Example 1 (Left):** The student adds the whole numbers (4 + 3 + 2 = 9) and then adds the numerators (2 + 3 + 3 = 8) over the denominator 15, resulting in $9\frac{8}{15}$. This is marked with a red 'X'.
- Example 2 (Middle):** The student writes "I don't understand the process of this" and includes the University of Fort Hare logo.
- Example 3 (Right):** The student incorrectly adds the whole numbers (4 + 3 + 2 = 9) and then adds the numerators (2 + 3 + 3 = 8) over the denominator 15, resulting in $9\frac{8}{15}$. This is marked with a red 'X'.

Fig. 4.5.2: Examples of some learners' incorrect calculations

For this question (number 5), 49 out of the 59 learners (about 83%) answered incorrectly. Some learners wrote that they did not know how to work with 3 mixed fractions. Other learners added the whole numbers, then added the numerators together and wrote their answer over the existing denominator or over an irrelevant denominator. The others got confused with how to add fractions with different denominators. Learners explained that they did not know how to work sums out with different denominators. There were learners who knew that they had to get the LCD but they did not convert the numerator when they changed the denominator. A learner found the LCD of 30 and converted correctly by multiplying both the numerator and denominator by the same number. But after this step the learner wrote that he/she did not understand what to do next. Some learners explained that they could not do the sum as they did not know how to convert the mixed fractions to an improper fraction and then back to a mixed fraction. This showed that learners did not know of other methods to work out the sum. Only a few learners knew that they had to find the LCD.

The incorrect answers given by the learners:

$$9\frac{8}{23}; 11\frac{2}{3}; 9\frac{8}{15}; 14; 9; 17,95; 9\frac{28}{15}; 9\frac{25}{45}; 10\frac{5}{15}; 23; 9\frac{8}{3}; 10\frac{4}{15}; 2\frac{3}{15}; 12\frac{1}{3}$$

4.1.2 The relevance of Section A to the research sub-questions

The results from this section of the learners' questionnaire refer to the research Sub-question 1. The information collected showed the learners' strengths and challenges. In the addition of like-fractions, the percentage of those who answered correctly were higher than that of the addition of unlike-fractions. The challenges were seen when learners had to add unlike-fractions and even more so when it was mixed fractions.

Section B: Subtraction

Question 1

$$\frac{3}{4} - \frac{2}{4} = \frac{1}{4}$$

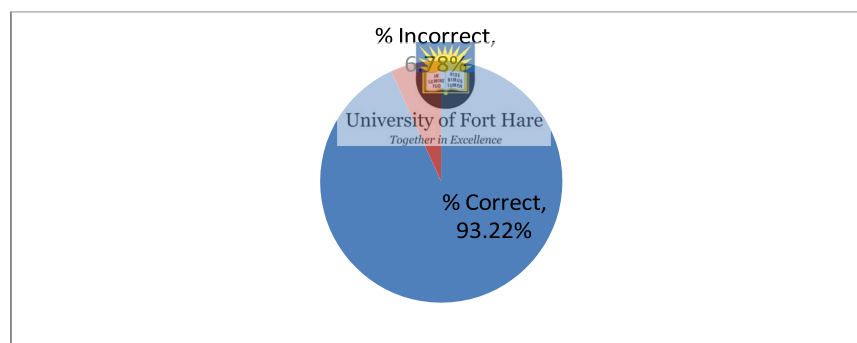


Fig. 4.6: Pie chart for correct and incorrect answers for Section B: Question 1

The correct steps:

The correct steps to be followed were:

$\frac{3}{4} - \frac{2}{4} = \frac{1}{4}$ The learners had to subtract the numerators from each other through holding the denominator as 4.

Examples of some learners' incorrect calculations:

$$1. \frac{3}{4} - \frac{2}{4} = ?$$

$$\frac{3}{4} - \frac{2}{4} = 0$$

$$1. \frac{3}{4} - \frac{2}{4} = ?$$

$$\frac{3}{4} - \frac{2}{4}$$

$$3 - 1 = 2$$

$$= \frac{2}{4}$$

$$1. \frac{3}{4} - \frac{2}{4} = ?$$

$$\frac{3}{4} - \frac{2}{4} = \frac{1}{2} - \frac{2}{4}$$

$$\frac{1}{2} - \frac{2}{4} = \frac{2}{4} - \frac{2}{4} = 0$$

Fig. 4.6.1: Examples of some learners' incorrect calculations

For this question (number 1), 4 out of 59 learners (about 7%) answered incorrectly. These learners subtracted incorrectly or wrote an answer over an irrelevant denominator. There were some learners who knew how to subtract the numerators from each other and leave the denominators as 4.

The incorrect answers given by the learners:

$$\frac{2}{4}, \frac{2}{2}, \frac{10}{0}, 0$$

Question 2



$$3 \frac{5}{7} - 1 \frac{2}{7} = 2 \frac{3}{7}$$

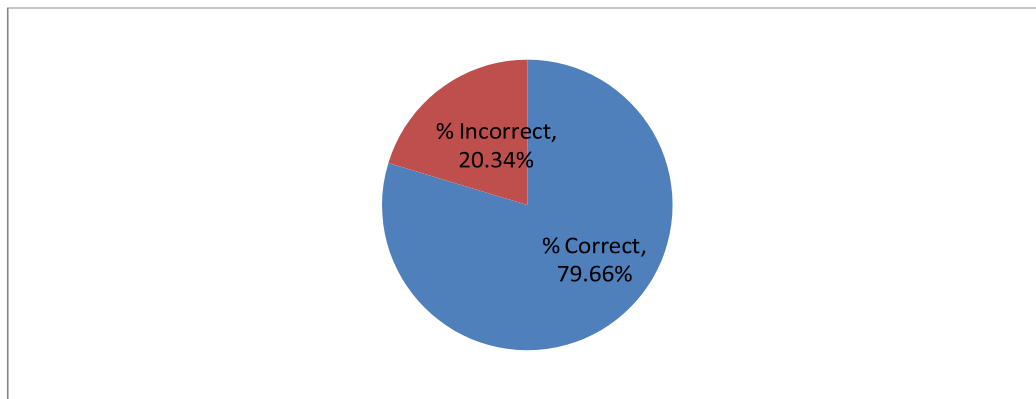


Fig. 4.7: Pie chart for correct and incorrect answers for Section B: Question 2

The correct steps:

The correct steps to be followed were:

2. $3\frac{5}{7} - 1\frac{2}{7} = ?$

Left side (Correct steps):

$$3 - 1 = 2$$

$$\frac{5-2}{7} = \frac{3}{7}$$

$$2\frac{3}{7}$$

Right side (Alternative correct steps):

$$\frac{26}{7} - \frac{9}{7}$$

$$= \frac{17}{7} = 2\frac{3}{7}$$

Fig 4.7.1: Correct steps

The contents on the left side of the above figure show the correct steps to calculate the answer. One could subtract the whole numbers first and, then find the LCD for both denominators as shown in fig. 4.7.1. Once converted, subtract the new numerators, write it above the denominator and then add the whole number to the fraction at the end.



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The contents on the right side of the above figure are alternative correct steps which could also be used. This method converts the fractions to improper fractions. Then, calculate the LCD and subtract the numerators. Convert the answer in improper fraction back to a mixed fraction, and in some cases, to its simplest form.

2. $3\frac{5}{7} - 1\frac{2}{7} = ?$

rr 2. $3\frac{5}{7} - 1\frac{2}{7} = ?$

2. $3\frac{5}{7} - 1\frac{2}{7} = ?$

Left side (Incorrect):

$$3\frac{5}{7} - 1\frac{2}{7} = 2\frac{4}{7}$$

Middle side (Incorrect):

$$3\frac{5}{7} - 1\frac{2}{7} = 2\frac{3}{7}$$

Right side (Incorrect):

$$3\frac{5}{7} - 1\frac{2}{7} = 2\frac{3}{7}$$

Fig. 4.7.2: Examples of some learners' incorrect calculations

For this question (number 2), 12 out of 59 learners (about 20%) answered incorrectly. There was a slight increase in the number of learners who got the mixed fraction question wrong even though the denominators were the same. Some learners forgot to add the whole number to their final fraction answer. Other learners simply subtracted incorrectly.

The incorrect answers given by the learners:

$$2\frac{4}{7}; 2\frac{2}{7}; 2; 2\frac{3}{1}; 2\frac{5}{7}$$

Question 3

$$2\frac{2}{5} - 1\frac{4}{5} = \frac{3}{5}$$

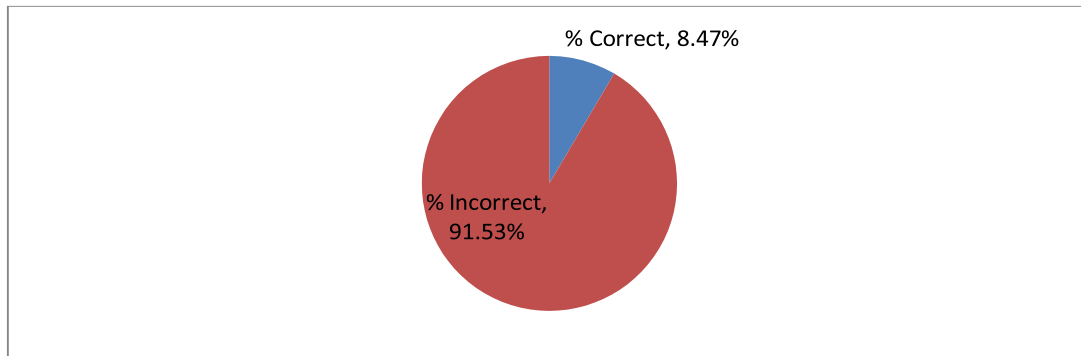


Fig. 4.8: Pie chart of correct and incorrect answers for Section B: Question 3

The correct steps:



The correct steps to be followed were.

$$3. \quad 2\frac{2}{5} - 1\frac{4}{5} = ?$$

$$2 - 1 = 1 = \frac{5}{5}$$

$$\frac{5+2}{5} - \frac{4}{5} = \frac{7}{5} - \frac{4}{5} = \frac{3}{5}$$

$$\frac{12}{5} - \frac{9}{5} = \frac{3}{5}$$

Fig. 4.8.1: Correct steps

The contents on the left side of the above figure show the correct steps to calculate the answer. One could subtract the whole numbers first, then find the LCD for both denominators as shown above. Once converted, subtract the new numerators, write it above the denominator and then add the whole number to the fraction at the end.

Examples of some learners' incorrect calculations:

$$3. 2\frac{2}{5} - 1\frac{4}{5} = ?$$

$$2\frac{2}{5} - 1\frac{4}{5} = 1$$

*I can not carry because I do not know borrow from my whole number.

$$3. 2\frac{2}{5} - 1\frac{4}{5} = ?$$

$$(2-1) + (2/5 - 4/5)$$

$$1 + (-2/5)$$

$$(5/5 - 4/5)$$

$$1 - 1 = 0$$

For this question (number 3), 54 out of 59 learners (92%) answered incorrectly. There was a significant difference between the number of learners who got this question incorrect versus Question 2 in this section. A possible reason for this could be that learners do not understand how to subtract the numerator 4 from 2. Alternatively, these learners were unsure of borrowing from the whole number method or how to convert to an improper fraction. Some learners explained that they did not understand the process of how to calculate the sum, while other learners just subtracted the whole numbers from each other and did not proceed further. Some learners mistakenly added instead of subtracting.

$$\frac{1}{5} \frac{2}{5}; \frac{1}{5} \frac{3}{5}; \frac{0.8}{5} \rightarrow \frac{3}{5} \frac{3}{5}; \frac{1}{5} \frac{0}{5}; 1; 1 \frac{1}{5}; \frac{8}{30} \frac{1}{5}; \frac{1}{5} \frac{9}{5}; 1; \frac{1}{5} \frac{1}{1}; \frac{1}{3} \frac{0}{5}$$

Question 4

$$5\frac{2}{3} - 1\frac{4}{6} = 4$$

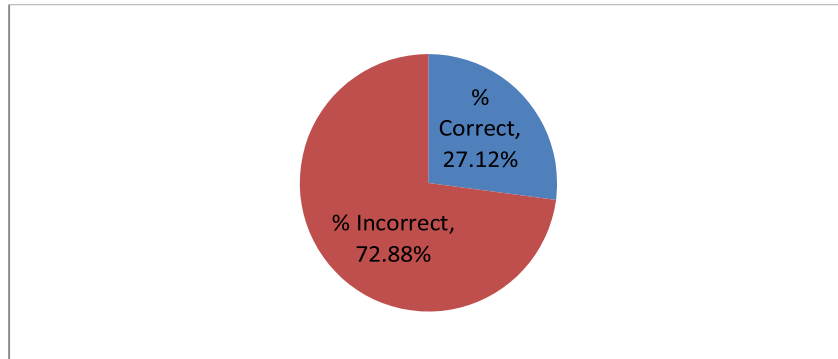


Fig. 4.9: Pie chat for correct and incorrect answers for Section B: Question 4

The correct steps:

The correct steps to be followed were:

4. $5\frac{2}{3} - 1\frac{4}{6} = ?$

Correct steps (Left side):

$$5 - 1 = 4$$
$$\frac{2 \times 2}{3 \times 2} = \frac{4}{6}$$
$$4 - 4 = 0$$
$$\frac{0}{6}$$
$$= 4$$

Alternative steps (Right side):

$$\frac{17 \times 2}{3 \times 2} = \frac{34}{6}$$
$$\frac{10}{6}$$
$$= \frac{24}{6}$$
$$= 4$$

Fig. 4.9.1: Correct steps

The contents on the left side of fig. 4.9.1 shows the correct steps to calculate the answer. One could subtract the whole numbers first and, then find the LCD for both denominators as shown above. Once converted, subtract the new numerators, write it above the denominator and then add the whole number to the fraction at the end.

The contents on the right side of fig. 4.9.1 are alternative correct steps which could also be used. This method converts the fractions to improper fractions. Then, calculate

the LCD and subtract the numerators. Convert the answer in improper fraction back to a mixed fraction, and in some cases, to its simplest form.

Examples of some learners' incorrect calculations:

The image shows three examples of student work for the problem $4.5\frac{2}{3} - 1\frac{4}{6} = ?$.
 Example 1: The student has written $4.5\frac{2}{3} - 1\frac{4}{6} = ?$ and has written 43 below it, with a large 'y' next to it.
 Example 2: The student has written $4.5\frac{2}{3} - 1\frac{4}{6} = ?$ and has written $4-1=3$ and $3\frac{2}{3} + \frac{2}{3} - \frac{4}{3} = \frac{2}{3}$ below it, with a large 'y' next to it.
 Example 3: The student has written $4.5\frac{2}{3} - 1\frac{4}{6} = ?$ and has written $(4) 5\frac{2}{3} - 1\frac{4}{6} = 4\frac{2}{3}$ below it, with a large 'y' next to it. To the right of this, the student has written 'I know what to do because I don't understand'.

Fig. 4.9.2: Example of some learners' incorrect calculations

For this question (number 4), 43 out of 59 learners (about 73%) answered incorrectly. Surprisingly, fewer learners got this question wrong compared to Question 3. Instead of seeing 6 as the LCD, some learners multiplied both denominators to get to 30. Some among the learners said they were unable to calculate the sum as the denominators were different. Explanations were given by the learners as to why they could not calculate the sum due to them not knowing how to borrow or how to change the denominators. Some learners subtracted the denominators and others said it was difficult for them to find the LCD.



The incorrect answers given by the learners:

$$4\frac{2}{3}, \frac{8}{43}; 3\frac{1}{3}, \frac{2}{6}; 4\frac{2}{6}, 4\frac{0}{12}; 4\frac{2}{4}, \frac{1}{4}; 3\frac{2}{4}, 3; 4\frac{2}{3}, 5; 6\frac{2}{9}, \frac{2}{3}, \frac{7}{7}$$

Question 5

$$7\frac{1}{2} - 2\frac{3}{10} = 5\frac{1}{5}$$

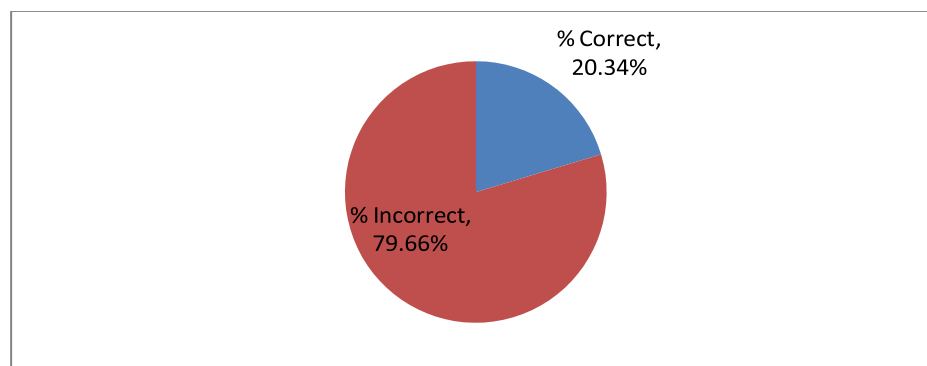


Fig. 4.10: Pie chart of correct and incorrect answers for Section B: Question 5

The correct steps:

The correct steps to be followed were:

5. $7\frac{1}{2} - 2\frac{3}{10} = ?$

Left side (Correct steps):

$$7 - 2 = 5$$

$$\frac{1 \times 5}{2 \times 5} = \frac{5}{10}$$

$$\frac{5}{10} - \frac{3}{10} = \frac{2}{10}$$

$$= \frac{2 \div 2}{10 \div 2} = \frac{1}{5}$$

$$= 5\frac{1}{5}$$

Right side (Alternative correct steps):

$$\frac{15 \times 5}{2 \times 5} = \frac{75}{10}$$

$$\frac{23}{10}$$

$$75 - 23 = 52$$

$$\frac{52}{10} = 5\frac{2}{10} = 5\frac{1}{5}$$

Fig. 4.10.1: Correct steps

The contents on the left side of the above figure show the correct steps to calculate the answer. One could subtract the whole numbers first and, then find the LCD for both denominators as shown in fig. 4.10.1. Once converted, subtract the new numerators, write it above the denominator and then add the whole number to the fraction at the end.

The contents on the right side of fig. 4.10.1 are alternative correct steps which could also be used. This method converts the fractions to improper fractions. Then, calculate the LCD and subtract the numerators. Convert the answer in improper fraction back to a mixed fraction, and in some cases, to its simplest form.

Examples of some learners' incorrect calculations:

5. $7\frac{1}{2} - 2\frac{3}{10} = ?$

Example 1 (Incorrect):

$$7\frac{1}{2} - 2\frac{3}{10} = 5\frac{2}{8}$$

Example 2 (Incorrect):

$$7\frac{1}{2} - 2\frac{3}{10} = 5$$

I don't know because the numbers at the bottom are different

Example 3 (Incorrect):

$$7\frac{1}{2} - 2\frac{3}{10} = 5$$

$\frac{1}{2} + \frac{1}{2} = 1$

Fig. 4.10.2: Examples of some learners' incorrect calculations

For this question (number 5), 47 out of 59 learners (about 80%) answered incorrectly. The results of this question were the complete opposite of Question 1 in this section. These learners appeared to be unaware that the LCD needed to be found. Some learners subtracted the whole numbers and did not proceed with the rest of the sum. Once again, the learners either subtracted the denominators or wrote their answer over an irrelevant denominator or used one of the existing denominators in their answer. There were others who explained that they did not know what to do after finding the LCD as they were unsure of how to calculate mixed fractions. A selected number of learners who got the answer correct, did not simplify their final answer.

The incorrect answers given by the learners:

$6\frac{2}{7}; 5; 3\frac{2}{6}; 6\frac{7}{8}; 6\frac{5}{10}; 5\frac{2}{5}; 5\frac{2}{8}; 5\frac{7}{10}; 8; 5\frac{4}{10}; 5\frac{1}{2}; 7\frac{7}{2}; 5\frac{2}{15}; \frac{4}{5}; \frac{3}{10}; 5\frac{1}{8}; \frac{2}{10}; 4; 5\frac{3}{8}; 5\frac{4}{20}$

4.1.3 The relevance of Section B to the research sub-questions

The results from this section of the learners' questionnaire refer to the research Sub-question 1. The analysed data show the learners' strengths and challenges. In the subtraction of like-fractions, the percentage of those who got correct answers were higher than that of subtraction of unlike-fractions. These challenges were seen when learners had to subtract unlike-fractions and even more so, when it was mixed fractions.

Section C: Multiplication and Division

Question 1

$$\frac{1}{2} \text{ of } 30 = 15$$

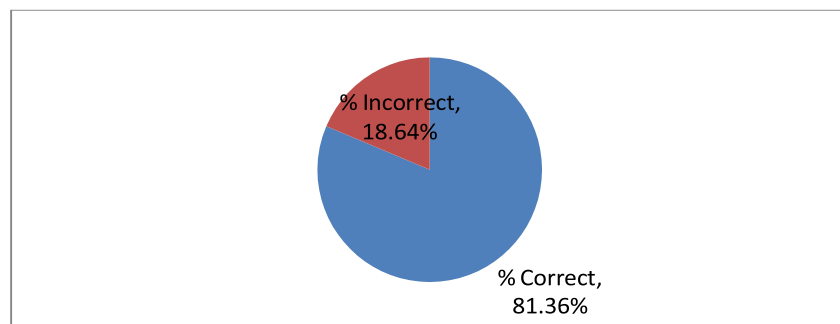


Fig. 4.11: Pie chart for correct and incorrect answers for Section C: Question 1

The correct steps:

The correct steps to be followed were:

1. $\frac{1}{2}$ of 30 = ?

Left side: $30 \div 2 = 15 \times 1$
 $= 15$

Right side: $30 \div 2 = 15$
 $15 \div 1 = 15$
 $= 15$

Fig. 4.11.1: Correct steps

The contents on the left side of fig. 4.11.1 show the correct steps to calculate the answer. One could divide the whole number by the denominator first, then multiply that answer by the numerator.

The contents on the right side of fig. 4.11.1 are alternative correct steps which could also be used. This method converts the whole number into an improper fraction and, simplify the opposite fractions, if possible. Then, cross multiply the numerators and denominators. Convert the improper fraction back to a mixed fraction, and in some cases, to its simplest form.

Examples of some learners' incorrect calculations:

1. $\frac{1}{2}$ of 30 = ?

Left side: $30 \times 2 = 60$

Middle side: $\frac{1}{2} \times 30 = 150$
 $5 \times 30 = 150$

Right side: $\frac{1}{2}$ of 30 = 157

Fig. 4.11.2: Examples of some learners' incorrect calculations

For this question (number 1), 11 out of 59 learners (about 19%) answered incorrectly. The learners who answered incorrectly divided 30 by 15 and got an answer of 2 instead of dividing 30 by 2 to get 15. Some learners multiplied 30 by 2 to get an answer.

Others wrote that they had forgotten how to do multiplication of fractions, while others stated that they could not remember the steps. The learners who got it correct knew what half of 30 was and answered correctly.

The incorrect answers given by the learners:

$$2 ; 60 ; \frac{15}{30} ; 6 ; 150 ; 15 \frac{2}{1} ; 15 \frac{2}{1} ; 3$$

Question 2

$$\frac{3}{4} \text{ of } 25 = 18 \frac{3}{4}$$

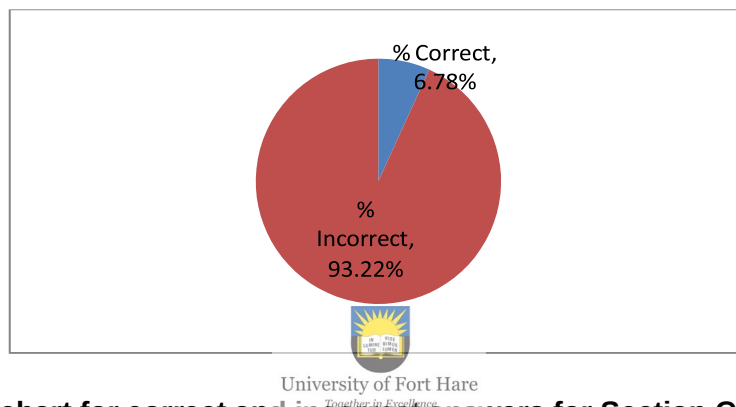


Fig. 4.12: Pie chart for correct and incorrect answers for Section C: Question 2

The correct steps:

The correct steps to be followed were:

2. $\frac{3}{4}$ of 25 = ?

Left side calculations:

$$25 \div 4 = 6.25 \times 3 = 18.75$$

$$\frac{3}{4} \times 25 = \frac{75}{4} = 18 \frac{3}{4}$$

Right side calculation:

$$\frac{3}{4} \times \frac{25}{1} = \frac{75}{4} = 18 \frac{3}{4}$$

Fig. 4.12.1: Correct steps

The contents on the left side of fig. 4.12.1 show the correct steps to calculate the answer. One could divide the whole number by the denominator first and, then multiply that answer by the numerator.

The contents on the right side of fig. 4.12.1 are alternative correct steps which could also be used. This method converts the whole number into an improper fraction. Simplify the opposite fractions, if possible. Then, cross-multiply numerators and denominators. Convert the improper fraction back to a mixed fraction, and in some cases, to its simplest form.

Examples of some learners' incorrect calculations:

2. $\frac{3}{4}$ of 25 = ?

$\frac{3}{4} \times 25 = 3750$

2 750
x 25
3750

2. $\frac{3}{4}$ of 25 = ?

I can't do this because
I get confused when
we have to multiply
at some point

2. $\frac{3}{4}$ of 25 = ?

$3 \times 6 = 18$
 $4 \times 6 = 24$

$18 \frac{1}{4}$

Fig. 4.12.2: Examples of some learners' incorrect calculations

For this question (number 2), 55 out of the 59 learners (about 93%) answered incorrectly. The learners who answered the question incorrectly, multiplied the denominator by the whole number and left that as their answer. Other learners explained that they were unable to do the sum due to them not knowing how to multiply or which steps to use. Out of the 4 learners' who answered the question correctly, 2 learners' wrote their answers as a decimal of 18,75 and the other 2 wrote their answer as a fraction $18 \frac{3}{4}$. Those who got a decimal answer worked their sum out using the method on the left in the Fig. 4.12.1. They did not convert this answer to a mixed fraction in the simplest form. The other two learners converted the whole number to an improper fraction using the steps on the right side of Fig. 4.12.1.

The incorrect answers given by the learners:

18 rem 3 ; $15 \frac{2}{1}$; 13 rem 3 ; 17 rem 1 ; 18 ; 5 ; 200 ; 8 rem 1 ; 24 rem 1 ; 15 rem 1 ; 33 rem 1 ; 2 ; 52 rem 2 ; 18,5 ; 21 ; 12 ; 6,5 ; 75 ; $6 \frac{1}{4}$; $13 \frac{1}{2}$; 15 ; 2 rem 1 ; 20 ; $13 \frac{3}{4}$; $3 \frac{1}{4}$; 3 750 ; 6 ; 1,60 ; 6,25 ; 100 ; 15 rem 5 ; 75 ; $6 \frac{1}{2}$; 19

$$\frac{4}{15} \text{ of } 40 = 10 \frac{2}{3}$$


The correct steps to be followed were:



The contents of fig. 4.13.1 are correct steps which could also be used. This method converts the whole number into a fraction. Simplify the opposite fractions, if possible. Then, cross-multiply the numerators and denominators. Convert the answer from an improper fraction back to a mixed fraction, and in some cases, to its simplest form.

2. $\frac{4}{15}$ of 40 = ? 60

$\frac{4}{15} \times 40 = \frac{4}{15} \times \frac{40}{1}$

$40 \times 4 \div 15 = 60$

3. $\frac{4}{15}$ of 40 = ? 160

40 40

$\times 15$ $\times 4$

200 160

+ 400

600

3. $\frac{4}{15}$ of 40 = ?

$\frac{4}{15}$ of 40 = 8 r 10

$4 \times 15 \div 40 = 8 \text{ rem } 10$

40

$\times 15$

200

+ 400

600

40

$\times 4$

160

8

10

56

For this question (number 3), 57 out of the 59 learners (about 97%) answered incorrectly. One among them wrote that they struggled with sums when the numbers were bigger. Some wrote that they were unsure of what needed to happen with the remainder. One of the learners who got this correct used the same steps as shown in Fig.4.13.1.

The incorrect answers given by the learners:

24 ; 39,9 ; 2 400 ; 8 ; 1 rem 10 ; 9 ; 60 ; 160 ; 10 rem 10 ; 8 rem 10 ; 2 rem 10 ; 12 ;
 10,10 ; 18 ; 10 ; 600 ; 4 ; 200 ; 2,9 ; $\frac{8}{30}$; 3 ; 6 rem 10 ; 2 ; 40 ; $2\frac{1}{2}$; 4,50 ; 7 ; 34 ; 8,10 ;
 20 ; 5 ; 150 rem 10

Question 4

$$\frac{25}{100} \text{ of } 250 = 62\frac{1}{2}$$

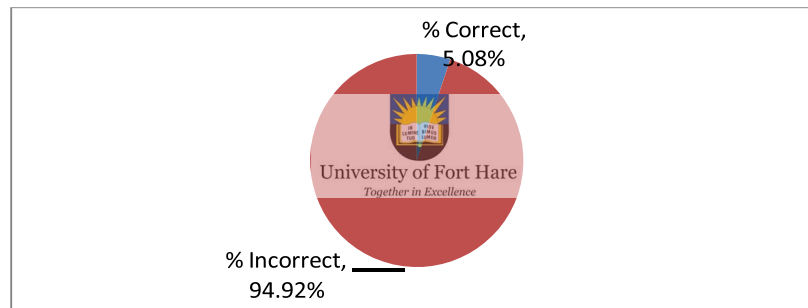


Fig. 4.14: Pie chart for correct and incorrect answers for Section C: Question 4

The correct steps:

The correct steps to be followed were:

Method 1 (Left):

$$250 \div 100 = 2,5$$

$$10 \overline{) 250} = 25$$

$$2,5 \times 25 = 62,5$$

$$62,5 = 62\frac{1}{2}$$

Method 2 (Right):

$$\frac{25}{100} \times \frac{250}{1} = \frac{25}{2} \times \frac{5}{1} = 125 \div 2 = 62\frac{1}{2}$$

Fig. 4.14.1: Correct steps

The contents on the left side of fig. 4.14.1 show the correct steps to calculate the answer. One could divide the whole number by the denominator first and, then multiply that answer by the numerator.

The contents on the right side of fig. 4.14.1 are alternative correct steps which could also be used. This method converts the whole number into an improper fraction. Simplify the opposite fractions, if possible. Then, cross multiply numerators and denominators. Convert the improper fraction back to a mixed fraction, and in some cases, to its simplest form.

Examples of some learners' incorrect calculations:

4. $\frac{25}{100}$ of 250 = ?
 $\frac{25}{100}$ of 250 = 25000
 I cannot show the working out because that is the only method I can do.

4. $\frac{25}{100}$ of 250 = ?
 $100 \overline{) 250}$
 200 25
 - 50 x 2
 60 r 50

4. $\frac{25}{100}$ of 250 = ?
 25
 $100 \overline{) 250}$
 $\frac{25}{100}$ of $\frac{250}{1} = \frac{1250}{100} = 12 \text{ r } 50$

Fig. 4.14.2: Examples of some learners' incorrect calculations



For this question (number 4), 56 out of the 59 learners (about 95%) answered incorrectly. Some learners multiplied the whole number by the denominator and left that as their answer. Others divided the whole number by the denominator and appeared to have lost direction with the steps which needed to be followed afterwards. Some learners were unsure of how to calculate the sum after dividing it or they were unsure what process to use while moving forward. Another group of learners wrote that they did not understand how to calculate the sum.

Two of the learners used the steps on the left in Fig. 4.14.1. The others used the steps on the right in Fig. 4.14.1.

The incorrect answers given by the learners:

6 rem 25 ; 625 ; 75 ; 175 ; 61 ; 50 ; 10 ; 50 rem 50 ; 50 ; 60 ; 2 rem 50 ; 50,5 ; 98 rem 2
 ; 100 ; 25 ; $2 \frac{3}{50}$; $6 \frac{250}{100}$; 100 rem 2 ; 25 ; 5 ; 1 000 ; 2 ; 62 rem 5 ; 1 ; 62 rem 50 ; $\frac{50}{200}$; 2,50
 ; 25 000 ; 100 ; 500 ; 622 ; 21

Question 5

$$\frac{13}{100} \text{ of } 320 = 41 \frac{3}{5}$$

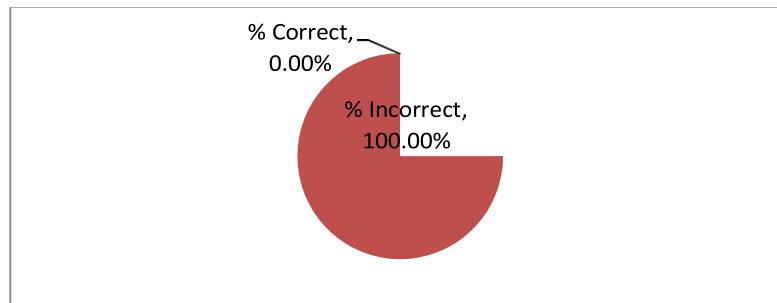


Fig. 4.15: Pie chart for correct and incorrect answers for Section C: Question 5

The correct steps:

The correct steps to be followed were:

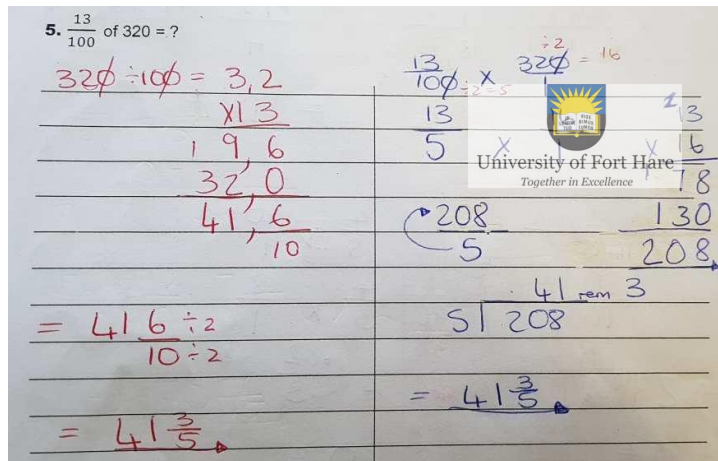


Fig. 4.15.1: Correct steps

The contents on the left side of fig. 4.15.1 show the correct steps to calculate the answer. One could divide the whole number by the denominator first and, then multiply that answer by the numerator.

The contents on the right side of fig. 4.15.1 are alternative correct steps which could also be used. This method converts the whole number into an improper fraction. Simplify the opposite fractions, if possible. Then, cross multiply numerators and

Examples of some learners' incorrect calculations

5. $\frac{13}{100}$ of 320 = ?

5. $\frac{13}{100}$ of 320 = ?

$\frac{13}{100} \times 320$

$4 \frac{1}{100} \times 4$

$500 \div 13 = 24$

$24 \times 13 = 312$

$60 \div 13 = 4$

$4 \times 13 = 52$

$60 - 52 = 8$

$12 \times 13 = 156$

13 of 320 = I don't understand this question because I don't know how to divide to get the answer.

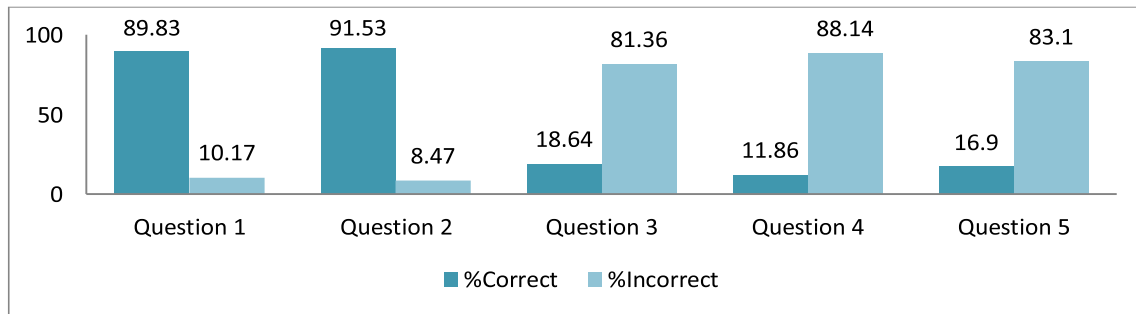
For this question (number 5), all the learners answered incorrectly. Some were close, but did not manage to complete the answer. They calculated the answer of 41 but with a remainder, not a fraction or decimal. Some learners wrote that they did not understand how to multiply or divide or when to use certain steps. It appeared that learners were unfamiliar with multiplication of fractions.

41 rem 60 ; 24 rem 7 ; 39 ; 128 ; 12,80 ; 105 ; 3,20 ; $\frac{24}{30}$; 39 rem 20 ; 12 rem 70 ; 117 ;
30 ; 3 rem 20 ; 4 160 ; 39,2 ; 24, 8 ; 16 ; 10 ; 30 ; 4 $\frac{1}{100}$; $\frac{13}{100}$; 26 ; 13 ; 32 ; 7 360 ; 20
; 12 rem 8 ; 25 ; $\frac{39}{300}$; 410 ; 32 000 ; 3 ; 36 ; 59 ; 360 ; 416 ; 3 200.

The results from this section of the learners' questionnaire refer to the research Sub-question 1. The analysed data show the learners' strengths and challenges.

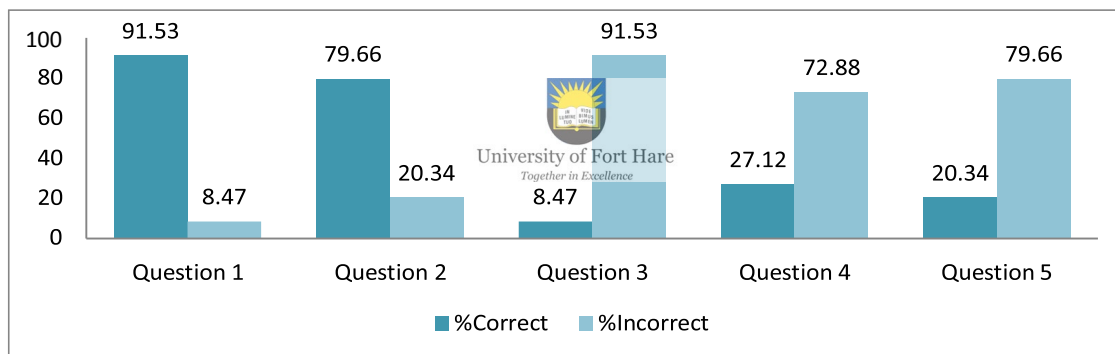
4.1.5 Summary for Sections A, B and C.

Table 4.1.5.1: Summary of Section A



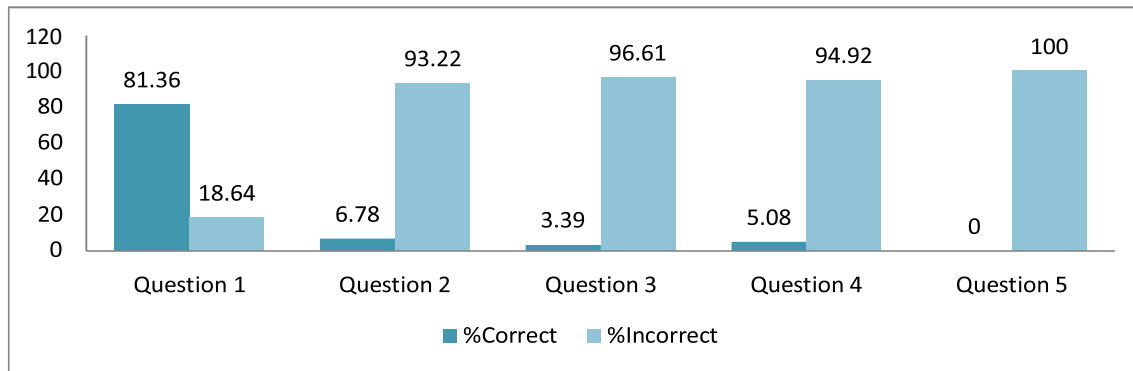
The above graph shows that majority of the learners knew how to add like-fractions. Learners struggled to calculate unlike-fractions, and even more so, mixed fractions. There was an inability to find the LCD. The number of learners who answered correctly from Questions 1-5 decreased.

Table 4.1.5.2: Summary of Section B



Results for Question 1 and 2 above were similar to Questions 1 and 2 in Section A, where the percentages were higher for the first two questions. Seemingly, majority of the learners answered like-fractions correctly compared to the unlike- and mixed-fractions. The learners were unable to find the LCD. The number of learners who answered correctly from Questions 1-5 decreased.

Table 4.1.5.3: Summary of Section C



This section appeared to be the most difficult for the learners. The only Question with the highest percentage for correct answer was question 1. From Questions 2-5 the number of correct answers decreased. Seemingly, the learners were unsure of how to do multiplication of fractions.

4.1.6 Relevance of this Section to the research Sub-questions

These results refer to the research Sub-question 1. The analysed data show the learners' strengths and challenges. According to the percentages shown in Tables 4.1.5.1 and 4.1.5.2, the strengths were on calculating like-fractions. The challenges were on calculating unlike-, mixed- and multiplication fractions. These challenge areas need to be focused on. Strategies are needed in order to address these challenges and improve the results in the operations of unlike-, mixed- and multiplication fractions. There were strategies suggested by the teachers who participated in the study; these can be found under Theme 4.5.5.

4.2 Data from learner interview questions

As stated earlier in Chapter Three, only 10 learners were interviewed. Examples of audio-transcriptions are given below:

4.2.1 Learners' rating of the different operations from interview questions

Each question which the learners had to rate will be analysed and discussed. Refer to Appendix 6 to see the learners' interview questions.

Question 1

All the learners said that they were able to add like-fractions. Majority (7/10; 70%) said that they were uncomfortable with adding unlike-fractions because they struggled to find the LCD.

“When there is a mixed fraction with different denominators I struggle to do that because I don’t know how to find the lowest common denominator.” Refer to Appendix 9 for the learners’ full response.

Question 2 – Table 4.2.1.1: Learners’ rating on difficulty subtracting types of fractions

Refer to Appendix 7 to see the learners’ interview questions. The learners had to rate the difficulty level of the fraction operation involving subtractions.

Subtraction	Rating	Number of learners out of 10	Percentage
1) like-fractions	Difficult	1	10%
	Easiest	9	90%
2) unlike-fractions	Easiest	2	20%
	Average	5	50%
	Difficult	3	30%
3) Mixed fractions	Easiest	1	10%
	Average	4	40%
	Difficult	5	50%

The following gives a summary of the learners’ rating of the types of fractions in Section B. Only one of the learners rated all types of questions in Section B to be easy. The majority of the learners (8/10; 80%) rated like-fractions to be easy and one rated like-fractions to be difficult. Two of the learners rated subtraction involving unlike-fractions as easy. Half of the learners (5/10; 50%) rated subtracting unlike-fractions to be of average difficulty and three learners rated subtraction of unlike-fractions to be difficult.

The same learner who rated like- and unlike-fractions to be easy also found subtraction of mixed fractions easy. This learner answered all questions in section B correctly. Half of the learners (5/10; 50%) rated subtraction of mixed fractions as difficult and 4 learners found it to be of average difficulty - that is, not as difficult as like-fractions and unlike-fractions.

Question 3 – Table 4.2.1.2: Learners' rating on the difficulty with fraction subtractions

Refer to Appendix 6 to see the learners' interview questions. The learners had to rate the difficulty level of the fraction operation involving subtractions.

Questions in Section B of learners' questionnaire	Rating	Number of learners out of 10	Percentage
Question 1	Easiest	9	90%
	Average	1	10%
Question 2	Second easiest	7	70%
	Average	1	10%
	Easiest	2	20%
Question 3	Difficult	2	20%
	Average	5	50%
	Second most difficult	1	10%
	Second easiest	1	10%
	Easiest	1	10%
	Second easiest	1	10%
	Second most difficult	6	60%

Question 4	Difficult	2	20%
	easiest	1	10%
Question 5	Difficult	5	50%
	Second most difficult	2	20%
	Average	2	20%
	Easiest	1	10%

The following gives a summary of the learners' rating of each question in Section B. A learner rated all the questions to be easy. Majority of the learners (9/10; 90%) rated question one and two to be the easiest. There were mixed responses for Question 3, half of the learners felt that it was an average question and, the other half felt that it was easy or difficult. Question 4 was rated second most difficult by 6 learners. Only 2 learners rated it most difficult, whereas 2 other learners rated it easy. Half of the learners (5/10; 50%) rated Question 5 most difficult, 2 learners rated it second most difficult and the other 2 rated it average.



Question 4

This question referred to section C of the learners' questionnaire. They had to explain the steps used to calculate their answers. Below are examples of learners' explanations:

"Question 2, I put 25 (whole number) over 1 then, I multiplied my numerators then my denominators. Then I divided 4 into 75 and got 18 with a remainder of 3." Refer to Appendix 8.

"I took the denominator and times it by the whole number and got 60 and then I got confused and did not know what to do afterwards." Refer to Appendix 10.

Question 5 – Table 4.2.1.3: Learners’ rating on difficulty with fraction multiplications

Refer to Appendices 8-17 to see the learners’ interview questions. The learners had to rate the difficulty level of the fraction operation involving multiplication.

Questions in Section C of the learners’ questionnaire	Rating	Number of learners out of 10	Percentage
Question 1	Did not understand	1	10%
Question 2	Did not understand	4	40%
Question 3	Did not understand	7	70%
Question 4	Did not understand	5	50%
Question 5	Did not understand	8	80%

The following gives a summary of the learners’ rating the questions in Section C. Only one of the learners rated Question 1 to be difficult. Majority of the learners (6/10; 60%) rated that they understood Question 2. Majority of the learners rated that they did not understand questions 3-5, their ratings were congruent with the result recorded in Figs. 4.13, 4.14 and, 4.15 or Table 4.1.2.3.

Question 6

This question referred to Section A of the learners’ questionnaire. They had to explain the steps used to calculate their answers. Below are examples of learners’ explanations:

“The mistake I made was I added my denominators together when I was not supposed to, I know that I must not do anything to the denominators when they are the same.”

Refer to Appendix 11 for full response.

"I first added my whole numbers in question 3 then I got stuck with the fractions because the denominators were very different." Refer to Appendix 12 for full response.

Question 7

This question referred to Section B of the learners' questionnaire. They had to explain the steps used to calculate their answers. Below is an example of a learner's explanation:

"I should have taken the second fraction and put it first then subtract the numerators from each other. Or I could have made my denominators bigger then I would have been able to subtract. I don't know about the borrowing method." Refer to Appendix 8 for the full response.

Question 8

This question referred to Section A of the learners' questionnaire. They had to explain if they got confused when adding different types of fractions. Below are examples of learners' explanations:



"It depends on how big the denominator number is, when it is too big it becomes confusing. If it is small then I can find the LCD." Refer to Appendix 9 for the full response.

"Yes, I get mixed up when the denominators are not the same." Refer to Appendix 8 for the full response.

Question 9

This question referred to Section B of the learners' questionnaire. They had to explain if they got confused when subtracting different types of fractions. Below are examples of learners' explanations:

"I get confused with fractions because I don't know what to do especially when the denominators are different." Refer to Appendix 10

"Yes, I would definitely get confused because I will have to take to the lowest common denominator in order to add or subtract." Refer to Appendix 13 for the full response.

Question 10

This question referred to Section C of the learners' questionnaire. They had to explain if they got confused when calculating multiplication fractions. Below is an example of a learner's explanation:

"No, I struggle with this, it depends on what the number is, like question 1 was easier but the others were not easy." Refer to Appendix 17.

Question 11

This question referred to Section B of the learners' questionnaire. Learners were asked if they knew of other methods that could have been used to calculate subtraction fractions. Below are examples of learners' explanations:

"No, I don't." Refer to Appendix 16

"No other method." Refer to Appendix 17

Question 12 – Table 4.2.1.4: Learners' rating on difficulty with fraction operations



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Refer to Appendices 11-20 to see the learners' interview questions. The learners had to rate the difficulty level of the fraction operation for each section.

	Rating	Number of learners out of 10	Percentage
Section A	Difficult	1	10%
	Average	4	40%
	Easiest	5	50%
Section B	Difficult	3	30%
	Average	5	50%
	Easiest	2	20%
Section C	Difficult	6	60%

	Average	1	10%
	Easiest	3	30%

The following gives a summary of learners' rating for Section A, B and C. Half of the learners (5/10; 50%) rated Section A to be easy, half of the learners (5/10; 50%) rated Section B average and majority (6/10; 60%) rated Section C most difficult.

Question 13 – Table 4.2.1.5: Learners' rating on difficulty with fractions additions

In this question, learners had to rate the questions in Section A from the easiest to the most difficult. Refer to Appendices 11-20 to see how individual learners rated. Table 4.2.1.5 shows the ratings.



Question 13 – Table 4.2.1.5: Learners' rating on difficulty with fractions

Questions in Section A of the learners' questionnaire	Rating	Number of learners out 10	Percentage
Question 1	Easiest	9	90%
	Second easiest	1	10%
Question 2	Second easiest	6	60%
	Easiest	2	20%
	Average	2	20%
Question 3	Most difficult	2	20%
	Average	5	50%
	Easiest	3	30%
Question 4	Most difficult	3	30%
	Second most difficult	3	30%
	Average	1	10%
	Second easiest	1	10%
	easiest	2	20%
Question 5	most difficult	5	50%
	Second most difficult	3	30%
	easiest	2	20%

The following gives a summary of the learners' rating of questions in Section A. The majority of the learners (9/10; 90%) rated Question 1 and 2 easy. Half of the learners (5/10; 50%) rated Question 3 to be average. Only one learner rated Question 4 to be average and half (8/10; 80%) rated Question 5 to be difficult.

Question 14

For this question, learners were asked if they thought there were enough exercises in their textbooks for them to practise fraction operations. Below are conflicting examples of learners' explanations:

"We have covered a lot of exercises on fractions, it is not like we do it often, it does not pop up every term as often so I can remember the concept or steps." Refer to Appendix 13 for the full response.

"No, I don't think there is enough exercises because we only do it now and then. We don't do fractions all the time." Refer to Appendix 16.

Question 15

For this question, learners were asked if they knew the different terminology used for fractions. They were asked to write down fractions and label it. All the learners wrote the correct descriptions similar to the one below.

$\frac{1}{4}$	Numerator	$\frac{17}{5}$	Improper fraction University of Fort Hare Together in Excellence	$3\frac{1}{3}$	Mixed fraction
	Denominator				

Question 16

For this question, learners were asked to provide strategies their teachers could adopt to help them understand fraction operations. The answers can be found under the learners' theme 4.3.1.9.

Table 4.2.1.6: Analysis of Mean, median and mode for the 10 learners who were interviewed

Learner no.	Section A correct answers out of 5	Section B correct answers out of 5	Section C correct answers out of 5	Total out of 15	%	Mean	Median	Mode
Learner 1	3	3	1	7	47%	2.33	3	3
Learner 2	2	1	3	6	40%	2	2	0
Learner 3	0	2	0	2	13%	0.66	0	0
Learner 4	0	1	1	2	13%	0.66	1	1
Learner 5	2	2	1	5	33%	1.66	2	2
Learner 6	5	4	1	10	67%	3.33	4	0
Learner 7	5	5	3	13	87%	4.33	5	5
Learner 8	1	1	3	5	33%	1.66	1	1
Learner 9	5	2	4	11	73%	3.66	4	0
Learner 10	2	2	1	5	33%	1.66	2	2
Totals	25	23	18	6.6	44%	2.2	23	0

The following gives a summary of learners' percentages for each section and a combined percentage. Majority of the learners (7/10; 70%) achieved under 50% overall. Common concerns mentioned by the learners were, (a) inability to find the

LCD, (b) inability to use operations involving addition and subtraction for unlike- or mixed fractions and, (c) inability to use operation involving multiplication.

4.3 Qualitative data

4.3.1 Learners themes

The themes which will be discussed come from the questionnaires and interviews of the learners who participated.

Main themes have been categorised into:

1. Learners' strengths
2. Learners' challenges
3. Learners' suggested strategies to assist in the teaching of fraction operations

Sub-themes

4.3.1.1 Learners' strengths in learning addition and subtraction of fractions

Addition and subtraction of like-fractions were said to be the easiest for learners to calculate. Learners did not struggle to calculate the fractions when they had to add or subtract numerators of like-fractions. The ratings for addition and subtraction of fractions are in Table 4.1.5.1 and 4.1.5.2.



4.3.1.2 Relevance of this theme to the research sub-question

This theme is relevant to research Sub-question 1. The learners' strengths were on addition and subtraction of like-fractions. This is evident from Tables 4.1.5.1 and 4.1.5.2 where learners answered the first two questions correctly in sections A and B.

4.3.1.3 Learners' strengths in learning multiplication of fractions

Learners reported multiplication of fractions to be the easiest as they could correctly calculate smaller numbers or calculate with no remainders. The ratings for multiplication of fractions are in Table 4.1.5.3. Proficiency in the knowledge of multiplication tables is important when calculating fractions of multiplication.

4.3.1.4 Relevance of this theme to the research sub-question

This theme is relevant to research Sub-question 1. The learners' strengths were on multiplication calculations with no remainder or on smaller numbers, for example, below 10. Most of the learners answered the first question correctly in Section C (refer to Table 4.1.5.3).

4.3.1.5 Learners' weaknesses in calculating different operations of unlike-fractions

The learners appeared to have struggled to calculate the questions involving addition and subtraction of unlike-fractions (refer to Tables 4.1.5.1 and 4.1.5.2). There were learners who added or subtracted the denominators of the unlike-fractions (refer to Figs. 4.4.2 and 4.8.2). Learners needed to provide a reason if they were unable to answer the question. Majority of them wrote that they were unsure of the steps to follow or how to calculate the LCD (see Tables 4.2.1.2 and 4.2.1.5). These learners also said they were unsure of which operation to use when trying to convert to an improper or mixed fractions.



Subtraction of mixed fractions where borrowing was needed appeared to be a challenge for the learners. The learners were unsure of an alternative method to subtract mixed fractions. When some learners borrowed from the whole number, they did not proceed further as they were unsure of what steps to use. Other learners either added or subtracted the whole numbers and left the calculation of fractions out. Referring to Ndulichako (2013), who concluded that learners were confused in the different operations in fractions together with treatment of numerators and denominators as separate entities would be in agreement with the findings above.

Lukhele, Murray and Olivier (1999) reported from a study in Port Elizabeth in South Africa that their learners added the numerators and denominators which revealed errors on adding fractions by treating each part of the fraction as separate entities. As such, the present study confirms their results even after almost two decades. For example, learners incorrectly added or subtracted fractions as separate entities (Refer to Tables 4.1.5.1 and 4.1.5.2). Figs. 4.1, 4.2, 4.6 and 4.7 show the percentage of learners who answered the questions correctly or incorrectly.

4.3.1.6 Relevance of this theme to the research sub-question

This theme is relevant to research Sub-question 1. The learners' challenges were on calculating unlike- and mixed fractions involving addition and subtraction. The learners' difficulty was on finding the LCD. Some learners said that they struggled because they did not understand how to find the LCD.

4.3.1.7 Learners' weaknesses in calculating multiplication of fraction

Majority of the learners said that they were unable to calculate multiplication fractions due to their being unsure of which method or steps to follow. There was a difficulty in calculating when there were remainders as they were unsure of what to do with remainders (refer to Table 4.1.5.3, and Figs. 4.12.2, 4.13.2, 4.14.2 and 4.15.2). Empson and Levi's (2011) and Bruce et al. (2013) too had reported on similar learners' challenges such as the difficulty in understanding and representing fraction relationships, the confusion of the role of the numerator and denominator and the relations between them. As such, the present results are confirmatory of the challenge as persistent.



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4.3.1.8 Relevance of this theme to the research sub-question

This theme is relevant to research Sub-question 1. Multiplication fractions appeared to be the most challenging for the learners. Many learners stated in their interview that they found multiplication of fractions difficult compared to addition and subtraction. Results of how the learners performed can be found on Table 4.1.5.3.

4.3.1.9 Learners' suggestions of strategies for teachers to improve learners' understanding of fraction operations

These are the suggested strategies from the 10 learners who were interviewed.

4.3.1.9.1 Using simpler words

"They could explain in shorter words or in words that are understandable. Teachers should explain methods in the simplest form of understanding."

Seemingly, learners need more practice with the terminology used for fractions. A possible strategy for teachers to use could be to allow learners to label and name the different types of fractions at the start of each lesson. The correct terminology should

be used when teaching fractions for learners to become familiar with it. Nieves (2009) reported that the CSA approach allowed the learners to develop their understanding around concepts and fluency in computations.

4.3.1.9.2 Learning simple steps to find the LCD

“To help me learn how to find the LCD.”

Evidently, the struggled to find the LCD. The present researcher’s personal experiences also showed that this was a common challenge for learners. Teachers ought to provide more ways to find the LCD and gave learners more options. Learners themselves also need to practise their multiplication tables daily. Once there is a foundation of understanding, learning to find the LCD should be easier for them. In Lukhele, Murray and Olivier’s (1999) study learners were asked to answer the following; $\frac{7}{8} + \frac{7}{8}$ and these were two of the answers $\frac{14}{16}$ and 30. Learners used algorithms which they knew for adding whole numbers. Similar response was given by Awuah (2016).



4.3.1.9.3 Develop a step by step method

“They need to help me learn how to add mixed fractions. I think my teacher could teach me an easier method.”

“My teacher could tell us to do our fractions step by step and give us different methods on a piece of paper.”

“My teacher could break down the fractions step by step.”

Learners need to have a foundation in understanding the basics of addition, subtraction and multiplication of fractions. They need to understand how to find the LCD. To avoid confusion, methods need to be taught separately and not in the same lesson. Repetition of steps is important when calculating answers. *Moss and Case (2001), Mary, Jill and Sara (2016) reported in their studies the importance of repetition of fraction exercises and the use of different methods.*

4.3.1.9.4 Converting to mixed fractions using set rules

“My teacher could explain how to convert the number into a mixed fraction.”

To understand how to convert an improper fraction back into a mixed fraction, learners need to divide. Should there be a remainder, learners need to be shown how to write it as a fraction. Repetition of these steps could help learners understand how to calculate and convert to a mixed fraction. Grider (1993) explains his cognitive learning theory as the way in which learning occurred, how information is perceived, organised, where it is stored cognitively and when it gets retrieved and used. Allow learners to be involved in their learning process. Their suggestions should be taken into consideration to assist the teaching and learning process. Learning fraction operations should be stored in their episodic memory which assists in the scaffolding of knowledge. Siegler, Duncan, Davis-Kean, Duckworth, Claessens, Engel, Susperruguy and Chen (2012), Booth, Newton and Twiss-Garrity (2014) and Siegler and Lortie-Forgues (2015) report that the understanding of fractions leads to success in Mathematics and it influences performance in more advanced courses in Mathematics.

4.3.1.10 Relevance of this theme to the research sub-question

This theme is relevant to research Sub-question 3. The suggestions made by the learners could improve the process of how operations of fractions are being taught. This could alleviate the challenges experienced by learners and it could enhance their strengths in fraction operations.

4.4 Biographical information of teachers

Biographical information was only collected from the teachers and not the learners.

Table 4.4.1: Biographical information

<u>Pseudo name:</u>	1. John Deer	2. Kim Lee	3. Maverick
<u>Years of teaching</u>	12	5	3
<u>Position at your school</u>	Grade 7 teacher	Grade 6 teacher	Grade 5 teacher

<u>Academic qualifications</u>	Bachelor of Education	BE.d and Honours in Educational Management	Bachelor of Education
<u>Age group</u>	Group 2	Group 1	Group 1

Age Groups: Group 1 (23-30), Group 2 (31-40), Group 3 (41-50), Group 4 (51-65)

The table 4.4.1 analysed, interpreted and compared information from the different teachers who participated. The interviewees were teachers in Grades 5, 6 and 7. Only two of the teachers are experienced in teaching Mathematics in Grade 6. The information collected answered the main research question: *How can the Grade 6 learners' as well as the teachers' specific strengths be enhanced and the challenges in the learning and teaching of fraction operations be addressed?*

There was a female teacher as well as two male teachers but this did not make a difference in the data collected. There were differences in the teaching experiences of the three teachers. This was reflected in the strategies given by the male teacher with 12 years of teaching experience. There were similarities and differences in their strengths and challenges.



4.5 Teachers themes

The themes below come from the teacher participants who took part in the interviews.

Main themes:

1. Teachers' strengths
2. Teachers' challenges
3. Teachers' teaching strategies

Sub themes:

4.5.1 Teachers' strengths in teaching addition and subtraction of fractions

One of their strengths of teaching fractions was on addition and subtraction through the use of physical demonstrations or pictures to show the learners how like-fractions would be represented. Reference to using a cake and cutting it up into pieces and, then allowing learners to identify the pieces by adding them together was suggested. The content of fractions must be related to the knowledge learners already have. This

is in agreement with Torbeyns, Schneider, Xin and Siegler (2015) where teachers adjust their teaching to their students' current skill levels.

One method mentioned was to ask the learners to identify the existing denominators and see if any could be the LCD. If not, the learners were taught to multiply the denominators together for the LCD. There is a saying which is taught to the learners, "what you do to the bottom you must do to the top" and all three teaches echoed this. This saying was taught when teaching learners how to find the LCD. Northcote's (2012, 2013) study was on the developing of pre-service teachers to enhance their abilities to demonstrate knowledge of mathematical concepts. Through this approach, future in-service teachers will be further developed to teach difficult concepts such as fraction operations.

Constant repetitions of step-by-step methods were used to teach addition and subtraction of fractions. This was achieved through demonstrations of several examples done on the board with the learners. Following a set of steps all the time created a sense of routine, knowing that these were the steps which needed to be followed every time they added or subtracted fractions. (Refer to Appendices 18-20 and Question 11 for teachers' detailed answers).

A method by which one of the teachers would teach subtraction of mixed fractions is;

" $2\frac{2}{5} - 1\frac{4}{5}$ $2 - 1 = 1$ but $2 - \frac{4}{5}$ we cannot do this because we going to get an integer fraction there which you don't want that case. So you know that there's the 1 whole left over so I'm going to borrow from that whole and I'm going to get $\frac{5}{5} + \frac{2}{5} = \frac{7}{5}$ $\frac{7}{5} - \frac{4}{5} = \frac{3}{5}$ this is a classic example of where a child needs to identify that the first fraction is smaller than the one I'm going to subtract from so I need to borrow. If they don't get the borrowing method correct then they must revert to the other method which is converting to improper fraction and finding a LCD." Full answer can be found in Appendix 18, Question 11.

4.5.1.1 Relevance of this theme to the research Sub-question

This theme is relevant to Sub-research Question 2. The common responses were teaching learners how to find the LCD, the use of tangible objects and the repetition of steps and methods. These teachers felt that constant drilling and repetition of doing calculations of fraction operations was a good method to use when trying to get learners understand fractions. An instructional approach used by teachers in Zimbabwe promoted procedural understanding of fractions. They believed that these methods would enhance learners' problem solving capabilities. But, Carpenter, Fennema and Romerg (1993) stated the complete opposite. They said the use of traditional drilling methods limited the learners' ability to understand the actual concept of fractions as they only relied on the rules being taught. It proved to be overwhelming for the learners as there were so many rules they had to learn for the various operations and this caused confusion. Yet, these teachers in 2018 feel that drilling procedural methods is beneficial for learners' conceptual understanding.

4.5.2 Teachers' strengths in teaching multiplication of fractions

Repetition of demonstrations on the board of how to calculate multiplication of fractions was a method used. Set methods and steps were reinforced by the teachers. The use of tangible objects was said to be visual stimuli to show how the concepts learnt are related to reality.

One teacher explained as follows:

"A nice way to do it is, you can actually show them physically with counters or beads excreta, half of 30 is 15. Then we separate by practically moving the beads around and seeing how much half of it is. But I definitely found that the little catch phrase 'divide by the bottom, times by the top' sticks with them and they are usual able to work out the 'of' sum from that" (refer to Appendix 18, Question 12).

More complex calculations, like Question 2 in Section C (Appendix 9), were taught as follows:

“I could change it to a decimal and say 0.75 of 25, which will be 75 over $100 \frac{75}{100}$ of 25, $25 \div 100 = 0.25 \times 75$ would be easier enough to solve. If I had to solve this specific sum, this is the method I would use” (refer to Appendix 18, Question 12 for full answer).

4.5.2.1 Relevance of this theme to the research sub-question

This theme is relevant to the research Sub-question 2. Visual stimuli and constant repetition of demonstrations were a common strength used by the teachers when multiplication of fractions was taught.

4.5.3 Teachers’ challenges in teaching addition and subtraction of fractions

A challenge was the difficulty in trying to get learners not to view fractions as a whole. Learners are conditioned from a young age to only add, subtract, multiply and divide whole numbers. Therefore, this created a challenge for learners to find the LCD or to calculate sums on fractions. Meyer and Land (2003) explained the fundamental concept of transformative as an opening to the new and previously inaccessible way of thinking about something. This is in agreement with the inability to find the LCD.



Another challenge, was trying to teach learners to remember the rules and steps needed to be used when calculating on addition or subtraction of unlike- or mixed fractions. The male teacher (John) explained as follows:

“So the fact that it is so complex, and if it is not second nature to them then they are going to battle with that because there’s so many different ways for the different operations, there’s a different method they have to apply every time” (refer to Appendix 18, Question 13 for the full answer).

The common challenge mentioned was trying to guide learners on ways to remember the rules and steps to finding the LCD. These teachers also mentioned that their learners’ inability to recall the multiplication tables was a contributing factor for them not being able to find the LCD. Jigyel and Afamasaga-Fuata’i (2007) reported on how learners viewed numerators and denominators as separate whole numbers, as done by the learners in this study.

“trying to get them to understand how to find the lowest common denominator that is a challenge” (refer to Appendix 19, Question13 for the full answer),

The UNESCO Institute for Statistics (2012) stated that the topic of fractions represented one of the most abstract and challenging concepts in primary school Mathematics. Teaching addition and subtraction of unlike- or mixed fractions was challenging for the teachers. The steps and methods involved in the explanation of these fraction operations were a common concern for the teachers in this study. Bruce et al. (2013) built on Empson and Levi's (2011) theory regarding how difficulties of teaching and learning fractions started earlier in the primary school years. This would be in agreement with this study.

Ndalichako (2013) also concluded that learners were confused about the different operations in fractions and ended up treating numerators and denominators as separate entities. It was stated that there was a lack of understanding of the appropriate procedures to apply in solving a problem. Evidently, the challenges encountered by the teachers in this study were similar to those mentioned above in other related studies.



4.5.3.1 Relevance of this theme to the research sub-question

This theme is relevant to the research Sub-question 2. Teachers encountered similar challenges such as, (a) difficulty in teaching learners how to find the LCD, (b) difficulty in trying to teach learners how to remember the different rules and steps used for the different types and operations of fractions, (c) difficulty in trying to teach learners to see fractions as pieces of a whole and not as separate entities and (d) the inability of learners to recall the multiplication tables when LCD needed to be found.

4.5.4 Teachers' challenges in teaching multiplication of fractions

To teach learners the rules of multiplication was challenging for the teachers because those differ from the rules of addition and subtraction. The difficulty was trying to recondition learners to adapt to the rules for calculating on multiplication of fractions. The male teacher (John) stated that this confused the learners because the rules were opposite to that of addition and subtraction. It challenged the learners to remember the

different rules. Ndalichako (2013) reported on the lack of learners' understanding of the appropriate procedures to apply in solving a problem, and it rings true with the learners in this study.

There was difficulty teaching learners the method which involved division, more so when there were remainders. The Grade 6 teacher explained that this concept confused the learners as decimals were a newly learnt method. The Grade 5 teacher mentioned that the learners did not do multiplication of fractions. Teaching of different methodologies involved in addition/subtraction and multiplication/division confused the learners. The European Commission (2011), Bruce et al. (2013), Ndalichako (2013) and Pienaar (2014) explained how individual teachers distinguished concepts differently based on their abstract knowledge of the topic.

4.5.4.1 Relevance of this theme to the research sub-question

This theme is relevant to the research Sub-question 2. The teachers encountered different challenges in the different grades. The Grade 7 teacher mentioned that teaching learners multiplication of fractions became complicated because rules changed. The Grade 6 teacher found it difficult as it was a new concept for the learners and they only learnt how to divide with decimals in this grade. The Grade 5 teacher explained that they did not cover multiplication of fractions in the grade.

4.5.5 Teacher strategies to help learners understand fraction operations

4.5.5.1 Using different methods to teach fractions

The Grade 7 teacher (John) suggested the following: Teaching different methods to learners so that they could use a methodology they felt most comfortable with and doing several examples with the learners before giving the learners exercises to do on their own are promoted. It is all about learning a skill. Drill the learners with examples to allow them to practice continuously, as practice makes perfect. Practising of fraction operations were compared to the practising of sports such as cricket and rugby.

“I will model a lot of a methodologies and do thousands and thousands of examples with them until I'm comfortable with them acquiring the skill to go do a couple on their own.” (refer to Appendix 18, Question 16).

The Grade 6 teacher said consistent repetition of the same steps is important. The teacher kept the steps the same which avoided confusion and the method was remembered by the learners as reiterated by Mary, Jill and Sara (2016) about the importance of repetition.

Below are the methods or steps the Grade 7 teacher used to teach his learners:

$$\begin{aligned}
 & 6\frac{2}{4} + 4\frac{4}{5} \\
 &= \frac{26 \times 5}{4 \times 5} + \frac{24 \times 4}{5 \times 4} = \frac{130 + 96}{20} \\
 & \begin{array}{r} 3 \\ 26 \\ \times 5 \\ \hline 130 \end{array} \quad \begin{array}{r} 1 \\ 24 \\ \times 4 \\ \hline 96 \end{array} \\
 &= \begin{array}{r} 130 \\ + 96 \\ \hline 226 \end{array} \\
 &= \frac{226}{20} = 11\frac{6}{20} \div 2 \\
 &= 11\frac{3}{10} \rightarrow
 \end{aligned}$$

Fig. 4.5.5.1.1: Example 1 of the teacher's calculation steps



$$\begin{aligned}
 & 6\frac{2}{4} + 4\frac{4}{5} \\
 &= 6 + 4 \\
 &= 10 \\
 & \frac{2 \times 5}{4 \times 5} + \frac{4 \times 4}{5 \times 4} = \frac{10 + 16}{20} \\
 &= \frac{26}{20} \\
 &= 1\frac{6}{20} \div 2 \\
 &= 1\frac{3}{10} \\
 &10 + 1\frac{3}{10} = 11\frac{3}{10} \rightarrow
 \end{aligned}$$

Fig. 4.5.5.1.2: Example 2 of the teacher's calculation steps

Refer to Appendix 18, Question 19 to read about the way in which the teacher explained the steps shown above.

Below are the methods or steps the Grade 6 teacher used to teach her learners:

①. $6\frac{2}{4} + 4\frac{4}{5}$

$10 \frac{10 + 16}{20} = 10 \frac{26}{20}$

$= 11 \frac{26}{20} - \frac{20}{20}$

$= 11 \frac{6}{20} \div 2 = 11 \frac{3}{10}$

Example 2

Example 1

Fig. 4.5.5.1.3: Examples of the teacher's calculation steps



Refer to Appendix 19, Question 19 to read about the way in which the teacher explained the steps shown above.

Below are the methods or steps the Grade 5 teacher used to teach his learners:

$6\frac{2}{4} + 4\frac{4}{5}$

$10 \frac{10 + 16}{20}$

$= 10 \frac{26}{20}$

$= 11 \frac{6}{20}$

$= 11 \frac{3}{10}$

Fig. 4.5.5.1.4: Example 1 of the teacher's calculation steps

$$\begin{aligned}
 & 6\frac{2}{4} + 4\frac{4}{5} = \frac{26 \times 5}{4 \times 5} + \frac{24 \times 4}{5 \times 4} = \frac{130}{20} + \frac{96}{20} \\
 & = \frac{226}{20} = 11\frac{6}{20} = 11\frac{3}{10}
 \end{aligned}$$

Fig. 4.5.5.1.5: Example 2 of the teacher's calculation steps

Refer to Appendix 20, Question 19 to read about the way in which the teacher explained the steps shown above.



4.5.5.2 Academy for extra support

At this Grade 7 teacher's school they had identified certain challenges such as fractions and set up an academy which was activated over weekends. The teachers focused on targeted areas to help the learners better their understanding. The extra tutoring helped the learners as there was time to focus on the difficult areas. In the past, seminars were held for parents and, they had lessons on how to calculate fraction operations. It helped parents to assist their children at home. In agreement to this, Bruce et al. (2013) reported on learners being exposed to newly innovative teaching strategies.

4.5.5.3 Peer learning

The Grade 6 teacher's (Kim) suggested strategy: Peer teaching is a method used by this teacher. Learners are allowed to sit with their peers and it assists in the understanding of the concepts. Constant examples of steps are done on the board

involving all learners in the lesson. The process is repeatedly taught on the board until learners stop asking for explanations,

“so it’s a constant showing them the steps, refreshing and allowing them to work with a peer so they are able to learn from one another.”

(Refer to Appendix 19, Question16 for the full response)

4.5.5.4 Reinforcing the foundations of basic fractions

The Grade 5 teacher’s (Maverick) suggested strategies: Priority is to build a solid foundation of the basics of fraction operations. This teacher ensured that problem areas were addressed individually. He also believed that Maths had to be practised constantly and individual examples were used, not textbook questions.

“Then they can apply the formula to any situation and not only the examples in the textbook. So it is about using the methods and their knowledge.” (Refer to Appendix 20, Question16 for the full response)

4.5.5.5 Visual Aids



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Another strategy reinforced by the teacher was the use of visual aids. The lessons were built around the use of tangible objects to assist learners to connect fractions to real life scenarios. He believed that it was important to bring visual stimuli as it brought fun back into the classroom. The integration of fun lessons was done to alleviate boredom from lessons. The more active the learners were in the lesson, the more information they retained. This is in agreement with the views of Bruce et al. (2013) that teachers needed to develop lessons which exposed and worked with the children’s intuitions.

4.5.6 Relevance of this theme to the research sub-question

This theme is relevant to the research Sub-question 3. Every teacher explained their personal strategies which they had used in their own classes and hoped that other teachers and learners would benefit from this. The following researchers, Fleisch (2008), Carnoy, Chishlom, and Chilisa (2011), Taylor and Taylor (2013) stated that the crisis in the results of primary schools was due to the lack of knowledge and understanding of the teachers. Their findings were not relevant to the three teachers

who were part of this study. They provided a variety of methods which were used to create a stimulating learning environment through the use of different strategies. This revealed that these teachers were not restricted to obsolete traditional methods but incorporated a variety of other teaching strategies.

Bruce et al. (2013) reported that teachers needed to develop lessons which exposed and worked with learner's intuitions about what they already understand about fractions. Evidently, the three teachers exposed their learners to a variety of different teaching styles as they used out-of-the-box teaching strategies.

4.6 Triangulation of learner questionnaire and learner interview

Section A

Learner no.1 responded: (Question 4 and 5)

Qu.4) I donot understand how to x the fraction to be the same denominator.

Qu.5) I don't understand this part $\frac{44}{30}$



In the above response, the learner got confused with the improper fraction answer and had no idea of what he/she needed to do. Firstly, the LCD was too high, it needed to be 15, and then he/she needed to convert it to a mixed fraction and simplify, if needed.

“...not so much when they are different, mixed fractions with the different denominators.” Refer to Appendix 8, Question 1 for full response.

“I do get confused with mixed fractions because the first number is always a big number...” Refer to Appendix 8, Question 8 for full response.

The learner's response to the questionnaire and interview questions on addition of fractions revealed his/her inability to find the LCD or calculate unlike- or mixed fractions. Refer to Appendix 8, questions 6 and 13 to find out how the learner answered the interview questions on section A.

Learner no. 2 responded: (Questions 3, 4 &5)

Qu.3) I can't do this because they don't have the same denominator.

Qu.4) I can't do this because they don't have the same denominator.

Qu.5) The denominators are not the same.

This learner did not attempt to solve these questions because it was unlike-fractions. This revealed that the learner was unsure of how to proceed further.

"I struggled a bit with different denominators. When there is a mixed fraction with different denominators I struggle to do that because I don't know how to find the lowest common denominator." Refer to Appendix 9, Question1 for the full response.

The learner's questionnaire and interview questions on addition of fractions revealed his/her inability to find the LCD or calculate unlike- or mixed fractions. Refer to Appendix 9, questions 6, 8 and 13 to find out how the learner answered the interview questions on Section A.

Learner no. 3 responded: (Question1-5)

This learner answered the whole section incorrectly. The learner added the denominators together, even with the like-fractions.

"I don't know how to add fractions as I've struggled to understand fractions since Grade 5." Refer to Appendix 10, Question 1 for full response.



Through this learner's response to the questionnaire and interview questions it was revealed that there had been no recall on the concept of fractions since the previous grade. Refer to Appendix 10, Questions 6, 8 and 13 to find out how the learner answered the interview questions on Section A.

Learner no. 4 responded: (Questions 1-5)

This learner attempted to solve the questions but unfortunately answered it incorrectly. This showed his/her inability to recall the basic understanding of addition of fractions.

"No, because I don't know how to change the denominators into the same denominators." Refer to Question1 of Appendix 11.

"The mistake I made was to add my denominators together when I was not supposed to." Learners response to answering Question 1 of the questionnaire incorrectly. Refer to Appendix 11, Question 6 for the full response.

The learner found it difficult to find the LCD. There was acknowledgement from the learner about the errors made in Question 1 and 2 of the questionnaire. Refer to

Appendix 11, Questions 8 and 13 to find out how the learner answered the interview questions on Section A.

Learner no. 5 responded: (Questions 3, 4 & 5)

Qu.3) I do not know how you add the fractions. Reason is I do not know how to find the lowest common denominator.

Qu.4) I do not know how to add different denominators.

Qu.5) I don't know to do when the denominators are different.

This learner did not attempt to solve the question on unlike-fractions. This showed the learners' inability to proceed further.

"No, I find it very challenging to find which denominator to use." Refer to Appendix 12, Question 1 for the full response.

"In Question 3 then I got stuck with the fractions because the denominators were very different." Refer to Appendix 12, Question 6 for the full response.

The learner mentioned how he/she struggled to find the LCD. The learner was challenged with calculation on unlike- and mixed fractions. Refer to Appendix 12, Questions 8 and 13 to find out how the learner answered the interview questions on Section A.

Learner no. 6

This learner answered all the questions correctly. The learner responded saying he/she found addition of fractions simple enough to do.

"With Question 3 you cannot add something with different denominators so you must look for the lowest denominator." Refer to Appendix 13, Question 6 for the full response.

The learner stated in his/her interview how easy they found addition of fractions. The learner appeared to be confident when asked about his/her ability to add fractions. Refer to Appendix 16, Questions 1, 6, 8 and 13 to find out how the learner answered the interview questions on Section A.

Learner no. 7

This learner answered all the questions correctly. He/she also found addition of fractions to be easy. The learner knew which steps needed to be followed in order to answer the questions.

"I first added my whole numbers in Question3 then I went to my fractions and find the LCD that could go into 3 and 4 which was 12, what I did to the bottom I did to the top."

Refer to Appendix 14, Question 6 for the full response.

The learner stated in his/her interview how easy he/she they found addition of fractions. The learner appeared to be confident when asked about his/her ability to add fractions. Refer to Appendix 14, Questions 1, 6, 8 and 13 to find out how the learner answered the interview questions on Section A.

Learner no. 8 (Questions 2- 5)

This learner only answered one question correctly. When asked about this section of questions, he/she responded as follows:



"No, I don't know what to do when the denominators are not the same." Refer to Appendix 15, Question 1 for the full response.

The learner mentioned how he/she got confused with unlike- and mixed fractions. He/she also mentioned his/her inability to find the LCD, and could not proceed further after calculating the whole numbers. Refer to Appendix 15, Questions 1, 6, 8 and 13 to find out how the learner answered the interview questions on Section A.

Learner no. 9

This learner answered all the questions correctly. The learner showed all his/her steps and how he/she found the LCD. It appeared that the learner understood addition of fractions.

"With Question3 the denominators are not the same so I had to find the LCD which is 12." Refer to Appendix 16, Question 6 for the full response.

The learner felt confident adding all types of fractions and said he/she was not confused with the questions. Refer to Appendix 19, Questions 1, 6, 8 and 13 to find out how the learner answered the interview questions on Section A.

Learner no. 10 (Questions 3, 4 & 5)

This learner only answered the first two like-fraction questions correctly. It was apparent that the learner was unable to calculate the unlike- and mixed fractions. This showed the learner's inability to find the LCD.

"It depends on how big the denominator is and then having to find the LCD and to get to the answer will take a lot of time and it is also confusing." Refer to Appendix 17, Question 1 for the full response.

It was stated by the learner that he/she was confused with big numbers and unlike-fractions. The learner found Question 4 in Section A to be most difficult as he/she was unable to find the LCD and one of the denominators was too big for the learner to calculate. Refer to Appendix 20, Questions 1, 6, 8 and 13 to find out how the learner answered the interview questions on Section A.



Section B

Learner no. 1

This learner answered the first two questions correctly. It appeared that the learner was unable to subtract unlike-fractions and was unsure of the borrowing concept.

"I found Question 3 difficult because I did not know when to multiply or divide and what to do with it." Refer to Appendix 8, Question 3 to see response.

The learner was unsure of which steps needed to be followed in Question 3, Section B, because the first numerator was smaller than the second. It was apparent the learner was unsure of how to borrow from the whole number and unable to find the LCD. Refer to Appendix, Questions 2, 3, 7, and 11 to find out how the learner answered the interview Questions on Section B.

Learner no. 2 responded: (Questions 3, 4 & 5)

Qu.4) I can't do this because the denominators are not the same.

Qu.5) I can't do this because the denominators are not the same.

This learner only answered Questions 1 and 2 correctly. The learner did not know or lost direction on what to do after subtracting the whole numbers. He/she appeared to be unaware of how to find the LCD.

“Question 3 was challenging because the first fraction was smaller than the one I had to subtract with. I don't know what to do when this happens.” Refer to Appendix 9, Question 7 to see full response.

It appeared that the learner was unsure of an alternative method to use or was unsure of how to borrow from the whole number. Refer to Appendix 9, Questions 2, 3, 7 and 11 to find out how the learner answered the interview questions on this section.

Learner no. 3



This learner only got Questions 1 and 2 correct. The learner did not provide reasons on his/her answer paper as to how he/she got the answer.

“Question 4 was so difficult to do because of the method I did not remember the method I needed to use.” Refer to Appendix 10, Question 3 for response.

It appeared that the learner was unsure of an alternative method to use or was unsure of how to borrow from the whole number. It appeared as though there was an inability to find the LCD. Refer to Appendix 10, questions 2, 3, 7 and 11 to find out how the learner answered the interview questions on this section.

Learner no. 4 responded: (Questions 2, 3, 4 & 5)

Qu.3) I can not carry because I do not know borrow from my whole number.

Qu.4) I can not carry on because I do not know how to borrow.

This learner answered Questions 1 and 2 incorrectly because he/she miscalculated the numerators. The learner did not know or lost direction on what to do after subtracting the whole numbers. He/she appeared to be unaware of how to find the LCD nor did he/she know of an alternative method to use.

“No, because I don’t know how to change the denominators into the same denominators.” Refer to Appendix 11, Question 1 to see full response.

In the learner’s interview, he/she said he/she knew of an alternative method which could have been used. However, the learner did not apply it to calculate his/her answer. Refer to Appendix 11, Questions 2, 3, 7 and 11 to find out how the learner answered the interview questions on this section.

Learner no. 5 responded: (Questions 3, 4 & 5)

Qu.3) I do not know how to subtract 2 from 4.

Qu.4) Trying to find the lowest common denominator is very difficult for me.

Qu.5) Subtraction numbers like $1\frac{3}{2} - \frac{3}{10}$ is very challenging for me when you have to find the lowest common denominator.



The learner subtracted the whole numbers only in Question 3 and could not continue with the rest of the sum. In Question 4, and 5 the learner only subtracted the whole numbers.

This learner only answered Questions 1 and 2 correctly. As seen above, there was an inability to find the LCD. The learner added the whole numbers and did not proceed further as he/she was unsure of what needed to happen next.

“Question 3 was challenging because the first fraction was smaller than the one I had to subtract with. I don’t know what to do when this happens.” Refer to Appendix 9, Question 7 to see full response.

The learner was unsure of how to proceed with the calculations as he/she was unsure of the correct steps to be used nor did he/she know of an alternative method. Refer to Appendix 12, Questions 2, 3, 7 and 11 to find out how the learner answered the interview questions on this section.

Learner no. 6

This learner answered all the questions correctly. This is the same learner who answered all the questions in Section A correct. The learner appeared to have had a good understanding of fractions and rules to be used.

"I don't really find anything that difficult, because it is easy to understand the methods."

Refer to Appendix 13, Question 2 for response.

"I didn't find anything difficult, it's the same as adding, I didn't find any sums difficult."

Refer to Appendix 13, Question 3 for response.

The learner knew of an alternative method which was used to calculate Question 3. The learner converted it to an improper fraction then proceeded further with the calculations. Refer to Appendix 13, questions 2, 3, 7 and 11 to find out how the learner answered the interview questions on this section.

Learner no. 7

This learner answered all the questions correctly. It was apparent that the learner had a good understanding of how to calculate subtraction of fractions.



"I find it difficult when there are mixed fractions, sometimes I forget to simplify. I also find it difficult when I have to borrow from the whole number then to convert that borrowed number to a fraction." Refer to Appendix 14, Question 3 for response.

According to the interview, the learner found it difficult with certain calculations of fractions but did not stop calculating. Instead, he/she used the recalled rules to answer the question and managed to answer the questions correctly. Refer to Appendix 14, Questions 2, 3, 7, and 11 to see how the learner answered questions on subtraction of fractions.

Learner no. 8 (Questions 1, 3, 4 & 5)

This learner only answered Question 2 correctly. With Question 1 the learner copied the sum down incorrectly. There was an inability to find the LCD and to calculate mixed fractions.

"I find it hard when the denominators are different." Refer to Appendix 15, Question 3 for response.

It appeared as though the learner was unsure of how to subtract unlike-fractions nor did he/she know of an alternative method which could have been used. Refer to Appendix 18, Questions 2, 3, 7 and 11 to see how the learner answered the interview questions on this section.

Learner no. 9 responded: (Questions 3, 4 & 5)

Qu.3) I do not know how to subtract 2 from 4 because I haven't learnt how to do it.

Qu.4) I also do not know how to this one.

Qu.5) I don't know how to do this one.

This learner only answered Questions 1 and 2 correctly. It appeared that the learner was unable to find the LCD or subtract unlike- or mixed fractions.

"Question 5 was so difficult because the first fraction had lower denominator and the second fraction was bigger. So finding the LCD was difficult." Refer to Appendix 16, Question 3 for response.



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The learner did not know of an alternative method which could have been used to calculate the answer. The whole numbers were subtracted, thereafter he/she did not proceed further as he/she was unsure of what steps needed to be followed. Refer to Appendix 16, Questions 2, 3, 7 and 11 to see how the learner answered the interview questions on this section.

Learner no. 10 responded: (Questions 4 & 5)

This learner answered Questions 1-3 correctly. The learner knew how to borrow from the whole number in Question 3 and work out the correct answer. The learner was unsure of how to find the LCD.

"When the denominator is very big then it confuses me." Refer to Appendix 17, Question 3 for response.

"... before the subtracted my whole numbers, I first make sure the numerators are not too small for me to subtract the other numerator from each other.

So with Question 3 I knew that I needed to borrow from my whole number before subtracting my numerators.” Refer to Appendix 17, Question 7 for full response.

The learner said that calculating denominators confused him/her. It appeared that there was an inability to subtract unlike fractions when numbers were too big for the learner. Refer to Appendix 20, Questions 2, 3, 7 and 11 to find out how the learner answered the interview questions based on this section.

Section C

Learner no. 1

This learner only answered Question 1 correctly. The learner calculated the whole number correctly in questions 2 and 5 but instead of writing the remainder as a fraction he/she wrote it as a whole. For example;

Question 2 = 18 r 3 and,

Question 5 = 41 r 60



“I’m not sure of multiplication of fractions.” Refer to Appendix 8, Question 10 for response.

The learner’s interview questions revealed his/her slight inability to calculate multiplication fractions. Refer to Appendix 8, Question 4, 5, 10 and 16c to see how the learner responded to questions on multiplication of fractions.

Learner no. 2

This learner answered questions 1-3 correctly. He/she calculated questions 4 and 5 incorrectly. He/she used the same method as in Question 3 but miscalculated along the steps. In the interview the learner responded as follows;

“I put my whole number over a 1, then I multiplied my numerators and denominators, got an improper fraction, then I divided my denominator into my numerator to get a mixed fraction.” Refer to Appendix 9, Question 4 for response.

It appeared that this learner had the knowledge of how to calculate multiplication fractions. Refer to Appendix 9, questions 4, 5, 10 and 16c to see how the learner responded to questions on multiplication of fractions.

Learner no. 3

This learner answered all the questions incorrectly. From the questionnaire and the interview, it was apparent there was an inability to calculate multiplication fractions.

"I took the denominator and times it by the whole number and got 60 and then I got confused and did not know what to do afterwards." Refer to Appendix 10, Question 4 for response.

"No, I don't know how to do multiplication sums because I don't understand how to do fractions." Refer to Appendix 10, Question 10 for response.

It appeared as though the learner lost direction as was unable to proceed further after dividing his/her calculations. Refer to Appendix 10, Questions 4, 5, 10 and 16c to see how the learner responded about multiplication of fractions.



Learner no. 4

This learner only answered Question 1 correctly. There was an attempt to answer the rest of the questions but these were answered incorrectly. The learner responded to the questions he/she could not answer in the questionnaire:

Qu3 I can not do this sum because I struggle when working with big numbers.

Qu5 I can not carry on because I do not know the next step.

Interview response:

"You turn the whole number into an improper fraction then I was not sure of what steps to use. I looked for the lowest common denominator, I find this section difficult." Refer to Appendix 11, Question 4 for response.

"I can only multiply fractions with smaller numbers, when it gets bigger numbers it becomes more difficult for me." Refer to Appendix 11, Question 10 for response.

It appeared as though the learner lost direction and could not proceed further with his/her calculations. Refer to Appendix 11, Questions 4, 5, 10 and 16c to see how the learner responded to questions on multiplication of fractions.

Learner no. 5

This learner only answered Question 1 correctly. There was an attempt to answer the rest of the questions but these were answered incorrectly. The learner responded to the questions he/she could not answer in the questionnaire:

Qu2 If there is a remainder what must you do.

Qu3 I was not sure when there was a remainder.

Interview response:

“Doing ‘of’ sums you have to divide you denominator into your whole number and you times your answer by the numerator. With Question 2, I needed to put my remainder as a fraction.” Refer to Appendix 12, Question 4 for response.

“I don’t understand how to multiply fractions. I struggle when there is a remainder, I don’t know what to do with it.” Refer to Appendix 12, Question 10 for response.

The learner explained the steps used to calculate the questions correctly. It appeared that the learner was confused about what needed to happen with the remainders. Refer to Appendix 12, Questions 4, 5, 10 and 16c to see how the learner responded to questions on multiplication of fractions.

Learner no. 6

This learner only answered Question 1 correctly. He/she converted the whole number into a fraction and could not proceed any further with the rest of the questions. The learner responded to the questions he/she could not calculate in the questionnaire:

Qu2 Forgot how to do the steps. Have second thoughts if it’s divide the bottom or multiply the top.

Interview response:

"I forgot how to do the method. So I was supposed to convert the whole number into an improper fraction. Then do the multiplication and then convert back into a mixed fraction." Refer to Appendix 13, Question 4 for full response.

"I do know how to do 'of' sums it's just sometimes hard to remember the steps." Refer to Appendix 13, Question 10 for full response.

It appeared that the learner lost directions about what steps needed to be followed to convert back into a mixed fraction. Refer to Appendix 16, Questions 4, 5, 10 and 16c to see how the learner responded to questions on multiplication of fractions.

Learner no. 7

This learner answered Questions 1 and 2 correctly. The learner wrote his/her answers in Question 2 and 5 as decimals. It appeared that he/she had the ability to calculate multiplication of fractions.



Interview response:

"...because there was a remainder I had to put in the decimal sign and converted to a decimal then I multiplied by the numerator and then wrote my answer as a decimal. I could have written my decimal over 100 and then simplified it." Refer to Appendix 15, Question 4 for full response.

"I think I know how to do 'of' sums but not fractions of multiplication." Refer to Appendix 15, Question 10 for response.

The learner knew what steps needed to be followed. It appeared that the learner was uncomfortable with calculating on multiplication fractions. Refer to Appendix 14, Questions 4, 5, 10 and 16c to see how the learner responded to questions on multiplication of fractions.

Learner no. 8

This learner answered Question 1, 2 and 4 correctly. The learner did not show his/her calculations for the researcher to check which method he used to solve the questions.

Interview response:

“So I divide the whole number by the denominator took the answer and multiplied by the numerator and put my remainder as a fraction.” Refer to Appendix 15, Question 4 for full response.

“I know how to work ‘of’ sums, but I don’t know how to do straight multiplication of fractions.” Refer to Appendix 15, Question 10 for response.

This learner answered the questions correctly but did not show any calculation steps. Refer to Appendix 15, Questions 4, 5, 10 and 16c to see how the learner responded to questions on multiplication of fractions.

Learner no. 9



This learner answered questions 1-4 correctly. The learner divided the whole number by the denominator, then multiplied the answer by the numerator and wrote the answer as a decimal. The learner miscalculated in Question 5. From the learner’s calculations, it appeared that he/she had the ability to calculate multiplication fractions.

Interview response:

Response to Question 4 and 10 of the interview questions:

“I should have written my decimal as a fraction over 100 and then simplified my answer.” Refer to Appendix 16, Question 4 for response.

“I don’t find it difficult to do multiplication of fractions at all.” Refer to Appendix 16, Question 10 for response.

It appeared that the learner felt confident to calculate multiplication fractions. Refer to Appendix 16, Questions 4, 5, 10 and 16c to see how the learner responded to questions on multiplication of fractions.

Learner no. 10

This learner only answered Question 1 correctly. The learner responded to the questions in the questionnaire which he/she could not calculate:

Qu3 I don't understand because the denominator is too big and the numerator is small.

Qu5 I do not understand because the denominator is too big.

Interview response:

“With Question 2 - 5 I was not really sure of my steps and what to do.” Refer to Appendix 17, Question 4 for response.

“No I struggle with this, it depends on what the number is like Question 1 was easier but the others were not easy.” Refer to Appendix 17, Question 10 for response.

It appeared the learner was unsure of what steps needed to be used in order to proceed further nor did he/she know of an alternative method which could have been used. Refer to Appendix 17, Questions 4, 5, 10 and 16c to see how the learner responded to questions on multiplication of fractions.



4.6.1 Summary

It appeared that the learners were able to calculate like-fractions with ease. Some learners did not know or lost direction on what to do after adding or subtracting the whole numbers with mixed fractions. They appeared to be unaware that the LCD needed to be found when calculating unlike-fractions. Multiplying fractions appeared to be most difficult for the learners as they were unsure of what steps needed to be followed.

The information revealed the areas of concern which existed in the learners' processes to acquire the understanding of operations regarding unlike- and mixed fractions. These concerns need to be communicated with teachers in order for strategies to be implemented. It should be addressed before learners move forward into the next grade. If not, it creates a gap of misunderstanding and further complications arise in the learners' progression to learning fractions at a higher level. As such, teachers need

to implement strategies which aim to alleviate as many concerns as possible before learners' progress to the next grade.

4.7 Triangulation of learner responses and teacher responses

4.7.1 Similarities

Learners and teachers mentioned the following similarities in their interviews:

a) There was inability for learners to find the LCD and difficulties to teach the concept of how to find the LCD. In agreement to this, the same challenges were posed by the learners in Awuah's (2016) study.

b) The learners responded to addition and subtraction of unlike- and mixed fractions as being difficult. The teachers mentioned how difficult it was to teach the learners different methods and steps which needed to be followed when calculating unlike- and mixed fractions. Another challenge involved conversions of mixed fractions to improper fractions. Also, the concept of borrowing from the whole number during subtraction of fractions was challenging to teach and to learn. Jigyel and Afamasaga-Fuata'i (2007) concur with this view.



c) The difficulty to teach and learn the different rules for the operations was challenging. A common response given by the teachers was that the concept of multiplication fractions had different rules to those of addition and subtraction. When teaching multiplication, finding the LCD no longer mattered. The learners were confused by the steps they needed to use to answer questions of multiplication fractions. Hecht and Vagi (2012) related learners' inability to find the LCD to the lack of conceptual knowledge.

d) The teaching of more than one method to calculate fraction operations was suggested by the teachers and the learners. Similarly, the teachers also stated the importance of teaching more than one method which would allow their learners to use the best among these. There was the feeling that teaching multiple methods could confuse learners if taught all at once. The timing process of teaching alternative methods needs to be considered to avoid confusion amongst the learners. Moss and Case (2001) reported on the importance of repetition to enhance learners' procedural understanding.

f) It was mentioned that more exercises needed to be practised in order to build a solid foundation in the understanding of calculating different types of fraction operations. The teachers and the learners felt that constant repetition would be beneficial. Moss and Case (2001), and Mary, Jill and Sara (2016) highlighted the importance of practising fraction exercises and using different methods.

4.7.2 Differences

A common concern mentioned by all the teachers was the inability of learners to recall their multiplication tables. The teachers felt that this was one of the main reasons for learners to be unable to do fraction operations and find the LCD. The learners in Awuah (2016) study also struggled to find the LCD.

The learners felt that there were too many rules to remember when calculating fraction operations. Even though they suggested constant repetition, the challenge was remembering all the steps and methods used for different operations.

The use of tangible objects when learning how to calculate fractions was suggested to be pertinent. This would be beneficial as it would help learners relate fractions to real life examples. A fun lesson is most remembered.

4.7.3 Summary

There were similarities and differences mentioned in the interviews with teachers as well as learners. The process of finding the LCD was a main challenge pointed out by both the teachers and the learners. The difference between the challenges mentioned by the learners and the teachers was the concerns around the learners' inability to recall their multiplication tables which formed a pivotal part towards understanding how to calculate unlike- and mixed fractions when LCDs needed to be found. Lillias and Chagwiza (2012) reported that instructional approach promoted the procedural understanding of mathematics. A possible solution for concerns about the inability to recall multiplication tables could be daily recapping on multiplication tables. This could reinforce the importance of recalling the tables.

4.8 Discussion of main trends emerged

4.8.1 Inability to recall Multiplication tables leading to the inability to find the LCD

A main concern mentioned by the teachers was the learners' inability to recall their multiplication tables. This resulted in the learners' difficulties to find the LCD for unlike- and mixed fractions. The teachers found it difficult to teach the concept of how to find the LCD for unlike- and mixed fractions. It was difficult for the learners to recall all the steps they needed to follow for fraction operations. The learners seemed confident about adding or subtracting like-fractions only. This trend would be in line with the research questions 1 and 2, regarding learners' and teachers' difficulties. Saxe, Taylor, McIntosh and Geahart (2005) mentioned how mastering and learning of fractions was still a huge problem in primary and middle schools. This would correlate with the results shown in Tables 4.1.5.1, 4.1.5.2 and 4.1.5.3.

Ndalichako (2013) mentioned the reason for learners' inability to grasp the concepts of fraction operations as the use of different rules for the different operations. Ndalichako (2013) also pointed out how the learners found it difficult to find the LCD. As stated before, the same challenges were highlighted in this study.

4.8.2 Confusing in remembering different methodologies

The different methodologies used confused the learners as they were unsure of which operations to use to proceed further with their calculations. The teachers stated how challenged the learners were to remember the different steps or methods. They further explained how the learners were only introduced to multiplication of fractions for the first time in Grade 6. Therefore, the learners had to recondition their thinking about the rules being opposite to that of addition and subtraction because finding the LCD no longer mattered. This trend would also be linked to the research questions 1 and 2 regarding the learners' and teachers' difficulties.

4.8.3 Benefits of grouping learners according to their Mathematics ability

One of the teachers mentioned that it would be beneficial to group the learners according to their strengths, specifically for Maths (refer to Appendix 20, Question 18 to see the teacher's response). The teacher's reasoning for this emerged from the fact

that the learners grasped concepts at different paces and the fast learners got bored and were not being stimulated when they had to learn at the same pace as the learners who took longer to understand. If learners are grouped according to their strengths, then teachers could plan properly for these groups of learners. This trend links with the research Question number 3, the strategies which could enhance the teaching and learning of fraction operations.

4.8.4 Continuous practice of calculating fractions

The teachers believed in practising a lot of examples with learners to allow learners to remember the steps. They also used one method constantly, before teaching alternative methods. The learners felt that they needed to practise doing fractions more so that they could develop a better understanding. One of the teachers' said, "*practice makes perfect*" (Appendix 18, Question 16). It was mentioned that teachers needed to find other examples other than the questions from the textbook to go through with the learners. Ball et al. (2008) mentioned the same about teachers finding better examples which would assist in deepening the understanding of the idea of fractions by the practical representation of fractions through real life examples.

Siegler, Carpenter, Fennell, Geary, Lewis, Okamoto and Wray (2010) stated that more research needed to be done on fractions in order to address the effectiveness of the teaching strategies and to find out if any of the teacher training programmes have managed to close any of the widening gaps amongst learners in primary schools, especially in Grade 6. A starting point for minimising challenges concerning fractions operations could be made through the suggested teacher and learner strategies in this study.

4.8.5 Use of Visual Aids

A common trend mentioned by the teachers was the need for tangible or visual aids to be used when teaching operations of fractions. Through the creation of interactive lessons, learners would be able to recall more information. Fun lessons needed to be planned to create a stimulating learning environment which would encourage learners to be interested in understanding fraction operations. The teachers also believed that peer learning would be beneficial for learners as the ones who understood could explain to those who did not as they would explain using language of the same level.

These types of integrated lessons would tie in with Vygotsky's socio-cultural theory and Bandura's theory on social and cognitive which relates to the personal factors, behaviours and environment. More strategies which could be used to enhance the teaching and learning of fraction operations, linking to the research question number 3.

4.8.6 Time limitations

Time constraints were a concern mentioned by the teachers. One teacher explained how the content needed to be compressed into a certain number of weeks as CAPS was not structured into a realistic time frame. It does not consider assessment times or exam periods. Teachers are under pressure to try and teach all the topics within the given time frame. Therefore, not having enough time to spend on concepts such as fractions is a challenge. Some teachers felt that there were not enough exercises for learners to practise fraction operations. Some learners said that fractions only appeared now and then.

4.8.7 The uncertainty of the CPTD system



The CPTD system was introduced as a means to help develop teachers' professional knowledge and self. However, the results reported from Mthethwa's (2017) study showed uncertainties around its procedure. There were complaints that the workshops and development programmes took up a lot of personal time as it was being done over weekends or holidays and the duration of the workshops was tedious. The workshop developers are unable to say whether or not the expected outcome information was attained by the teachers. There should be development workshops or training programmes which assess the session and establish if it was beneficial or not for the teachers.

From the interviews, there seemed to be a gap in the operation of fractions from Grades 5-6 and from Grades 6-7. Multiplication fractions were first introduced in Grade 6 in 'as' or 'of' questions. Learners in Grade 7 calculate straight multiplication of fractions. It appeared that the transition between the different grades was not clear or smooth.

4.8.8 Summary

This section reported on the main trends highlighted by the teachers and the learners. It included the concerns about the learners' inability to recall multiplication tables and to find the LCD. The confusion around different methodologies and rules to be taught and learnt, the pressure of time constraints to complete the syllabus before terms end. The effectiveness of the implementation of CPTD and whether it is a beneficial strategy to develop teachers. The teachers stated that there is no smooth transition between the three grades creating gaps in the learning process.



CHAPTER FIVE

CONCLUSIONS AND RECOMMENDATION

This section addresses the conclusions and recommendations of the findings focused on in this research. Based on the literature review and theoretical framework, the following challenges were relevant for the study, (a) the learners' inability to view fractions as a whole and not separate entities, (b) the learners' inability to find the LCD, (c) the uncertainty of and how to calculate unlike- and mixed fractions, (d) the difficulties in retaining all the steps used for the different methods and, (e) time constraints for teaching.

5.1 Conclusion on the learners' inability to view fractions as a whole

This study revealed the learners' inability to view fractions as a whole, instead fractions were being treated as separate entities. In Awuah's (2016) study, it was revealed how learners perceived fractions as separate whole numbers and this resulted in the separate adding or subtracting of numerators and denominators. The learners' inability to recall the rules when it came to calculating fraction operations was evident of them not using their long-term memory resulting in the misunderstanding.

5.1.1 Recommendation

The teachers who were interviewed in this study recommended the use of constant practice and drilling of one method at a time, as one said, "*practice makes perfect*" (refer to Appendices 18-20, Questions 10-12). As stated by the teachers, fraction is a concept which needs to be practiced continuously and many examples need to be done with learners until they grasp the concepts. The mastering of fraction operations was compared to sport. The analogy meant by this was the fact that sports people practise constantly to become skilful, the same should happen in Mathematics as it is about learning a skill. This strategy of drilling and practising was also mentioned in Awuah's (2016) study.

5.2 Conclusion on finding the LCD

In Table 4.1.5.1, it was revealed that an average of 84% of the learners were unable to add unlike- and mixed fractions. It was concluded that the learners were unable to

find the LCD and this led to the uncertainty of how to calculate unlike- and mixed fractions. this inability of learners would link to the research Question 1, the challenges learners have when doing operations of fractions. For this reason, the percentage of learners who were unable to answer the questions on addition, added the numerators together as well as the denominators. A similar percentage of learners were unable to subtract the above mentioned types of fractions and the reasons provided were the same.

This study is in agreement with the study conducted by Ndalichako (2013) who concluded that the possible reasons for challenges were due to the many rules associated with the computation of fractions which were more complicated than those related to natural or whole numbers.

Hecht and Vagi (2012) related the learners' inability to find the LCD to the lack of conceptual knowledge that governed this concept. This was a challenge identified and revealed through the questionnaire and interview processes in this study.

5.2.1 Recommendations



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In this study, the teachers stressed the importance of learners building a foundation in their conceptual understanding of the multiplication tables as this was key to finding the LCD. This recommendation would relate to the research Question number 3 with strategies to enhance the teaching and learning of fraction operations. The concrete/semi-concrete/abstract (CSA) approach founded by the American institute for Research (2007) was cited by Awuah (2016). This method is used to enhance the overall performance of learners who struggle with Mathematics, specifically in the learning of fraction operations. The CSA approach allowed the learners to develop their understanding around concepts and fluency in computations as explained by Nieves (2009).

5.3 Conclusion on the inability of learners to calculate unlike- and mixed fractions

Evidence from the current study indicated the learners' inability to calculate unlike- and mixed fractions. In Figs. 4.3-4.5 and 4.8-4.10, it was noticed that there was an increase in the percentages of learners who answered the unlike- and mixed fractions

incorrectly. The reasoning for this is in congruence with the learners' inability to find the LCD. A reason for this confusion was the fact that the learners were unsure of what steps needed to be used with unlike- and mixed fractions. Jigyel and Afamasaga-Fuata'i (2007) mentioned in their study how the learners viewed numerators and denominators as separate whole numbers, as done by the learners in this study.

The learners in this study provided reasons for their inability to add or subtract unlike- and mixed fractions. They mentioned their inability to convert fractions or to find the LCD. The confusion of the different rules and operations which needed to be used and remembered was also mentioned. This refers to the research Question number 1.

In this study, Table 4.1.2.3 revealed the high percentage of learners who answered the questions incorrectly. A possible reason for the outcome is that multiplication of fractions is a new concept introduced in Grade 6. This led to learners confusing the rules for addition and subtraction with multiplication and vice versa. This often happened with the learners when they were confused about the methodologies used for the different operations of fractions.

5.3.1 Recommendations



The teachers in this study recommend continuous repetition of rules to be used when doing specific calculations of fractions. The use of one method at a time was suggested, in order for learners to develop an understanding through the use of the same method. A standard set of steps to use when calculating fractions should be developed. Learners need more time to practise calculating fraction operations. This recommendation relates to the research Question number 3, strategies to enhance the teaching and learning.

5.4 Conclusion on the difficulties to retain all the steps used for the different methods

In this study, it was suggested by the teachers to ensure lessons start from basics to help refresh their memories before introducing new concepts on fractions. When teaching fractions, tangible tools should be used to enhance learners' ability to remember what is practised. Constant repetition of examples needs to be done with learners to enforce the procedural steps. Mary, Jill and Sara (2016) reiterate this fact

by stating how this method represented concise teaching and perfecting a skill or procedures needed for fraction operations.

5.4.1 Recommendations

Moss and Case (2001) explained how instructional approach would promote acquisition of knowledge or skill through repetitive practice. This strategy has the likelihood to increase learners' abilities to permanently remember new information. The teachers in this study suggested that step-by-step methods should be strategically taught and constantly used to build a solid foundation of the method and steps learners needed to recall.

5.5 Conclusion on time constraints for teaching

Another challenge identified by the teachers in this study was concerns around the constrictions on time experienced from term to term. It was stated that there was insufficient time to focus on fractions as the curriculum needed to be completed before the end of every term. The pressure to complete the syllabus puts teachers in a position where they cannot often spend more time needed on the areas of concern. The learners also mentioned how fractions do not appear often through the various terms to allow for practice and recall.

It was also mentioned that there was a gap between Grades 5-7. In Grade 5 they do addition and subtraction of like-fractions and briefly touch on the concept of addition and subtraction of unlike-fractions. The Grade 5s do not calculate multiplication fractions. In Grade 6 the learners are supposed to have an understanding of how to add or subtract unlike-fractions as there are more unlike-fractions done in Grade 6. The first time learners are exposed to multiplication fractions is in Grade 6. By the time the learners are in Grade 7, there should have been a solid foundation in the application of fraction operations. It was stated by the teachers that the transition of the concept between the grades was not level. In agreement to the above challenge, a study done in France and Italy by the European Commission (2011) stated that the time spent on the calculations of whole numbers was far more than the time spent on addition, subtraction, multiplication and division of fractions.

5.5.1 Recommendations

The curriculum does not consider the other activities which are done throughout the term such as assessments, test or examinations. Curriculum developers need to relook at the expectations of content which needs to be covered within a term and to factor in the realistic activities which need to be done within that given term.

In the recommendations from teachers for the curriculum developers, it was highlighted that concepts on fraction operations needed more time and exercises for continual exposure and practising. Fractions are a complex concept for learners to grasp, therefore, the different operations should be practised every term.

5.6 Summary

This section discussed the strengths and challenges experienced in the teaching and learning of fraction operations. Recommendations were given for each challenge experienced. New and innovative methods need to be created to motivate the learners to understand the concept of fractions. Repetition of many examples need to be done in class, the more practise the learners do the better their understanding will be. For this to occur, more time needs to be dedicated to the teaching and learning of fraction operations in the respective grades. The recommendations mentioned in this chapter refer to the research questions 1 and 2 regarding the strengths and challenges teachers and learners have with fraction operations. The conclusions provide strategies which could enhance the teaching and learning of fraction operations and these refer to the research Question number 3.

5.7 Areas for future research

Future researchers can investigate other challenges and strengths related to fraction operations in Grade 6. Alternative strategies could be discovered and shared through their findings in order to assist Mathematics teachers and all learners. They could also investigate matters on division of fractions which was not focused on in this study.

REFERENCES

- Alan, F. (n.d). Research from Within: Moral and Ethical Issues and Dilemmas. Research Domain: *Research Methodology*. 1-6. 0108.
- Annual National Assessment (ANA): A SADTU perspective. Executive summary. 1-5. Retrieved on 12 May 2017. Available from http://www.google.co.za/url?sa=t&rct=j&q=&esrc=s&source=web&cd=6&cad=rja&uact=8&ved=0ahUKEwijupz694_TAhVHI8AKHQekDcYQFgg0MAU&url=http%3A%2F%2Fwww.sadtu.org.za%2Fdocs%2Fdisc%2F2014%2Fana.pdf&usg=AFQjCNEHrTGXhl9dbmZArew0YGkVYgzvxA
- Askew, M., Venkat, H. & Matthews, C. (2012). Coherence and consistency in South African primary mathematics lessons. In T. Tso (Ed.), *36th Conference of the International Group for the Psychology of Mathematics Education Opportunities to learn in mathematics education*. Taipei, Taiwan: IPME. 27-34.
- Assalahi, H. (2015). The Philosophical Foundations of Educational Research: a Beginners Guide. *American Journal of Educational Research*, 3(3), 312-317.
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- Awuah, B.P. (2016) *The effectiveness of Concrete/ Semi-Concrete/ Abstract and Drill-Practice on Grade 10 learners' ability to solve algebraic fractions*. M.Ed dissertation, University of Fort Hare. 1-156.
- Ball, D. L., Lubienski, S. T. & Mewborn, D. S. (2001). Research on teaching mathematics: the unsolved problem of teachers' mathematical knowledge. V. Richardson (Ed.). *Handbook of research on teaching*, (4), 433-456. Washington, D.C. American Educational Research Association.
- Ball, D. L., Thames, M. H. & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, 59(5), 389–407.
- Bandura, A. (2001). Social cognitive theory: an Agentic perspective. Department of psychology, 52; 1-26. Stanford University, California.
- Barber, M. & Mourshed, M. (2007). How the world's best-performing school systems come out on top. London: McKinsey and Company.

Begle, E. G. (1979). *Critical variables in mathematics education: Findings from a survey of the empirical literature*. Washington, D. C. Mathematical Association of America.

Bonner, A. & Tolhurst, G. (2002). Insider-outsider perspectives of participant observation. *Nurse Researcher*, 9(4), 7-19.

Booth, J.L. & Newton, K.J. (2012). Fractions: Could they really be the gatekeeper's doorman? *Contemporary Educational Psychology*, 37(4), 247-253.

Booth, J.L., Newton, K.J. & Twiss-Garrity, L.K. (2014). The impact of fraction magnitude knowledge on algebra performance and learning. *Journal of Experimental Child Psychology*, 118, 110-118.

Burns, N. & Grove, S. K. (2001). *The practice of nursing research: conduct, critique and utilization*. 4, 12-17. Philadelphia: WB Saunders.

Breen, L.J. (2007). The researcher 'in the middle': Negotiating the insider/outsider dichotomy. *Centre of Social Research*, 19(1), 163-174. Edith Cowan University.

Brogan, M. (n.d). What's the deal with algorithms? Your 101 guide to the computer codes that are shaping the way we live. Slate, New America and ASU. Accessed on the 25 August 2018:

http://www.slate.com/articles/technology/future_tense/2016/02/what_is_an_algorithm_an_explainer.html

Bruce, C., Chang, D., Flynn, T. & Yearley, S. (2013). Foundations to learning and teaching fractions: Addition and subtraction. 1-54. Retrieved on 16 May 2017. available from: <http://www.edugains.ca/resources/ProfessionalLearning/FoundationstoLearningandTeachingFractions.pdf>

Bryman, A. (2004). *Social Research Methods*. (2nd ed). Oxford: Oxford University Press.

Cai, J., Perry, B., Wong, N. Y. & Wang, T. (2009). *What is effective teaching: A study of experienced mathematics teachers from Australia*. University of Hamburg. Rotterdam.

Carnoy, M., Chisholm, L., Addy, N., Arends, F., Baloyi, H. & Irving, M. (2011). The process of learning in South Africa: *The quality of Mathematics teaching in North West province*. Pretoria: Human Sciences Research Council.

Carnoy, M., Chisholm, L. & Chilisa, B. (2012). The Low Achievement Trap: Comparing Schooling in Botswana and South Africa. Cape Town: HSRC Press.

Carpenter, T. P., Fennema, E. & Romberg, T. A. (1993). *Towards a unified discipline of scientific inquiry. Rational numbers: An integration of research*. 1-11. NJ: Lawrence Erlbaum Associates.

Clarke, D. (2006). Fractions as division: The forgotten notion. *Australian Primary Mathematics Classroom*, 11(3), 4-10.

Cobuild, C. (n.d). Definition of 'learner'. English Dictionary. Accessed on the 25 August 2018 from: <https://www.collinsdictionary.com/dictionary/english/learner>

Coetzee, J & Mammen, K.J. (2017). Science and engineering students' difficulties with fractions at entry-level to university. *IEJME Mathematics Education*, 12(4), 281-310.

Cohen, L., Manion, L. & Morrison, K. (2011). *Research Methods in Education*, 7. London: Routledge-Taylor and Francis Group.

Cousin, G. (2006). An introduction to threshold concepts. 17, (4-5). Accessed on the 21 August 2018. <http://www.gees.ac.uk/planet/p17/gc.pdf>

Creswell, J. W. (2008). Mixed Methods Research: Design and Procedures. Department of Educational Psychology. *Journal of Mixed Methods Research*. 1-46.

Crossan, F. (2003). Research philosophy: Towards an understanding. *Nurse Researcher*. 11(1): 46-55.

Department of Basic Education. (2012). *National Senior Certificate 2012: National diagnostic report on learners' performance*. Pretoria, South Africa: Department of Basic Education. Retrieved on 22 August 2018 from <http://www.education.gov.za>.

Doyle, A. (2018). What is a Semi-structured interview? Accessed on the 5 September 2018, from: <https://thebalancecareers.com/what-is-a-semi-structured-interview-2061632>

Drew, C.J., Hardman, M.L. & Hosp, J.L. (2008). *Designing and Conducting Research in Education*. California: Sage Publications, Inc.

Empson, S. & Levi, L. (2011). *Extending children's mathematics: Fractions and decimals: Innovations in cognitively guided instruction*. Portsmouth, NH: Heinemann. 178-216.

European Commission. (2011). *Mathematics Education in Europe: Common Challenges and National Policies*. 1-184. 978-92-9201-221-2. Publications Office of the European Union.

Fleisch, B. (2008). *Primary education in Crisis: Why South African schoolchildren underachieve in reading and mathematics*. Cape Town: Juta.

Fouka, G. & Mantzourou, M. (2011).  What are the Major Ethical Issues in Conducting Research? Is there a Conflict between the Research Ethics and the Nature of Nursing?. *Health Science Journal*, 5(1): 3-14.

Fraction Facts! (2011), EAP, 1. LSC-0. Retrieved on 16 May 2018. Available from <https://www.google.co.za/url?sa=t&source=web&rct=j&url=http://www.lsc.edu/learningcenter/fraction-facts.pdf&ved=0ahUKEwiKscqz24PVAhXFC8AKHWFyCr0QFggaMAA&usg=AFQjCNFSgb1jjn4mE1NOlyRECRmfqeo1Lg>.

Gerber, M. (2011). *Pedagogical Experiences of Educators Implementing Mathematical Literacy in Three FET Colleges*. University of Fort Hare, South Africa: East London.

Gould, P., Outhred, L. N. & Mitchelmore, M. C. (2006). Conference proceedings of the 2006 meeting of the Mathematics Education Research Group of Australia (MERGA): Australia.

- Grider, C. (1993). *Foundation of Cognitive Theory: A Concise Review*. ED 372 324. 1-15. Washing, DC.
- Guerriero, S. (2009). Teachers' Pedagogical Knowledge and the Teaching Profession. Background Report and Project Objectives. OECD: Better policies for better lives. 2-7.
- Gunawan, J. (2015). Ensuring Trustworthiness in Qualitative Research. *Belitung Nursing Journal*. 1 (1), 10-11: 2477 -4073.
- Harel, G. & Lim, K. H. (2004). Mathematics teachers' knowledge base: Preliminary results. Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education. Bergen, Norway.
- Hecht, S. A. & Vagi, K. J. (2010). Sources of group and individual differences in emerging fraction skills. *Journal of Educational Psychology*, 102(4), 843-859.
- Hecht, S. A. & Vagi, K. J. (2012). Patterns in children's knowledge about fractions. *Journal of Experimental Child Psychology*, 111, 212-229.
- Hoadley, U. (2007). The reproduction of social class inequalities through mathematics pedagogies in South African primary schools. *Journal of Curriculum Studies*, 679-706.
- Hodgen, J., Brown, M., Küchemann, D. & Coe, R. (2011). Why have educational standards changed so little over time: the case of school mathematics in England. The British Educational Research Association (BERA). Institute of Education, London.
- Jigyel, K. & Afamasaga-Fuata'i, K. (2007). Students' conceptions of models of fractions and equivalence. *Australian Mathematics Teacher*. 63, 17-25.
- Kaboub, F. (2004). *Oxford English Dictionary. In Encyclopedia*.
- Khalo, X. (2014). Analysis of Grade 10 Mathematical Literacy Students' errors in Financial Mathematics. University of Fort Hare, South Africa: East London.
- Land, R., Meyer, J.H.F. & Baillie, C. (2010). Editors' Preface: Threshold Concepts and Transformational Learning. (ix-xlii).
- Latham, B. (2007). *Sampling: What is it? Quantitative research methods*. Spring.

Levine, R.J. (1976). *Preliminary Papers prepared for the commission*. The Belmond Report. DHEW Publication, Washington DC.

Lillias, H.N. & Chagwiza, C.J. (2012). Teaching Fractions at Ordinary Level: A Case Study of Mathematics Secondary School Teachers in Zimbabwe. *Educational Research and Reviews*, 7(4); 90-101.

Lloyd, G. M., Remillard, J. T. & Herbel-Eisenman, B. A. (2009). Teachers' use of curriculum materials: an emerging field. Remillard, J. T., Herbel, B. A. & Lloyd, G. M. Mathematics teachers at work: connecting curriculum materials and classroom instruction. 3-14. New York: Routledge.

Lortie-Forgues, H., Tian, J. & Siegler, R.S. (2015). Why Is Learning Fraction and Decimal Arithmetic So Difficult. *Developmental Review*. 38, 201-221.

Lukhele, R.B., Murray, H. & Olivier, A. (1999). Learners' understanding of the addition of fractions. *Proceedings of the Fifth Annual Congress of the Association for Mathematics Education of South Africa: 1*, 87-97. Port Elizabeth: Port Elizabeth Technikon. MLATI.



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Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' knowledge of fundamental mathematics in China and the United States*. Mahwah, NJ: Erlbaum.

Maharaj, A., Brijlall, D. & Molebale, J. (2007). Teachers' views of practical work in the teaching of fractions: A case study. *South African Journal of Education*, 27(4), 597-612.

Marriem-Webster. (1828). Definition of Numeracy. Accessed on the 25 August 2018 from: <https://www.merriam-webster.com/dictionary/numeracy>

Mary, B., Jill, J. & Sara, F. (2016). American Psychology Association. Washington, DC. 20002 – 4242.

Mayer, R.E. (1989). Models for Understanding. *Review of Educational Research*. 59(1); 43-64. California, Santa Barbara.

Meyer, J.H.F. and Land, R. (2003). Threshold concepts and troublesome knowledge: linkages to ways of thinking and practising, in: Rust, C. (ed.), Improving Student

Learning - Theory and Practice Ten Years On. Oxford: Oxford Centre for Staff and Learning Development (OCSLD), 412-424.

Meyer, J.H.F. & Land, R. (2005). Threshold concepts: Undergraduate teaching, Post graduate training, Professional development and School Education. Accessed on the 19 August 2018 at: <https://www.ee.ucl.ac.uk/~mflanaga/thresholds.html>

Monk, D. H. (1994). Subject area preparation of secondary mathematics and science teachers and student achievement. *Economics of Education Review*, 13(2), 125-145.

Moss, J. & Case, T. (1999). Developing children's understanding of the rational numbers: A new model and an experimental curriculum. *Journal for Research in Mathematics Education*, 30, 122-148.

Moss, J. & Case, R. (2001). Developing children's understanding of the rational numbers: A new model and an experimental curriculum. *Journal for Research in Mathematics Education*, 30(2), 122-147.

Mthethwa, N. V. (2017). An assessment of the implementation of state-led continuing professional teacher development (CPTD) in the Gert Sibande Education district of Mpumalanga province in South Africa. University of Fort Hare.



Mundy, J. F., Floden, R., McCrory, R., Burrill, G. & Sandow, D. (2015). *Running Head: Knowledge for Teaching School Algebra. A Conceptual Framework for Knowledge for Teaching School Algebra*. 1-30.

National Assessment of Educational Progress. (2005). *The nation's report card: Mathematics 2005*. Retrieved on 22 May 2017. Available from <http://nces.ed.gov/nationsreportcard/pdf/main2005/2006453.pdf>

Ndalichako, J. L. (2013). Analysis of Pupils' Difficulties in Solving Questions Related to Fractions: *The Case of Primary School Leaving Examination in Tanzania*, 4(9), 69-73.

Nieves, T.A. (2009). What teaching strategies can I employ to improve students' achievement in the addition of fractions with different denominators? In M.S.

Plakhotnik, S.M. Nielsen, & D.M. Pane (Eds), Proceeding of the Eight Annual College of Education & GSN Research Conference (pp. 85-91). Miami: Florida International University. Accessed on the 13 September 2018, from: <http://coeweb.fin.edu/researchconference>

Northcote, M. (2012, 2013). Threshold concepts and attitudes in mathematics education: Listening to students' past, present and projected stories. Avondale College of Higher Education, Australia.

Oblinger, D. G. (2005). Learners, Learning, & Technology. The FEducause learning initiative. 66-75.

O'Donoghue, J. (2002). Numeracy and Mathematics. Irish Maths, 48, 47-55.

Organisation for Economic Cooperation and Development. (2009). Creating effective teaching and learning environments: First results from TALIS. Paris: Organization for Economic Co-Operation and Development. Accessed on the 15 July 2018, available at:

http://www.oecd.org/document/54/0,3343,en_2649_39263231_42980662_1_1_1_1_00.html.



Perkins, D. (1999, 2006). The many faces of constructivism. *Educational Leadership*, 57(3), 311.

Piaget, J. (1971). Cognitive development. An introduction for students of psychology and education. Accessed on the 9 July 2018, from: https://www.google.co.za/url?sa=t&rct=j&q=&esrc=s&source=web&cd=8&cad=rja&uact=8&ved=2ahUKEwiT_PKLhaLdAhVklcAKHWnIDpEQFjAHegQIAhAC&url=https%3A%2F%2Ffiles.eric.ed.gov%2Ffulltext%2FEJ841568.pdf&usq=AOvVaw23NqndpD-989uWedVp6nan

Pienaar, E. (2014). Learning about and Understanding fractions and their role in the High School Curriculum. 1-55.

Prediger, S. (2008). The relevance of didactic categories for analysing obstacles in conceptual change: Revisiting the case of multiplication of fractions. *Learning and Instruction*, 18(1), 3-17.

Polit, D.F., Beck, C.T. & Hungler, B.P. (2001). *Essentials of nursing research: Methods, appraisal, and utilization*, 5. Philadelphia: Lippincott.

Raiker, A & Procter, R. (2012). Threshold concepts and their use in the professional development of mathematics teachers: a methodology for collaboration across four European countries. University of Bedfordshire. 1-14.

Rowan, B., Chiang, F.-S. & Miller, R.: (1997). Using research on employees' performance to study the effects of teachers on students' achievement. *Sociology of Education*, 70, 256-284..

Sandelowski, M. (1993). Rigor or rigor mortis: the problem of rigor in qualitative research revisited. *ANS. Advances in nursing science*, 16(2), 1-8.

Saunders, M., Lewis, P. & Thornhill, A. (2009). *Research Methods for Business Students*. New York. Pearson..

Saxe, G. B., Taylor, E..V., McIntosh, C. & Geahart, M. (2005). Representing fractions with standard notations: A development analysis. *Journal for Research in Mathematics Education*, (36): 137-157.



SEACMEQ. (2007). *SACMEQ: Manual for National Research Co-ordinators: Main study*. SEACMEQ.

Seah, T. K. R. (2004). An investigation of the depth and breadth of students' knowledge of multiplication as a basis for the development of multiplicative thinking. Unpublished MEd. Thesis. Griffith University.

Siegler, R., Carpenter, T., Fennell, F., Geary, D., Lewis, J., Okamoto, Y., Thompson, L., & Wray, J. (2010). *Developing effective fractions instruction: A practice guide* (NCEE #2010-009). Washington, DC: National Center for Education Evaluation and Regional Assistance, Institute of Education Sciences, U.S. Department of Education. Accessed on 24 October 2017, form: <http://ies.ed.gov/ncee/wwc/publications/practiceguides/>.

Siegler, R.S., Duncan, G.J., Davis-Kean, P.E., Duckworth, K., Claessens, A., Engel, M., Susperreguy, M.I. & Chen, M. (2012). Early Predictors of High School Mathematics Achievement. *Psychological Science*, 23(7), 691-697.

Siegler, R.S. & Lortie-Forgues, H. (2015). Conceptual Knowledge of Fraction Arithmetic. *Journal of Educational Psychology*, 107(3), 909-918.

Siemon, D. (2004). Partitioning – The missing link in building fraction knowledge and confidence.

Smith, J. (2006). Lost in translation: Staff and students negotiating liminal spaces. Liverpool.

Soicher, L. & Vivaldi, F. (2004). Algorithmic Mathematics. A web-book. The University of London. 1-85. Retrieved on 30 May. Available from http://www.google.co.za/url?sa=t&rct=j&q=&esrc=s&source=web&cd=3&cad=rja&uact=8&ved=0ahUKEwiYnO7N5Y_TAhWoJsAKHVScBX0QFggoMAI&url=http%3A%2F%2Fwww.maths.qmul.ac.uk%2F~leonard%2Fambook.pdf&usq=AFQjCNEootQR4mlwPcmfLp8B2iWMWDgg8g



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Sukamolson, S. (2007). Fundamentals of quantitative research. Language Institute Chulalongkorn University, 1-20.

Taylor, N. & Vinjevold, P. (1999). Getting Learning Right: Report of the President's Education Initiative Research Project. Joint Education Trust.

Taylor, N. & Taylor, S. (2013). Teacher knowledge and professional habitus. In Taylor, N., Van der Berg, S. & Mabogoane, T. *What makes schools effective?* Report of the National Schools Effectiveness Study. 202-232. Cape Town: South Africa. Pearson Education.

Torbeyns, J., Schneider, M., Xin, Z. & Siegler, R.S. (2015). Bridging the gap: Fraction understanding is central to mathematics achievement in students from three different continents. *Learning and Instruction*, 37, 5-13.

UNESCO Institute for Statistics. (2012). *Primary School Curricula on Reading and Mathematics in Developing Countries*, (8), 92- 978.

Usiskin, Z. (2008). The Arithmetic Curriculum and the Real World. In De Bock, D., Søndergaard, B. D., Gómez, B. A. & Cheng, C. C. L. 916.

Van Teijlingen & Hundley. (2001). Baker in Nursing Standard, 1(5), 33-34. Pilot Study.

Venkat, H. & Spaul, N. (2015). What do we know about primary teachers' mathematical content knowledge in South Africa? *International Journal of Educational Development*. 41, 121-130.

vom Hofe, R., Kleine, M., Blum, W. & Pekrun, R. (2006). The effect of mental models ("Grundvorstellungen") for the development of mathematical competencies. First results of the longitudinal study PALMA. (4), 142-151.

Voss, T. (2014). *Teachers' General Pedagogical/psychological knowledge: Conceptualization and Test Construction*. Eberhard Karls Universität Tübingen. Brussels. 2-28.

Vygotsky, L. (1978, 1981). Socio-cultural theory. Accessed on the 17 July 2018, from:

https://www.researchgate.net/publication/279279381_Vygotsky%27s_socio-cultural_theory_of_literacy_Scaffolding_children_to_read_and_write_at_an_early_age



Xuhui, L. (2011). Mathematical Knowledge for Teaching Algebraic Routines: A Case Study of Solving Quadratic Equations. *Journal of Mathematics Education*, 4(2), 1-16. California State University at Long Beach.

Yin, R. K. (2014). Case Study Research Design and Methods. Thousand Oaks, CA: Sage. (5):282. ISBN 978-1-4522-4256-9.

Zombwe, G., Missokia, E., Saiguran, S. & Mihayo, R. (2008-2013). Who is a Teacher? Quality teachers for Quality education. Faculty of Education. 1-15. University of Dares Salaam, Tanzania.

Appendix 1 – Ethical Clearance letter



Province of the
EASTERN CAPE
EDUCATION

STRATEGIC PLANNING POLICY RESEARCH AND SECRETARIAT SERVICES

Steve Vukile Tshwete Complex • Zama B • Zwelitsha • Eastern Cape
Private Bag X0032 • Bisho • 5805 • REPUBLIC OF SOUTH AFRICA
Tel: +27 (0)40 606 4773/4035/4537 • Fax: +27 (0)40 606 4574 • Website: www.ecdoe.gov.za

Enquiries: B Pamla

Email: bajalewa.pamla@ecdoe.gov.za

Date: 09 May 2018

Ms. Ashnee Chetty

4 Hillside Gardens Norden Street

Quigney

East London

5201

Dear Ms. Chetty

PERMISSION TO UNDERTAKE MASTER'S THESIS: ASSESSING THE LEARNING AND TEACHING OF FRACTION OPERATIONS IN GRADE 6

1. Thank you for your application to conduct research.
2. Your application to conduct the above mentioned research involving 101 participants from George Randell Primary School, Port Alfred City District under the jurisdiction of the Eastern Cape Department of Education (ECDoE) is hereby approved based on the following conditions:
 - a. there will be no financial implications for the Department;
 - b. consent will be sought from parents of minor children;
 - c. institutions and respondents must not be identifiable in any way from the results of the investigation;
 - d. you present a copy of the written approval letter of the Eastern Cape Department of Education (ECDoE) to the Cluster and District Directors before any research is undertaken at any institutions within that particular district;
 - e. you will make all the arrangements concerning your research;
 - f. the research may not be conducted during official contact time;



- g. should you wish to extend the period of research after approval has been granted, an application to do this must be directed to Chief Director: Strategic Management Monitoring and Evaluation;
 - h. your research will be limited to those institutions for which approval has been granted, should changes be effected written permission must be obtained from the Chief Director: Strategic Management Monitoring and Evaluation;
 - i. you present the Department with a copy of your final paper/report/dissertation/thesis free of charge in hard copy and electronic format. This must be accompanied by a separate synopsis (maximum 2 – 3 typed pages) of the most important findings and recommendations if it does not already contain a synopsis;
 - j. you present the findings to the Research Committee and/or Senior Management of the Department when and/or where necessary;
 - k. you are requested to provide the above to the Chief Director: Strategic Management Monitoring and Evaluation upon completion of your research;
 - l. you comply with all the requirements as completed in the Terms and Conditions to conduct Research in the ECDoE document duly completed by you.
 - m. you comply with your ethical undertaking (commitment form).
 - n. you submit on a six-monthly basis, from the date of permission of the research, concise reports to the Chief Director: Strategic Management Monitoring and Evaluation
3. The Department reserves the right to withdraw the permission should there not be compliance to the approval letter and continue to monitor the Terms and Conditions to conduct Research in the ECDoE.
4. The Department will publish the completed Research on its website.
5. The Department wishes you well in your undertaking. You can contact the Director, Ms. NY Kanjana on the numbers indicated in the letterhead or email nelisa.kanjana@ecdce.gov.za should you need any assistance.



NY KANJANA
DIRECTOR: STRATEGIC PLANNING POLICY RESEARCH & SECRETARIAT SERVICES
FOR SUPERINTENDENT-GENERAL: EDUCATION



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ETHICAL CLEARANCE CERTIFICATE
REC-270710-028-RA Level 01

Certificate Reference Number: MAM121SCHE01

Project title: **Assessing the learning and teaching of fraction operations in grade six.**

Nature of Project  **Masters in Education**

Principal Researcher:  **Arjun Chetty**
University of Fort Hare
Together in Excellence

Supervisor: **Prof K.J Mammen**

Co-supervisor: **N/A**

On behalf of the University of Fort Hare's Research Ethics Committee (UREC) I hereby give ethical approval in respect of the undertakings contained in the above-mentioned project and research instrument(s). Should any other instruments be used, these require separate authorization. The Researcher may therefore commence with the research as from the date of this certificate, using the reference number indicated above.

Please note that the UREC must be informed immediately of

- Any material change in the conditions or undertakings mentioned in the document
- Any material breaches of ethical undertakings or events that impact upon the ethical conduct of the research

The Principal Researcher must report to the UREC in the prescribed format, where applicable, annually, and at the end of the project, in respect of ethical compliance.

Special conditions: Research that includes children as per the official regulations of the act must take the following into account:

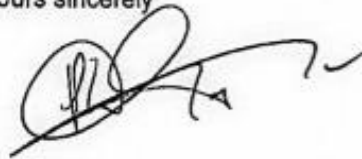
Note: The UREC is aware of the provisions of s71 of the National Health Act 61 of 2003 and that matters pertaining to obtaining the Minister's consent are under discussion and remain unresolved. Nonetheless, as was decided at a meeting between the National Health Research Ethics Committee and stakeholders on 6 June 2013, university ethics committees may continue to grant ethical clearance for research involving children without the Minister's consent, provided that the prescripts of the previous rules have been met. This certificate is granted in terms of this agreement.

The UREC retains the right to

- Withdraw or amend this Ethical Clearance Certificate if
 - Any unethical principal or practices are revealed or suspected
 - Relevant information has been withheld or misrepresented
 - Regulatory changes of whatsoever nature so require
 - The conditions contained in the Certificate have not been adhered to
- Request access to any information or data at any time during the course or after completion of the project.
- In addition to the need to  comply with the highest level of ethical conduct principle investigators must report back annually as an evaluation and monitoring mechanism on the progress being made by the research. Such a report must be sent to the Dean of Research's office

The Ethics Committee wished you well in your research.

Yours sincerely



Professor Pumla Dineo Gqola
Dean of Research

05 March 2018

Appendix 2 – Introductory letter from the supervisor



FACULTY OF EDUCATION

Alice (main) Campus:

Private Bag X1314, King William's Town Road, Alice, 5700, South Africa

East London Campus:

Private Bag X9083, East London, 5200, RSA



Professor K. John Mammen B.Sc., LLB., M.Sc., M.Ed., PhD. (East London Campus)

13 May 2018

TO WHOM IT MAY CONCERN

This is to confirm that Ms. Ashnee Chetty (Student number 201001275) is a Master of Education degree candidate at the University of Fort Hare. For fulfilment of her studies, she needs to undertake research to complete a dissertation.

She has already been granted Ethical Clearance Certificates by the University of Fort Hare and South African Provincial Department of Basic Education. She is due to collect data during the month of May, 2018. Kindly assist her with your permission if she chooses your school as a research site.

I may be contacted, if need arises, on email kmammen@ufh.ac.za or through cell 071 580 6954

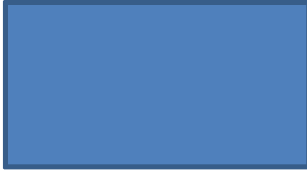
Sincerely

Professor K. J. Mammen

Research Supervisor

Appendix 3: Letter of entry to the principal of the chosen school

4 Hillside Garden
Norden Street
Quigney, 5201
East London



14 May 2018

Dear Mr 

Request for permission to conduct research

I am a student at the University of Fort Hare. I am currently doing Master of Education degree and conducting research to write a dissertation is a requirement. My research topic is on “**Assessing the learning and teaching of fraction operations in Grade 6**”. I am writing to request your permission to conduct research at your school.



If permission is granted, I would like to interview one educator from Grades 5-7. I would also like to send a letter out to parents of the Grade 6 learners at your school asking them permission to allow their children to take part in my research by answering a set of questions based on fraction operations. From the findings I would also like to interview 10 Grade 6 learners who have answered the questionnaire to find out what the learners understanding is of the concept fractions. I seek to understand the strategies used by teachers to teach fraction operations and how learners comprehend the concept.

All information about the school and information given by participants will remain anonymous and confidential.

Yours sincerely

Miss A. Chetty

Appendix 4: Participants' consent letter

4 Hillside Garden
Norden Street
Quigney, 5201
14 May 2018

INFORMED CONSENT FORM FOR TEACHERS RESEARCH CONDUCTED BY: Miss Ashnee Chetty

I write to request your consent to interview you about your understanding of ***Assessing the learning and teaching of fraction operations in Grade 6***. I am a student at the University of Fort Hare and the Research is in part fulfilment of the degree Master in Educations.

Participation in this research will be entirely voluntary. As a participant in this research you will be free to choose not to participate. Should you choose to participate, you are free not to respond to any question you do not wish to respond to, or can withdraw at any time without consequences of any kind. As a participant you will remain anonymous in the study and the raw data from interviews will remain confidential.

In terms of the ethical requirements of the University of Fort Hare I now invite you to complete the form below as an indication of your voluntary acceptance to take part in this research:

I voluntarily consent to participate in the study researching the ***Assessing the learning and teaching of fraction operations in Grade 6***. I fully understand the procedures of the study as explained to me. I am aware that I am under no obligation to participate in this study and may withdraw at any time without negative consequences to me.

SIGNATURE OF PARTICIPANT

DATE

SIGNATURE OF RESEARCHER

DATE

Dear Parents of Grade 6 learners

I, Miss A. Chetty, is a MEd student at the University of Fort Hare. I seek your permission to allow your child to be part of my research on ***Assessing the learning and teaching of fraction operations in Grade 6.***

Participation in this research will be entirely voluntary. All that would be required of your child is to answer 15 questions on fraction operations. As a participant, your child will remain anonymous in the study and the raw data.

Further consent will be requested from a selected number of parents to allow me to interview your child based on the questionnaire they will have answered. Once again, they will remain anonymous and information collected will not affect your child or their results in anyway.

In terms of the ethical requirements of the University of Fort Hare I now invite you to complete the form below as an indication of you giving voluntary permission for your child to take part in this research:



I, _____, parent of _____, give/ do not give permission for my child to participate in the above mentioned research. I understand that my child's identity will remain anonymous and will not be harmed in any way. I also understand that my child is not being forced to be part of this research and is free to not take part in it if they do not wish to.

Parent's Signature

Date

A handwritten signature in black ink, appearing to read 'A. Chetty', written over a horizontal line.

Miss A. Chetty

28/05/2018
Date

University of Fort Hare
MEd Student
A. Chetty
East London
28 May 2018

Dear Parents of Grade 6 learners

I, Miss A. Chetty, is a MEd student at the University of Fort Hare. I would like to thank you for allowing your child to answer the questionnaire. In the previous consent letter it stated that I would seek further consent from a selected number of parents to interview your child based on how they answered the questionnaire.

As mentioned before, your child will not be harmed or no negative outcome will affect their results. All information collected is purely for research purposes. Participation in this research will be entirely voluntary. As a participant, your child will remain anonymous in the study. I seek permission for you to allow me to interview your child, they will be asked questions based on what they understood or did not from the questions which they had answered. The interview will be recorded using a cellular phone device and only I, the researcher will have access to this.



In terms of the ethical requirements of the University of Fort Hare I now invite you to complete the form below as an indication of you giving voluntary permission for your child to take part in this next part of the research:

Please tick or circle whether or not you give permission.

I, _____ parent of _____ **[give / do not]**
give permission for my child to be interviewed. I am also aware that the interview will be recorded and only the researcher will use the information given. I understand that my child's identity will remain anonymous and will not be harmed in any way. I also understand that my child is not being forced to be part of this research and is free to not take part in it if they do not wish to.

Parent's Signature

Miss A. Chetty

Date

28/05/2018

Date

Appendix 5: Learners' questionnaire

Instructions: No calculators are allowed. Please show all your working out. Please do not leave any questions unanswered. If you do not know how to answer the question, using the lines provided, explain why you are unable to do the calculation.

Questionnaire – Learners

Name: _____

SECTION A: ADDITION

1. $\frac{1}{4} + \frac{2}{4} = ?$

2. $\frac{3}{6} + \frac{2}{6} = ?$



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3. $4\frac{1}{4} + 1\frac{2}{3} = ?$

SECTION B: Subtraction

1. $\frac{3}{4} - \frac{2}{4} = ?$

2. $3\frac{5}{7} - 1\frac{2}{7} = ?$

3. $2\frac{2}{5} - 1\frac{4}{5} = ?$



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[illegible]

SECTION C: Multiplication and Division

1. $\frac{1}{2}$ of 30 = ?

2. $\frac{3}{4}$ of 25 = ?



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3. $\frac{4}{15}$ of 40 = ?

4. $\frac{25}{100}$ of 250 = ?

[illegible]

5. $\frac{13}{100}$ of 320 = ?

[illegible]

Appendix 6: Learners' interview questions

Interview schedule – Learner

Participant no: _____

1. Refer to Section A of the questionnaire. Do you agree that you know how to add i) like-fractions, ii) proper fractions or, iii) mixed fractions? What about it if you do or don't understand?
2. Write the order of fractions you find most difficult to easiest in Section B. All you have to do is write 1-3 under the column, 1 being the most difficult and 3 being the easiest.

Table 6.1

Like fraction (Same denominator)	Proper/unlike-fractions (Different denominators)	Mixed Fractions

3. Looking at **Section B** of the questionnaire you answered, what sum did you find **most difficult** to do and why? Rate it from the most difficult to the easiest using the same number system as above.



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Table 6.2

Question 1	Question 2	Question 3	Question 4	Question 5

4. Referring to Section C of the questionnaire, please explain your thinking process of how you multiplied each question.
5. Tick in the table below any questions from **Section C** of the questionnaire which you did not understand. Please explain why you did it.

Table 6.3

6. Please take me through the steps you used to answer the questions in **Section A**.

Question 1	Question 2	Question 3	Question 4	Question 5

7. Can you tell me what steps you used to do the calculations in **Section B**?
8. Are there times when you get confused when adding i) like-fractions, ii) proper fractions, iii) improper fractions or iv) mixed fractions? Why is this?
9. Are there times when you get confused when subtracting i) like-fractions, ii) proper fractions iii) improper fractions or iv) mixed fractions? Please tell me why this happens.
10. Are there times when you get confused when multiplying i) like-fractions, ii) proper fractions or iii) mixed fractions?
11. Do you know of other methods you could have used to answer **question 5 in Section B**? If you do, please write it down.
12. Please rate which section you found the easiest by rating it from 1-3, 1 being the most difficult, 3 being the easiest.

Table 6.4

Section A	Section B	Section C
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13. Which fraction calculations in **Sections A** did you find the most difficult to do, and why? Rate the questions from the most difficult to the easiest using the numbers 1-5, 1 being most difficult.

Table 6.5

Question 1	Question 2	Question 3	Question 4	Question 5

Please explain why you felt certain questions were so difficult and why you found the others to be easier.

14. Do you think the textbooks you use have enough exercises for you to practice doing fraction operations in order to understand it better? Please explain your answer with an example of why you have said yes or no.

15. Do you understand the different terms used when working with fractions? For instance, what is the denominator, numerator, an improper fraction, mixed fraction and the line between the numerator and denominator? Please write down any fraction and label it with the headings provided in this question.

16. What methods do you think your teacher could use to better your understanding of fraction operations in:

a) Addition



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b) Subtraction

c) Multiplication

Appendix 7: Teacher Interview questions

Interview questions – Teacher

1. Biographical Information

Table 7

<u>Pseudo name:</u>	1.	2.	3.
<u>Years of teaching</u>			
<u>Position at your school</u>			
<u>Academic qualifications</u>			
<u>Age group</u>			

Age Groups: Group 1 (23-30), Group 2 (31-40), Group 3 (41-50), Group 4 (51-65)

2. What strengths do learners present when calculating like-fractions, fractions with same denominators in



a) Addition?

b) Subtraction?

c) Multiplication?

3. Have you identified strengths presented by your learners when doing proper fractions/unlike-fractions, fractions with different denominators, in

a) Addition?

b) Subtraction?

c) Multiplication?

4. Can you mention the strengths learners present when calculating improper fractions, fractions where the numerator is bigger than the denominator, in

a) Addition?

b) Subtraction?

c) Multiplication?

5. What strengths do your learners show when doing mixed fractions, where there are whole numbers and fractions, in

a) Addition?

b) Subtraction?

c) Multiplication?

6. What challenges do the learners experience when calculating like-fractions, fractions with the same denominators, in

a) Addition?

b) Subtraction?

c) Multiplication?

7. List the challenges your learners face when calculating unlike/proper fractions, fractions with different denominators, in



a) Addition.

b) Subtraction.

c) Multiplication.

8) What challenges do your learners present when calculating improper fractions, fractions where the numerator is bigger than the denominator, in

a) Addition?

b) Subtraction?

c) Multiplication?

9) Please name the challenges your learners show when calculating mixed fractions, whole numbers and fractions, in

a) Addition.

b) Subtraction.

c) Multiplication.

10. As a Mathematics teacher, what are your specific strengths in teaching the fraction operation of addition? Refer to Section A of the learners' questionnaire. What strategies would you use to teach those questions?

11. Name the various strengths you have when teaching the fraction operation of subtraction? Refer to Section B of the learners' questionnaire. How would you teach those questions?

12. List specific strategies you use to teach the fraction operation of multiplication. Using the questions from Section C of the learners' questionnaire to help you, illustrate the strategies you use.

13. When teaching fraction operation of addition, what challenges do you find difficult to explain, and why?

14. Do you face challenges when teaching fraction operations of subtraction and multiplication? Please elaborate your answer using examples for each operation.

15.1 In your opinion, please elaborate your thoughts on whether or not you think learners struggle to do like-fractions in

a) Addition.

b) Subtraction.

c) Multiplication.

15.2 In your opinion, please elaborate your thoughts on whether or not you think learners struggle to do proper fractions in

a) Addition.

b) Subtraction.

c) Multiplication.

15.3 In your opinion, please elaborate your thoughts on whether or not you think learners struggle to do improper fractions in

- a) Addition.
- b) Subtraction.
- c) Multiplication.

15.4 In your opinion, please elaborate your thoughts on whether or not you think learners struggle to do mixed fractions in

- a) Addition.
- b) Subtraction.
- c) Multiplication.

16. How do you address the challenges learners struggle with when it comes to learning fraction operations, referring to questions

- a) 15.1?
- b) 15.2?
- c) 15.3?
- d) 15.4?



17. What strategies could you use to address the specific challenges of teaching fraction operations identified in questions

- a) 15.1?
- b) 15.2?
- c) 15.3?
- d) 15.4?

18. Do you think there are enough practical exercises and time allocated for learners to understand the concept of fraction operations of addition, subtraction and multiplication? Please elaborate your answer with an example for validation.

19. Look at the following sum: $6\frac{2}{4} + 4\frac{4}{5} = ?$ How could you possibly teach this sum? If you have more than one method please write them all done. Do you teach from the unknown to the known? Or when do you start?



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Appendix 8: Learner no.1 - interview

Interview schedule – Learner

Participant no: 1

1. Refer to Section A of the questionnaire. Do you agree that you know how to add i) like-fractions, ii) proper fractions or, iii) mixed fractions? What about it if you do or don't understand?

Yes, I can add when they are the same, not so much when they are different, mixed fractions with different denominators.

2. Write the order of fractions you find the most difficult to the easiest in Section B. All you have to do is write 1-3 under the column. 1 being most difficult and 3 being the easiest.

Table 8.1

Like fraction (Same denominator)	Proper/unlike-fractions (Different denominators)	Mixed Fractions
3	2	1

3. Looking at **Section B** of the questionnaire you answered, what sum did you find **most difficult** to do and why? Rate it from the most difficult to the easiest using the same number system as above.

Table 8.2

Question 1	Question 2	Question 3	Question 4	Question 5
5	3	1	4	2

I found question 3 difficult because I did not know when to multiply or divide and what to do with it.

4. Referring to Section C of the questionnaire, please explain your thinking process of how you multiplied each question.

First I divided the 2 (denominator) into 30 (whole number) then got my answer in question 1. With question 2, I put 25 (whole number) over 1 then, I multiplied my numerators then my denominators. Then I divided 4 into 75 and got 18 with a remainder of 3.

5. Tick in the table below any questions from **Section C** of the questionnaire which you did not understand. Please explain why you did it.

Table 8.3

Question 1	Question 2	Question 3	Question 4	Question 5
		√		

6. Please take me through the steps you used to answer the questions in **Section A**.

Question 1 and 2 I just added my numerators then I didn't add my denominators. Question 3, I first added my whole numbers then worked out my fraction, I had to change my denominators to 12 and then added my answers.

7. Can you tell me what steps you used to do the calculations in **Section B**?

With question 3, I subtracted the whole numbers then I should have taken the second fraction and put it first then subtract the numerators from each other. Or I could have made my denominators bigger then I would have been able to subtract. I don't know about the borrowing method.



8. Are there times when you get confused when adding i) like-fractions, ii) proper fractions, iii) improper fractions or iv) mixed fractions? Why is this?

I don't get confused when the denominators are the same, no I don't get confused when they are different, I do get confused with mixed fractions because the first number is always a big number and when the numerator is bigger, I don't know how to convert it back into a mixed fraction.

9. Are there times when you get confused when subtracting i) like-fractions, ii) proper fractions iii) improper fractions or iv) mixed fractions? Please tell me why this happens.

I don't get confused when the denominators are the same, sometimes I get confused when the denominators are different and when the second fraction is smaller than the first, I don't know what to do here. Yes, I get confused with mixed fractions because I don't know what to do when the denominators are different.

10. Are there times when you get confused when multiplying i) like-fractions, ii) proper fractions or iii) mixed fractions?

I'm not too sure about multiplication of fractions.

11. Do you know of other methods you could have used to answer **question 5 in Section B**? If you do, please write it down.

I don't know of another method besides subtracting whole numbers first.

12. Please rate which section you found the easiest by rating it from 1-3. 1 being the most difficult, 3 being the easiest.

Table 8.4

Section A	Section B	Section C
1	2	3

13. Which fraction calculations in **Sections A** did you find the most difficult to do, and why? Rate the questions from the most difficult to the easiest using the numbers 1-5, 1 being most difficult.

Table 8.5

Question 1	Question 2	Question 3	Question 4	Question 5
5	4	3	1	2

Please explain why you felt certain questions were so difficult and why you found the others to be easier.

Because I could not add because the denominators were not the same and I couldn't find the perfect lowest common denominator, I tied 28 but I could not do it because the first answer would have been bigger.

14. Do you think the textbooks you use has enough exercises for you to practice doing fraction operations in order to understand it better? Please explain your answer with an example of why you have said yes or no.

No, because the exercises are short and the methods they show us to use are not the exact method or they do not give a method that is easier for us to understand.

15. Do you understand the different terms used when working with fractions? For instance, what is the denominator, numerator, an improper fraction, mixed fraction and the line between the numerator and denominator? Please write down any fraction and label it with the headings provided in this question.

$\frac{13}{30}$	Numerator	$\frac{17}{5}$	Improper fraction	$3\frac{1}{3}$	Mixed fraction
	Denominator				

16. What methods do you think your teacher could use to better your understanding of fraction operations in:

a) Addition

They could explain in shorter words or in words that are understandable. Teachers should explain methods in the simplest form of understanding.

b) Subtraction



In subtraction they should use more and more examples, because it is very difficult to understand quickly.

c) Multiplication

Teachers should use mixed fractions in multiplication because it is the only way it is realistic and helps. They should use improper fractions in multiplication so we know how to put improper fractions into proper fractions.

Appendix 9: Learner no.2 - interview

Interview schedule – Learner

Participant no: 2

1. Refer to Section A of the questionnaire. Do you agree that you know how to add i) like-fractions, ii) proper fractions or, iii) mixed fractions? What about it if you do or don't understand?

I know how to add fractions with the same denominator, I struggle a bit with different denominators. When there is a mixed fraction with different denominators I struggle to do that because I don't know how to find the lowest common denominator.

2. Write the order of fractions you find the most difficult to the easiest in Section B. All you have to do is write 1-3 under the column. 1 being most difficult and 3 being the easiest.

Table 9.1

Like fraction (Same denominator)	Proper/unlike-fractions (Different denominators)	Mixed Fractions
1	3	2

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3. Looking at **Section B** of the questionnaire you answered, what sum did you find **most difficult** to do and why? Rate it from the most difficult to the easiest using the same number system as above.

Table 9.2

Question 1	Question 2	Question 3	Question 4	Question 5
5	4	3	2	1

I found question 3-5 difficult because they don't have the same denominators.

4. Referring to Section C of the questionnaire, please explain your thinking process of how you multiplied each question.

I put my whole number over a 1, then I multiplied my numerators and denominators, got an improper fraction, then I divided my denominator into my numerator to get a mixed fraction.

5. Tick in the table below any questions from **Section C** of the questionnaire which you did not understand. Please explain why you did it.

Table 9.3

Question 1	Question 2	Question 3	Question 4	Question 5
			√	√

6. Please take me through the steps you used to answer the questions in **Section A**.

Question 1 and 2 were easier to do because the denominators are the same. Questions 3-5 were not the same as the denominators are different and it becomes challenging to find the lowest common denominator.

7. Can you tell me what steps you used to do the calculations in **Section B**?

When the denominators are the same, it is easier for me to do. Question 3 was challenging because the first fraction was smaller than the one I had to subtract with. I don't know what to do when this happens.



8. Are there times when you get confused when adding i) like-fractions, ii) proper fractions, iii) improper fractions or iv) mixed fractions? Why is this?

i) No

ii) Yes, different denominators

iii) Yes, I get confused with different denominators

iv) Yes, because of the different denominators and finding the LCD.

9. Are there times when you get confused when subtracting i) like-fractions, ii) proper fractions iii) improper fractions or iv) mixed fractions? Please tell me why this happens.

i) No

ii) Yes, different denominators

iii) Yes, different denominators

iv) Yes, finding the LCD

10. Are there times when you get confused when multiplying i) like-fractions, ii) proper fractions or iii) mixed fractions?

I understand how to do multiplication of fractions.

11. Do you know of other methods you could have used to answer **question 5 in Section B**? If you do, please write it down.

No, I don't know any other method I could use to do the sum.

12. Please rate which section you found the easiest by rating it from 1-3. 1 being the most difficult, 3 being the easiest.

Table 9.4

Section A	Section B	Section C
3	2	1

13. Which fraction calculations in **Sections A** did you find the most difficult to do, and why? Rate the questions from the most difficult to the easiest using the numbers 1-5, 1 being most difficult.

Table 9.5

Question 1	Question 2	Question 3	Question 4	Question 5
5	4	 3	2	1

Please explain why you felt certain questions were so difficult and why you found the others to be easier.

Finding the LCD.

14. Do you think the textbooks you use have enough exercises for you to practice doing fraction operations in order to understand it better? Please explain your answer with an example of why you have said yes or no.

No, I don't feel that there is enough exercises. I feel the exercises are too short and I don't feel that we did a lot of fraction calculations in term 2.

15. Do you understand the different terms used when working with fractions? For instance, what is the denominator, numerator, an improper fraction, mixed fraction and the line between the numerator and denominator? Please write down any fraction and label it with the headings provided in this question.

Numerator	$\frac{40}{10}$	Improper fraction	$2\frac{1}{3}$	Mixed fraction
Denominator	$\frac{40}{10}$			

16. What methods do you think your teacher could use to better your understanding of fraction operations in:

a) Addition _____

To help me learn how to find the LCD.

b) Subtraction _____

To help me understand how to find the LCD.

c) Multiplication ____

Teach me any methods.



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Appendix 10: Learner no.3 - interview

Interview schedule – Learner

Participant no: 3

1. Refer to Section A of the questionnaire. Do you agree that you know how to add i) like-fractions, ii) proper fractions or, iii) mixed fractions? What about it if you do or don't understand?

I don't know how to add fractions as I've struggled to understand fractions since Grade 5.

2. Write the order of fractions you find the most difficult to the easiest in Section B. All you have to do is write 1-3 under the column. 1 being most difficult and 3 being the easiest.

Table 10.1

Like fraction (Same denominator)	Proper/unlike-fractions (Different denominators)	Mixed Fractions
3	2	1



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3. Looking at **Section B** of the questionnaire you answered, what sum did you find **most difficult** to do and why? Rate it from the most difficult to the easiest using the same number system as above.

Table 10.2

Question 1	Question 2	Question 3	Question 4	Question 5
5	4	2	1	3

Question 4 was so difficult to do because of the method I did not remember the method I needed to use.

4. Referring to Section C of the questionnaire, please explain your thinking process of how you multiplied each question.

I took the denominator and times it by the whole number and got 60 and then I got confused and did not know what to do afterwards.

5. Tick in the table below any questions from **Section C** of the questionnaire which you did not understand. Please explain why you did it.

Table 10.3

Question 1	Question 2	Question 3	Question 4	Question 5
		√		√

6. Please take me through the steps you used to answer the questions in **Section A**.

I added my denominators and my numerators to get my answer. I realised too late that I didn't need to add my denominator. I tried to convert to an improper fraction in question 3 but then I got confused of what steps to use.

7. Can you tell me what steps you used to do the calculations in **Section B**?

For the first question I didn't add the denominators only the numerators. With question 3, I subtracted the whole numbers then I didn't know what to do with the fraction because the denominators were different.



8. Are there times when you get confused when adding i) like-fractions, ii) proper fractions, iii) improper fractions or iv) mixed fractions? Why is this?

I don't understand how to 'of' sums, we do not do a lot of this in my Grade.

9. Are there times when you get confused when subtracting i) like-fractions, ii) proper fractions iii) improper fractions or iv) mixed fractions? Please tell me why this happens.

I get confused with fractions because I don't know what to do especially when the denominators are different.

10. Are there times when you get confused when multiplying i) like-fractions, ii) proper fractions or iii) mixed fractions?

No, I don't know how to do multiplication sums because I don't understand how to do fractions.

11. Do you know of other methods you could have used to answer **question 5 in Section B**? If you do, please write it down.

I don't know of another method.

12. Please rate which section you found the easiest by rating it from 1-3. 1 being the most difficult, 3 being the easiest.

Table 10.4

Section A	Section B	Section C
3	2	1

13. Which fraction calculations in **Sections A** did you find the most difficult to do, and why? Rate the questions from the most difficult to the easiest using the numbers 1-5, 1 being most difficult.

Table 10.5



Question 1	Question 2	Question 3	Question 4	Question 5
4	3	5	1	2

Please explain why you felt certain questions were so difficult and why you found the others to be easier.

I did not understand the methods and I was struggling with the questions.

14. Do you think the textbooks you use has enough exercises for you to practice doing fraction operations in order to understand it better? Please explain your answer with an example of why you have said yes or no.

It has have exercises but not to my level of understanding. I would like to practise doing more fraction sums so I can learn to understand it better.

15. Do you understand the different terms used when working with fractions? For instance, what is the denominator, numerator, an improper fraction, mixed fraction and

the line between the numerator and denominator? Please write down any fraction and label it with the headings provided in this question.

$\frac{1}{2}$	Numerator	$\frac{23}{3}$	Improper fraction	$4\frac{3}{5}$	Mixed fraction
	Denominator				

16. What methods do you think your teacher could use to better your understanding of fraction operations in:

a) Addition _____

[Need help learning how to add mixed fractions.](#)

b) Subtraction _____

[Need help with improper fractions.](#)

c) Multiplication _____

[Need help understanding how to multiply mixed fractions.](#)



Appendix 11: Learner no.4 - interview

Interview schedule – Learner

Participant no: 4

1. Refer to Section A of the questionnaire. Do you agree that you know how to add i) like-fractions, ii) proper fractions or, iii) mixed fractions? What about it if you do or don't understand?

i) Yes I know how to add when the denominators are the same.

ii) No, because I don't know how to change the denominators into the same denominators.

iii) Yes I can.

2. Write the order of fractions you find the most difficult to the easiest in Section B. All you have to do is write 1-3 under the column. 1 being most difficult and 3 being the easiest.

Table 11.1

Like fraction (Same denominator)	Proper/unlike-fractions (Different denominators)	Mixed Fractions
3	 1	2

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3. Looking at **Section B** of the questionnaire you answered, what sum did you find **most difficult** to do and why? Rate it from the most difficult to the easiest using the same number system as above.

Table 11.2

Question 1	Question 2	Question 3	Question 4	Question 5
5	4	1	2	3

Question 3 was difficult because I did not know how to borrow from my whole number.

4. Referring to Section C of the questionnaire, please explain your thinking process of how you multiplied each question.

You turn the whole number into an improper fraction then I was not sure of what steps to use. I looked for the lowest common denominator, I find this section difficult.

5. Tick in the table below any questions from **Section C** of the questionnaire which you did not understand. Please explain why you did it.

Table 11.3

Question 1	Question 2	Question 3	Question 4	Question 5
✓	✓	✓	✓	✓

6. Please take me through the steps you used to answer the questions in **Section A**.

The mistake I made was I added my denominators together when I was not supposed to, I know that I must not do anything to the denominators when they are the same. With question 3, I added the whole numbers first then work with the fractions, I did not know how to find the lowest common denominator.

7. Can you tell me what steps you used to do the calculations in **Section B**?

I use the same steps as I used in section



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8. Are there times when you get confused when adding i) like-fractions, ii) proper fractions, iii) improper fractions or iv) mixed fractions? Why is this?

i) No
 ii) Yes, I get mixed up when I have to make the denominators the same.
 iii) I find it challenging, I am used to working with the same denominators, and with smaller numerators to the denominator.
 iv) Yes I find it difficult because I make mistakes when I find my lowest common denominator.

9. Are there times when you get confused when subtracting i) like-fractions, ii) proper fractions iii) improper fractions or iv) mixed fractions? Please tell me why this happens.

i) No
 ii) Yes, I struggle to find the lowest common denominator
 iii) Yes, I get mixed up finding the denominators.
 iv) Yes, I also get confused with this when the denominators are different.

10. Are there times when you get confused when multiplying i) like-fractions, ii) proper fractions or iii) mixed fractions?

I can only multiply fractions with smaller numbers, when it gets bigger numbers it becomes more difficult for me.

11. Do you know of other methods you could have used to answer **question 5 in Section B**? If you do, please write it down.

Yes, I know of another method, when you times the whole number to the denominator and then add the numerator, converting to an improper fraction.

12. Please rate which section you found the easiest by rating it from 1-3. 1 being the most difficult, 3 being the easiest.

Table 11.4

Section A	Section B	Section C
3	2	1

13. Which fraction calculations in **Sections A** did you find the most difficult to do, and why? Rate the questions from the most difficult to the easiest using the numbers 1-5, 1 being most difficult.

Table 11.5

Question 1	Question 2	Question 3	Question 4	Question 5
5	3	2	4	1

Please explain why you felt certain questions were so difficult and why you found the others to be easier.

Question 5 was difficult to do because there were three fractions with different denominators which made it difficult to find the common denominator.

14. Do you think the textbooks you use has enough exercises for you to practice doing fraction operations in order to understand it better? Please explain your answer with an example of why you have said yes or no.

I don't feel that it is enough, because we don't do a lot of fractions we do a lot of other concepts. So it is difficult for me to understand.

15. Do you understand the different terms used when working with fractions? For instance, what is the denominator, numerator, an improper fraction, mixed fraction and the line between the numerator and denominator? Please write down any fraction and label it with the headings provided in this question.

$\frac{2}{4}$	Numerator	$\frac{30}{1}$	Improper fraction	$2\frac{4}{5}$	Mixed fraction
	Denominator				

16. What methods do you think your teacher could use to better your understanding of fraction operations in:

a) Addition _____

I think my teacher could teach me more easier methods.

b) Subtraction



I think my teacher could teach me the method when converting them into improper fractions.

c) Multiplication _____

I think my teacher could use the method when using improper fractions.

Appendix 12: Learner no.5 - interview

Interview schedule – Learner

Participant no: 5

1. Refer to Section A of the questionnaire. Do you agree that you know how to add i) like-fractions, ii) proper fractions or, iii) mixed fractions? What about it if you do or don't understand?

i) Yes

ii) No, I find it very challenging to find which denominator to use.

iii) No, because I must find the lowest common denominator.

2. Write the order of fractions you find the most difficult to the easiest in Section B. All you have to do is write 1-3 under the column. 1 being most difficult and 3 being the easiest.

Table 12.1

Like fraction (Same denominator)	Proper/unlike-fractions (Different denominators)	Mixed Fractions
3	2	1



3. Looking at **Section B** of the questionnaire you answered, what sum did you find **most difficult** to do and why? Rate it from the most difficult to the easiest using the same number system as above.

Table 12.2

Question 1	Question 2	Question 3	Question 4	Question 5
5	4	3	1	2

I found question four very difficult to do because the denominators are different and I had to find the lowest common denominator.

4. Referring to Section C of the questionnaire, please explain your thinking process of how you multiplied each question.

Doing 'of' sums you have to divide you denominator into your whole number and you times your answer by the numerator. With question 2, I needed to put my remainder as a fraction.

5. Tick in the table below any questions from **Section C** of the questionnaire which you did not understand. Please explain why you did it.

Table 12.3

Question 1	Question 2	Question 3	Question 4	Question 5
	✓	✓	✓	✓

I found this section difficult because I did not know what to do with my remainders.

6. Please take me through the steps you used to answer the questions in **Section A**.

You do not add denominators, you just add the numerators. Questions 1 and 2 were easy for me because it has the same denominators. I first added my whole numbers in question 3 then I got stuck with the fractions because the denominators were very different.



7. Can you tell me what steps you used to do the calculations in **Section B**?

I used the same steps as I did in section A. In question 3 I first subtracted my whole numbers then I couldn't do the fractions because the 2 was smaller than the 4. I don't know how to borrow to make the first fraction bigger than the second one.

8. Are there times when you get confused when adding i) like-fractions, ii) proper fractions, iii) improper fractions or iv) mixed fractions? Why is this?

- i) No
- ii) Yes, because I have to find the LCD
- iii) Yes, I do not know how to do it, the steps are very challenging.
- iv) Yes, I don't understand how to do the steps.

9. Are there times when you get confused when subtracting i) like-fractions, ii) proper fractions iii) improper fractions or iv) mixed fractions? Please tell me why this happens.

- i) No
- ii) Yes, because I struggle to find the LCD

iii) Yes, I don't know how to find the lowest common denominator

iv) Yes, same as the other reason

10. Are there times when you get confused when multiplying i) like-fractions, ii) proper fractions or iii) mixed fractions?

I don't understand how to multiply fractions. I struggle when there is a remainder, I don't know what to do with it.

11. Do you know of other methods you could have used to answer **question 5 in Section B**? If you do, please write it down.

I don't know of another method.

12. Please rate which section you found the easiest by rating it from 1-3. 1 being the most difficult, 3 being the easiest.

Table 12.4

Section A	Section B	Section C
2	 1	3

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13. Which fraction calculations in **Sections A** did you find the most difficult to do, and why? Rate the questions from the most difficult to the easiest using the numbers 1-5, 1 being most difficult.

Table 12.5

Question 1	Question 2	Question 3	Question 4	Question 5
5	4	3	2	1

Please explain why you felt certain questions were so difficult and why you found the others to be easier.

Question 5 was so difficult to do because the denominators being very different and there were 3 mixed fractions.

14. Do you think the textbooks you use has enough exercises for you to practice doing fraction operations in order to understand it better? Please explain your answer with an example of why you have said yes or no.

No, it is because the examples in the textbook are different to the way we do fractions.
I feel that we don't practise fractions enough.

15. Do you understand the different terms used when working with fractions? For instance, what is the denominator, numerator, an improper fraction, mixed fraction and the line between the numerator and denominator? Please write down any fraction and label it with the headings provided in this question.

$\frac{1}{2}$	Numerator	$\frac{5}{3}$	Improper fraction	$2\frac{1}{2}$	Mixed fraction
	Denominator				

16. What methods do you think your teacher could use to better your understanding of fraction operations in:

a) Addition

My teacher could tell us to do our fractions step by step and give us different methods on a piece of paper.

b) Subtraction



My teacher could give us examples on how to do the things and go through the sum step by step to better understand.

c) Multiplication

My teacher could give us a lot of practises on how to do the things, for example; if the denominators are different then my teacher could explain how to find the lowest common denominator.

Appendix 13: Learner no.6 - interview

Interview schedule – Learner

Participant no: 6

1. Refer to Section A of the questionnaire. Do you agree that you know how to add i) like-fractions, ii) proper fractions or, iii) mixed fractions? What about it if you do or don't understand?

i) Yes
ii) Yes, you just convert by finding the LCD
iii) Yes

2. Write the order of fractions you find the most difficult to the easiest in Section B. All you have to do is write 1-3 under the column. 1 being most difficult and 3 being the easiest.

Table 13.1

Like fraction (Same denominator)	Proper/unlike-fractions (Different denominators)	Mixed Fractions
None	None	None

I don't really find anything that difficult, because it is easy to understand the methods.

3. Looking at **Section B** of the questionnaire you answered, what sum did you find **most difficult** to do and why? Rate it from the most difficult to the easiest using the same number system as above.

Table 13.2

Question 1	Question 2	Question 3	Question 4	Question 5
None	None	None	None	None

I didn't find anything difficult, it's the same as adding, I didn't find any sums difficult.

4. Referring to Section C of the questionnaire, please explain your thinking process of how you multiplied each question.

I didn't understand how to do the rest of the questions, only question 1 I could do. I forgot how to do the method. So I was supposed to convert the whole number into an

improper fraction. Then do the multiplication and then convert back into a mixed fraction.

5. Tick in the table below any questions from **Section C** of the questionnaire which you did not understand. Please explain why you did it.

Table 13.3

Question 1	Question 2	Question 3	Question 4	Question 5
	√	√	√	√

I forgot how to do the method, but afterwards I remembered what I needed to do.

6. Please take me through the steps you used to answer the questions in **Section A**.

With question 1 and 2 I just added the numerators because you never add the denominators. With question 3 you cannot add something with different denominators so you must look for the lowest denominator. Then changed both to 12, what you do to the bottom you do to the bottom and carry on with the sum.



7. Can you tell me what steps you used to do the calculations in **Section B**?

Same steps just different inverse operation. Question 3, I could not subtract 4 from 2, I said 2 times 5 plus 2 is 12 so you say 12 over 5 changed into an improper fraction and do the same with the second fraction then I was able to subtract and got 3 over 5 and I could not simplify further and that was my final answer.

8. Are there times when you get confused when adding i) like-fractions, ii) proper fractions, iii) improper fractions or iv) mixed fractions? Why is this?

i) No I don't get confused when they are the same.

ii) I would get confused when they are different because then I have to process the steps and it is a bit long to do.

iii) No I wouldn't get confused.

iv) I would get confused because I would need to look for the LCD.

9. Are there times when you get confused when subtracting i) like-fractions, ii) proper fractions iii) improper fractions or iv) mixed fractions? Please tell me why this happens.

i) No
 ii) Yes I would definitely get confused because I will have to take to the lowest common denominator in order to add or subtract.
 iii) No I didn't struggle
 iv) No I just have to change the denominators to the same. What might be difficult for me is remembering all the steps.

10. Are there times when you get confused when multiplying i) like-fractions, ii) proper fractions or iii) mixed fractions?

I'm not too familiar with multiplication of fractions. I do know how to do 'of' sums it's just sometimes hard to remember the steps.

11. Do you know of other methods you could have used to answer **question 5 in Section B**? If you do, please write it down.

I do, you could convert the mixed fraction to an improper fraction then find the LCD and then do the sum and then convert back to a mixed fraction.



12. Please rate which section you found the easiest by rating it from 1-3. 1 being the most difficult, 3 being the easiest.

Table 13.4

Section A	Section B	Section C
3	2	1

13. Which fraction calculations in **Sections A** did you find the most difficult to do, and why? Rate the questions from the most difficult to the easiest using the numbers 1-5, 1 being most difficult.

Table 13.5

Question 1	Question 2	Question 3	Question 4	Question 5
None	None	None	None	None

Please explain why you felt certain questions were so difficult and why you found the others to be easier.

I did not find any questions difficult.

14. Do you think the textbooks you use has enough exercises for you to practice doing fraction operations in order to understand it better? Please explain your answer with an example of why you have said yes or no.

Yes we have lots and lots of examples and exercises. We have covered a lot of exercises on fractions, it is not like we do it often, it does not pop up every term as often so I can remember the concept or the steps.

15. Do you understand the different terms used when working with fractions? For instance, what is the denominator, numerator, an improper fraction, mixed fraction and the line between the numerator and denominator? Please write down any fraction and label it with the headings provided in this question.

$\frac{1}{3}$	Numerator	$\frac{18}{9}$	Improper fraction	$3\frac{1}{3}$	Mixed fraction
	Denominator				

16. What methods do you think your teacher could use to better your understanding of fraction operations in:



a) Addition _____

None, the two we were given is easy to understand.

b) Subtraction _____

None, the two we were given is easy to understand.

c) Multiplication _____

None, the two we were given is easy to understand.

Appendix 14: Learner no.7 - interview

Interview schedule – Learner

Participant no: 7

1. Refer to Section A of the questionnaire. Do you agree that you know how to add i) like-fractions, ii) proper fractions or, iii) mixed fractions? What about it if you do or don't understand?

i) Yes I do know how to add like-fractions.

ii) Yes, sometimes it is confusing, I forget to simplify and to find the LCD you have to times to get it but sometimes I divide instead of times.

iii) Yes

2. Write the order of fractions you find the most difficult to the easiest in Section B. All you have to do is write 1-3 under the column. 1 being most difficult and 3 being the easiest.

Table 14.1

Like fraction (Same denominator)	Proper/unlike-fractions (Different denominators)	Mixed Fractions
3	 1	2

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3. Looking at **Section B** of the questionnaire you answered, what sum did you find **most difficult** to do and why? Rate it from the most difficult to the easiest using the same number system as above.

Table 14.2

Question 1	Question 2	Question 3	Question 4	Question 5
5	4	3	2	1

I find it difficult when there are mixed fractions, sometimes I forget to simplify. I also find it difficult when I have to borrow from the whole number then to convert that borrowed number to a fraction.

4. Referring to Section C of the questionnaire, please explain your thinking process of how you multiplied each question.

So I took the whole number and divided it by the denominator and multiplied by the numerator in question 1. In question 2 I used the same method as question 1, because there was a remainder I had to put in the decimal sign and converted to a decimal then I multiplied by the numerator and then wrote my answer as a decimal. I could have written my decimal over 100 and then simplified it.

5. Tick in the table below any questions from **Section C** of the questionnaire which you did not understand. Please explain why you did it.

Table 14.3

Question 1	Question 2	Question 3	Question 4	Question 5
				✓

I got confused when I needed to divide and I did not know where to write my answers.

6. Please take me through the steps you used to answer the questions in **Section A**.

I first added my whole numbers in question 3 then I went to my fractions and find the LCD that could go into 3 and 4 which was 12 what I did to the bottom I did to the top. Then I added my fractions and then added my fractions to the whole number.



7. Can you tell me what steps you used to do the calculations in **Section B**?

I used the same method as I did in addition.

8. Are there times when you get confused when adding i) like-fractions, ii) proper fractions, iii) improper fractions or iv) mixed fractions? Why is this?

i) No

ii) Yes sometimes, when I have to add different denominators, I find the LCD first, I sometimes get confused if I must multiply the numerator as well after converting to the LCD.

iii) Not really, I must convert my final answer back to a mixed fraction.

iv) Nope

9. Are there times when you get confused when subtracting i) like-fractions, ii) proper fractions iii) improper fractions or iv) mixed fractions? Please tell me why this happens.

i) No

ii) No

iii) _____ No
 iv) Yes when I have to borrow from the whole number then convert that into a fraction.

10. Are there times when you get confused when multiplying i) like-fractions, ii) proper fractions or iii) mixed fractions?

I think I know how to do 'of' sums but not fractions of multiplication.

11. Do you know of other methods you could have used to answer **question 5 in Section B**? If you do, please write it down.

No

12. Please rate which section you found the easiest by rating it from 1-3. 1 being the most difficult, 3 being the easiest.

Table 14.4

Section A	Section B	Section C
3	2	1



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13. Which fraction calculations in **Sections A** did you find the most difficult to do, and why? Rate the questions from the most difficult to the easiest using the numbers 1-5, 1 being most difficult.

Table 14.5

Question 1	Question 2	Question 3	Question 4	Question 5
5	4	3	1	2

Please explain why you felt certain questions were so difficult and why you found the others to be easier.

Finding the LCD.

14. Do you think the textbooks you use has enough exercises for you to practice doing fraction operations in order to understand it better? Please explain your answer with an example of why you have said yes or no.

Yes, before we try new unit, the textbook shows you an example of how to do it and its explained with different colours each step.

15. Do you understand the different terms used when working with fractions? For instance, what is the denominator, numerator, an improper fraction, mixed fraction and the line between the numerator and denominator? Please write down any fraction and label it with the headings provided in this question.

$\frac{1}{4}$	Numerator	$\frac{17}{5}$	Improper fraction	$3\frac{1}{3}$	Mixed fraction
	Denominator				

16. What methods do you think your teacher could use to better your understanding of fraction operations in:

a) Addition _____

They can tell the children to break up the number into hundreds, tens, units.

b) Subtraction _____



They could also break down the numbers.

c) Multiplication.

Nothing.

Appendix 15: Learner no.8 - interview

Interview schedule – Learner

Participant no: 8

1. Refer to Section A of the questionnaire. Do you agree that you know how to add i) like-fractions, ii) proper fractions or, iii) mixed fractions? What about it if you do or don't understand?

i) Yes
ii) No because I don't know what to do when the denominators are not the same.
iii) No, sometimes it confuses me, I don't know what to do when the denominators are different.

2. Write the order of fractions you find the most difficult to the easiest in Section B. All you have to do is write 1-3 under the column. 1 being most difficult and 3 being the easiest.

Table 15.1

Like fraction (Same denominator)	Proper/unlike-fractions (Different denominators)	Mixed Fractions
3	2	1



3. Looking at **Section B** of the questionnaire you answered, what sum did you find **most difficult** to do and why? Rate it from the most difficult to the easiest using the same number system as above.

Table 15.2

Question 1	Question 2	Question 3	Question 4	Question 5
3	5	4	2	1

I find it harder when the denominators are different.

4. Referring to Section C of the questionnaire, please explain your thinking process of how you multiplied each question.

I just halved 30 and got 15 in question 1. So I divide the whole number by the denominator took the answer and multiplied by the numerator and put my remainder as a fraction.

5. Tick in the table below any questions from **Section C** of the questionnaire which you did not understand. Please explain why you did it.

Table 15.3

Question 1	Question 2	Question 3	Question 4	Question 5
		√		√

6. Please take me through the steps you used to answer the questions in **Section A**.

In question 1 I knew denominators were the same so I just added the numerators.
With question 3 I just added the whole numbers and denominators because they were different. I don't know to find the LCD.

7. Can you tell me what steps you used to do the calculations in **Section B**?

I used the same steps to answer question 1 and 2 as I did in addition. when it got to question 3, I subtracted the whole numbers first then worked out the fractions.

8. Are there times when you get confused when adding i) like-fractions, ii) proper fractions, iii) improper fractions or iv) mixed fractions? Why is this?



i) No
 ii) Yes, because I don't know if I must change it or leave it or to just added it when they are not the same denominators.
 iii) Yes, because the numerator is bigger than the numerator it confuses me because I don't know if I must leave it like that or carry it over.
 iv) Yes, not sure what to do when I have to do when the denominators are different.

9. Are there times when you get confused when subtracting i) like-fractions, ii) proper fractions iii) improper fractions or iv) mixed fractions? Please tell me why this happens.

i) Yes
 ii) No same as addition
 iii) Same reason as addition.
 iv) Same reasons as addition.

10. Are there times when you get confused when multiplying i) like-fractions, ii) proper fractions or iii) mixed fractions?

I know how to work 'of' sums, but I don't know how to do straight multiplication of fractions.

11. Do you know of other methods you could have used to answer **question 5 in Section B**? If you do, please write it down.

No other method.

12. Please rate which section you found the easiest by rating it from 1-3. 1 being the most difficult, 3 being the easiest.

Table 15.4

Section A	Section B	Section C
2	1	3

13. Which fraction calculations in **Sections A** did you find the most difficult to do, and why? Rate the questions from the most difficult to the easiest using the numbers 1-5, 1 being most difficult.



Table 15.5

Question 1	Question 2	Question 3	Question 4	Question 5
5	4	2	3	1

Please explain why you felt certain questions were so difficult and why you found the others to be easier.

I struggle when the denominators are not the same.

14. Do you think the textbooks you use has enough exercises for you to practice doing fraction operations in order to understand it better? Please explain your answer with an example of why you have said yes or no.

We have practiced a lot of fractions, I don't understand mixed fractions especially when the denominators are not the same. I feel there is enough exercises.

15. Do you understand the different terms used when working with fractions? For instance, what is the denominator, numerator, an improper fraction, mixed fraction and the line between the numerator and denominator? Please write down any fraction and label it with the headings provided in this question.

$\frac{5}{8}$	Numerator	$\frac{15}{11}$	Improper fraction	$2\frac{4}{7}$	Mixed fraction
	Denominator				

16. What methods do you think your teacher could use to better your understanding of fraction operations in:

a) Addition _____

My teacher could break down the fractions step by step.

b) Subtraction _____

My teacher could break down the fraction step by step.

c) Multiplication _____



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My teacher could teach me how to use the division method.

Appendix 16: Learner no.9 - interview

Interview schedule – Learner

Participant no: 9

1. Refer to Section A of the questionnaire. Do you agree that you know how to add i) like-fractions, ii) proper fractions or, iii) mixed fractions? What about it if you do or don't understand?

i) Yes

ii) Yes

iii) Yes

2. Write the order of fractions you find the most difficult to the easiest in Section B. All you have to do is write 1-3 under the column. 1 being most difficult and 3 being the easiest.

Table 16.1

Like fraction (Same denominator)	Proper/unlike-fractions (Different denominators)	Mixed Fractions
3	1	2



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3. Looking at **Section B** of the questionnaire you answered, what sum did you find **most difficult** to do and why? Rate it from the most difficult to the easiest using the same number system as above.

Table 16.2

Question 1	Question 2	Question 3	Question 4	Question 5
5	4	3	2	1

Question 5 was so difficult because the first fraction had lower denominator and the second fraction was bigger. So finding the LCD was difficult.

4. Referring to Section C of the questionnaire, please explain your thinking process of how you multiplied each question.

So I first divided the whole number by the denominator then multiplied my answer by the numerator. I should have written my decimal as a fraction over 100 and then simplified my answer.

5. Tick in the table below any questions from **Section C** of the questionnaire which you did not understand. Please explain why you did it.

Table 16.3

Question 1	Question 2	Question 3	Question 4	Question 5
None	None	None	None	None

6. Please take me through the steps you used to answer the questions in **Section A**.

Because the denominators were the same I just added the numerators. With question 3 the denominators are not the same so I had to find the LCD which is 12, then what you do to the bottom you also do to the top



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7. Can you tell me what steps you used to do the calculations in **Section B**?

I used the same methods as I used in section A.

8. Are there times when you get confused when adding i) like-fractions, ii) proper fractions, iii) improper fractions or iv) mixed fractions? Why is this?

i) No

ii) No

iii) No

iv) No

9. Are there times when you get confused when subtracting i) like-fractions, ii) proper fractions iii) improper fractions or iv) mixed fractions? Please tell me why this happens.

i) No

ii) Yes, when the one denominator is bigger than the other.

iii) No

iv) No

10. Are there times when you get confused when multiplying i) like-fractions, ii) proper fractions or iii) mixed fractions?

I don't find it difficult to do multiplication of fractions at all.

11. Do you know of other methods you could have used to answer **question 5 in Section B**? If you do, please write it down.

No I don't.

12. Please rate which section you found the easiest by rating it from 1-3. 1 being the most difficult, 3 being the easiest.

Table 16.4

Section A	Section B	Section C
3	1	2

13. Which fraction calculations in **Sections A** did you find the most difficult to do, and why? Rate the questions from the most difficult to the easiest using the numbers 1-5, 1 being most difficult.

Table 16.5

Question 1	Question 2	Question 3	Question 4	Question 5
None	None	None	None	None

Please explain why you felt certain questions were so difficult and why you found the others to be easier.

I did not find any question difficult.

14. Do you think the textbooks you use has enough exercises for you to practice doing fraction operations in order to understand it better? Please explain your answer with an example of why you have said yes or no.

No I don't think there is enough exercises because we only do it now and then. We don't do fractions all the time.

15. Do you understand the different terms used when working with fractions? For instance, what is the denominator, numerator, an improper fraction, mixed fraction and the line between the numerator and denominator? Please write down any fraction and label it with the headings provided in this question.

$\frac{2}{4}$	Numerator	$\frac{15}{14}$	Improper fraction	$4\frac{2}{4}$	Mixed fraction
	Denominator				

16. What methods do you think your teacher could use to better your understanding of fraction operations in:

a) Addition _____

There is nothing because I understand how to add fractions.

b) Subtraction

I think my teacher should give us sums that need subtracting fractions for homework and then she can show us where we went wrong.

c) Multiplication _____



I haven't yet done multiplication with fractions.

Appendix 17: Learner no.10 - interview

Interview schedule – Learner

Participant no: 10

1. Refer to Section A of the questionnaire. Do you agree that you know how to add i) like-fractions, ii) proper fractions or, iii) mixed fractions? What about it if you do or don't understand?

i) Yes
ii) It depends on how big the denominator is and then having to find the LCD and to get to the answer will take a lot of time and it is also confusing.
iii) Yes I find it difficult.

2. Write the order of fractions you find the most difficult to the easiest in Section B. All you have to do is write 1-3 under the column. 1 being most difficult and 3 being the easiest.

Table 17.1

Like fraction (Same denominator)	Proper/unlike-fractions (Different denominators)	Mixed Fractions
3	 2	1

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3. Looking at **Section B** of the questionnaire you answered, what sum did you find **most difficult** to do and why? Rate it from the most difficult to the easiest using the same number system as above.

Table 17.2

Question 1	Question 2	Question 3	Question 4	Question 5
5	4	3	2	1

When the denominator is very big then it confuses me.

4. Referring to Section C of the questionnaire, please explain your thinking process of how you multiplied each question.

So I divide my whole number by my denominator then divide by my numerator. With question 2 - 5 I was not really sure of my steps and what to do.

5. Tick in the table below any questions from **Section C** of the questionnaire which you did not understand. Please explain why you did it.

Table 17.3

Question 1	Question 2	Question 3	Question 4	Question 5
	√	√	√	√

6. Please take me through the steps you used to answer the questions in **Section A**.

When the denominators are the same it is easier, when the denominators are different I add the whole numbers first, I got confused because the denominators were not the same then I just realised I was not sure how to make the denominators the same so it was confusing.

7. Can you tell me what steps you used to do the calculations in **Section B**?

I used the same steps as I did in Section A, but before the subtracted my whole numbers, I first make sure the numerators are not too small for me to subtract the other numerator from each other. So with question 3 I knew that I needed to borrow from my whole number before subtracting my numerators.



8. Are there times when you get confused when adding i) like-fractions, ii) proper fractions, iii) improper fractions or iv) mixed fractions? Why is this?

No I don't get confused when the denominators are the same. It depends on how big the denominator number is, when it is too big it becomes confusing, if it is small then I can find the LCD.

9. Are there times when you get confused when subtracting i) like-fractions, ii) proper fractions iii) improper fractions or iv) mixed fractions? Please tell me why this happens.

i) Not confused.

ii/iii/iv) it is confusing when the denominators are not the same to do subtraction with these fractions.

10. Are there times when you get confused when multiplying i) like-fractions, ii) proper fractions or, iii) mixed fractions?

No I struggle with this, it depends on what the number is like question 1 was easier but the others were not easy.

11. Do you know of other methods you could have used to answer **question 5 in Section B**? If you do, please write it down.

No other method.

12. Please rate which section you found the easiest by rating it from 1-3. 1 being the most difficult, 3 being the easiest.

Table 17.4

Section A	Section B	Section C
2	3	1

13. Which fraction calculations in **Sections A** did you find the most difficult to do, and why? Rate the questions from the most difficult to the easiest using the numbers 1-5, 1 being most difficult.



Table 17.5

Question 1	Question 2	Question 3	Question 4	Question 5
5	4	3	1	2

Please explain why you felt certain questions were so difficult and why you found the others to be easier.

Question 4 was difficult was because all of the fractions did not have the same denominator and the last fraction had a big denominator so I found it a bit confusing to see what LCD to use. If it was two fractions it would be easier to find the LCD but because there were 3 it made it more difficult to see.

14. Do you think the textbooks you use has enough exercises for you to practice doing fraction operations in order to understand it better? Please explain your answer with an example of why you have said yes or no.

I think it does have enough, but when it comes to explaining in the textbook it doesn't explain in it the correct or easy way for me to understand, it explains how to do it for itself and not for children to understand.

15. Do you understand the different terms used when working with fractions? For instance, what is the denominator, numerator, an improper fraction, mixed fraction and the line between the numerator and denominator? Please write down any fraction and label it with the headings provided in this question.

$\frac{5}{10}$	Numerator	$\frac{5}{3}$	Improper fraction	$4\frac{1}{3}$	Mixed fraction
	Denominator				

16. What methods do you think your teacher could use to better your understanding of fraction operations in:

a) Addition _____

My teacher could explain how to convert the number into a mixed fraction.

b) Subtraction _____



How to subtract big numbers.

c) Multiplication _____

Make it more specific to my understanding.

Appendix 18: Raw data interview – John Deer

Interview questions – Teacher

1. Biographical Information

Table 18

<u>Pseudo name:</u>	1. John Deer	2.	3.
<u>Years of teaching</u>	12		
<u>Position at your school</u>	Deputy Principal		
<u>Academic qualifications</u>	Bachelor of Education		
<u>Age group</u>	Group 2		

Age Groups: Group 1 (23-30), Group 2 (31-40), Group 3 (41-50), Group 4 (51-65)

2. What strengths do learners present when calculating like-fractions, fractions with same denominators in

- a) Addition?
- b) Subtraction?
- c) Multiplication?

With the same denominators it is easier for them, you can show them pictures or a physical demonstration where you would cut a cake or a pie where they can physically see it cut into a number of pieces and adding it to another pie or cake cut into the same number of pieces and they will be able to identify by just simply adding the pieces and seeing how many of the total they have.

3. Have you identified strengths presented by your learners when doing proper fractions/unlike-fractions, fractions with different denominators, in

- a) Addition?

b) Subtraction?

c) Multiplication?

When they have to add fractions with different denominators it becomes more challenging. No strengths as they really battle with it. I wouldn't say they have many strengths it is not an easy concept for them to work with because of the fact that it is a foreign concept, they don't see pieces of a whole, they just see wholes added or subtracted because in their whole school life they have been conditioned like that. There would be strengths if they know their times tables well. If they know their tables well then basically they are finding and LCD then all you are doing is using tables. So as I was saying, if they know their times tables they have a distinct advantage in calculating the LCD because calculating LCD is knowledge of times tables. With multiplications strengths would be if learners know their multiplications and division times tables, yes.

4. Can you mention the strengths learners present when calculating improper fractions, fractions where the numerator is bigger than the denominator, in



a) Addition?

b) Subtraction?

c) Multiplication?

Once again, all fraction calculations would require them to know their times tables up to at least 12x tables. For instances, when you calculate improper fraction you still finding an LCD in calculating it, and in the end you still have to convert it back it to a mixed fraction which is once again division, finding out what is left over and then putting it over the original denominator. So once again, it is multiplication and division times tables. And I think that division and multiplication tables is the key to mathematics in every section that we are doing, whether it is decimal fractions or whether it is fractions, it is all based around your times tables and strong children in mathematics normally have good command of their times tables.

5. What strengths do your learners show when doing mixed fractions, where there are whole numbers and fractions, in

- a) Addition?
- b) Subtraction?
- c) Multiplication?

Well, when calculating a mixed fraction you could apply two methods. One you could convert the mixed fraction to an improper fraction and then find the LCD as per normal or you could add the whole numbers and then add the fractions but you are still having to find the LCD. So in other words doing a bond calculation then doing a fraction calculation where you find the LCD. Both of them are once again, as I have echoing in previous questions once again are all reliant on whether the child knows their times tables, what command they have over their multiplication and division. If they are strong at that then they will excel in this section, if they have been taught the correction, step by step on how to work out the problem. But with addition, subtraction or multiplication is all reliant on their ability to multiply and divide.

6. What challenges do the learners experience when calculating like-fractions, fractions with the same denominators, in



- a) Addition?
- b) Subtraction?
- c) Multiplication?

Addition and subtraction will obviously be easier because they don't have to find a LCD. So with like-fractions it is just simple bonds, so at Grade 6 or 7 level surely the child should know their bonds, if they don't, or if they are battling with addition and subtraction of like-fractions then that means there is an issue, there is some gaps because they can't do a bonds calculation.

With multiplication if their multiplication ability is poor then you got to cross multiply when multiplying like-fractions and if it is poor then obviously they would experience challenges in the fact that they can simply not compute the multiplication. And then at the end obviously, if you get an improper fraction you going to need to convert it back to a mixed number, this could be a problem for them if they do not have a good

command over their division so the challenges there are once again, are they able to multiply and are they able to divide.

With addition and subtraction as I've said before, it is simple bonds, it is very easy.

7. List the challenges your learners face when calculating unlike/proper fractions, fractions with different denominators, in

- a) Addition.
- b) Subtraction.
- c) Multiplication.

So if they have different denominators then obviously it becomes more difficult with addition because they have to find the LCD and once again, that would require knowledge of tables, multiplication and division structures.

With subtractions it is really the same linked with addition. it will be more difficult with same denominator but with different denominators it will just make it more complicated.



With multiplication the challenges would be very similar to what I said in question 6. It is all cross multiplying and it will all depend on their ability to multiply and divide.

8) What challenges do your learners present when calculating improper fractions, fractions where the numerator is bigger than the denominator, in

- a) Addition?
- b) Subtraction?
- c) Multiplication?

So basically as I've said, if they have issues with multiplication and division as per answer to the previous questions

9) Please name the challenges your learners show when calculating mixed fractions, whole numbers and fractions, in

- a) Addition.

b) Subtraction.

c) Multiplication.

I think it's the fact that they don't see it as a whole number, they see it as something foreign so the fact that it has a whole number and a fraction with a denominator they battle in terms of converting it, they don't see that it is easier to convert it into an improper fraction and then look for the LCD.

In addition they don't sometimes separate it, in addition and subtraction, they should separate the whole numbers, separate the fractions and at the end add the whole numbers and fractions back together with simplification if they need to do so.

With subtraction, it sometimes becomes a problem is when they adding or subtracting mixed numbers is that if the fraction that is left over is smaller than the other one they are subtracting, they don't sometimes realise they need to borrow from the whole number and bring it across and that often causes or proposes a challenge for them. So if the child is battling with that, I will propose the first method where they first rather convert the mixed number to an improper fraction and then solve it. if they have no problem with that then I will say go for the whole numbers and then go for the fractions and if they are able to recognise that the fraction on the left is smaller than the one on the right then they need to borrow, for example if the denominator was 8, then they would need to take 8 over 8 ($\frac{8}{8}$) as a whole and bring it across.

Multiplication would be the same, they would need to convert it to an improper fraction they would need to recognise that it is one number and there they would definitely need to convert it to an improper fraction before the cross multiply. You can't cross multiply when it is in mixed number form. So the challenges are if their multiplications are weak they are going to have problems in converting.

10. As a Mathematics teacher, what are your specific strengths in teaching the fraction operation of addition? Refer to Section A of the learners' questionnaire. What strategies would you use to teach those questions?

The first question is very easy, its simply bonds so $\frac{1}{4} + \frac{2}{4}$ obviously it is one piece plus two pieces is three pieces out of four. The children just have to realise that the denominator is indicating that a whole has been cut into four pieces and if they

recognise that one piece plus two pieces then we say we are back in sub A, just as I teach then, as we are doing simple bonds calculation. One piece plus two pieces is equal to three pieces.

Strength wise I think you need to be quite practical with the children, you can't leave them to do things or figure it out on their own, you got to show them step by step in a methodical approach how to solve the problem and that takes a lot of sweat on the board it takes, you practically showing them if you maybe want to have a fun lesson you could order some pizzas and cut them up and let them eat it at the end, they would really enjoy that, or a cake or something like that if you wanted to show them practically. But there are certainly some creative ideas you could come up with teaching that, but it is very simple the first and second sum in section A because the denominators are the same so it is simple bonds. The first one would be $1 + 2 = 3$, and the second one is $3 + 2 = 5$. That would be straight forward, I think anyone could probably teach that. Even a foundation phase teacher could teach that as there is no real complicity to teaching those two questions. I don't think you would need a specific strength in teaching adding or subtracting of fractions with like denominators



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There is two methods you could use to do questions 3-5. So if you are a teacher who is able to recognise to do both methods I think that will be a strength in itself because then you are offering the children two options. Some children would prefer to convert question 3 to an improper fraction by saying 4×4 is $16 + 1 = 17$ over 4 $\frac{17}{4} \pm 1 \times 3$ is $3 + 2 = 5$ over 3 $\frac{5}{3}$ and then find the LCD as they have normally been conditioned to do.

Convert the numerators according to what the LCD is and then solve it and then simplify it back into a whole number with simplification of the fraction if need be.

The other the approach, you could separate the wholes and the fractions, I could say $4 + 1 = 5$ and then I can say $\frac{1}{4} + \frac{2}{3}$ finding my LCD of 12 solving that as a fraction, finding the answer, simplifying and then adding it back to my 5 to get and whatever I would get if I had to solve it. So the strength of knowing both methods well, being able to teach the children both methods and then saying to the children you choose whichever one is easier for you and whichever one makes sense to you.

Previously as I've explained to you how knowing your times tables will assist in finding the LCD, for example; $\frac{4}{7} + \frac{3}{4}$ I need to know if $7 \times 4 = 28$ that is due to my times table, so I normally ask them to ask themselves, what is the lowest number that both the two numbers can go into equally, if they have good knowledge of tables then they can work it out quicker than the child who doesn't.

11. Name the various strengths you have when teaching the fraction operation of subtraction? Refer to Section B of the learners' questionnaire. How would you teach those questions?

I would explain it identically to them the way I've explained addition. first of all with the like-fractions it goes down to bonds $3 - 2 = 1$ piece left, to explain it how many pieces, I have 3 pieces of cake or lets get a cake and cut it into pieces, I have 3 pieces and you take 2 pieces from me and obviously I have one piece left.

With in terms of mixed numbers, once again there is two methodologies we could use, either convert it to an improper fraction and then minus the two via finding the LCD. Or minus the wholes then minus the fractions



In this case given, question 2, is quite an easy one because there is whole numbers and a fraction with like denominators. So then it would be $3 - 1 = 2$ and $\frac{5}{7} - \frac{2}{7}$ is $\frac{3}{7}$ add that to the whole number and your answer is $2\frac{3}{7}$ this is quite a nice one.

Question 3 is quite a nice one because now you bringing in the tough situation with subtraction where you are using the whole method approach. Where you taking the whole number and then the fractions. you've given $2\frac{2}{5} - \frac{4}{5}$. $2 - 1 = 1$ but $2 - \frac{4}{5}$ we cannot do this because we going to get an integer fraction which you don't want that case. So you know that there's the 1 whole left over so I'm going to borrow from that whole and I'm going to get $\frac{5}{5} + \frac{2}{5} = \frac{7}{5}$ $\frac{7}{5} - \frac{4}{5} = \frac{3}{5}$ this is a classic example of where a child needs to identify that the first fraction is smaller than the one I'm going to subtract from so I need to borrow. If they don't get the borrowing method correct then they must revert to the other method which is converting to improper fraction and finding a LCD.

12. List specific strategies you use to teach the fraction operation of multiplication. Using the questions from Section C of the learners' questionnaire to help you, illustrate the strategies you use.

How I teach them is I teach them a little phrase, divide by the bottom times by the top. That's just the computing way so half of 30, 30 divided by 2 is 15 and 15×1 is 15, so half of 30 is 15. That's just the easier way to do it. But a nice way to do it is, you can actually show them physically with counters or beads etc., half of 30 is 15. Then we separate by practically moving the beads around and seeing how much half of it is. But I definitely found that the little catch phrase 'divide by the bottom, times by the top' sticks with them and they are usual able to work out the 'of' sum from that.

Question 2, 25 is not divisible by 4, so in this case maybe I would change that to an equivalent fraction to get it to a number that is divisible by 25, not sure if that is possible. We can't divide 25 by 4 because there is going to be a remainder which will give you a decimal fraction. But you could actually find the decimal by continuing to use the box method of division, you could get it as a decimal then times decimal by the top.



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Looking at it off hand Grade 7s know how to find the decimals manually without using the calculator so my children would know that method in order to solve that. Grade 6s do decimal fractions so they should be familiar working out via division getting a decimal fraction from that.

I could change it to a decimal and say 0.75 of 25, which will be 75 over $100 \frac{75}{100}$ of 25, $25 \div 100 = 0.25 \times 75$ would be easier enough to solve. If I had to solve this specific sum, this is the method I would use.

13. When teaching fraction operation of addition, what challenges do you find difficult to explain, and why?

As I've said before, children don't view a fraction as a piece of a whole, they are conditioned to add only wholes, subtract, multiply and divide wholes, so that's where the challenge of the LCD comes into play, sometimes what children want to do, they simply want to add the tops and add the bottoms which is not the correct way of doing things, that's only when you multiplying. It is challenging for them because in one way

you are saying to them when you multiplying you must multiply the tops and the bottoms right, now when you teaching them addition and subtraction they mustn't do that they must find a LCD, so that obviously throws them into confusion and then furthermore, is when you go into division of fractions you got to invert and multiply so that throws them off even more. So for addition and subtraction is the same method, for your multiplication you are crossing and for your division you are inverting and multiplying so it is quite confusing for them to understand the various operations and how they operate.

So the fact that it is so complex, and if it is not second nature to them then they are going to battle with that because there's so many different ways for the different operations, there's a different method they have to apply every time.

14. Do you face challenges when teaching fraction operations of subtraction and multiplication? Please elaborate your answer using examples for each operation.

When I'm telling the children when you plus or minus you have to find the LCD because you can't just plus the tops and the bottoms that wouldn't work, so now they trying to find the correlation between the two then it throws them even into more turmoil when it comes to division.



Addition and subtraction are hand in hand, so finding the LCD. Same challenges I face as when I teach addition.

15.1 In your opinion, please elaborate your thoughts on whether or not you think learners struggle to do like-fractions in

- a) Addition.
- b) Subtraction.
- c) Multiplication.

It will definitely be easier for them to understand as I've said in previous questions, they are adding like a bonds sums. With multiplication maybe slightly more difficult because they are multiplying on the top and on the bottom and then they have to simplify so because the method changes in the multiplication maybe it is a bit more difficult. With addition and subtraction, If you get them into the concept of showing

them practically, for instance a piece cut into four and you are adding pieces of that four, they seem to get it easier.

15.2 In your opinion, please elaborate your thoughts on whether or not you think learners struggle to do proper fractions in

- a) Addition.
- b) Subtraction.
- c) Multiplication.

they would struggle more with addition and subtraction more now because they have to find the LCD and then your multiplication and division concepts come in to find that LCD so it will make it more difficult. So yes, it will be slightly more difficult because they have to understand the multiplication and division principals.

15.3 In your opinion, please elaborate your thoughts on whether or not you think learners struggle to do improper fractions in

- a) Addition.



- b) Subtraction.
- c) Multiplication.

If the denominators are the same then it will be easier, if they are different then my answer will be the same as they would need to find the LCD.

15.4 In your opinion, please elaborate your thoughts on whether or not you think learners struggle to do mixed fractions in

- a) Addition.
- b) Subtraction.
- c) Multiplication.

Shown examples when answered questions from the questionnaire from section A and B.

16. How do you address the challenges learners struggle with when it comes to learning fraction operations, referring to questions

- a) 15.1?
- b) 15.2?
- c) 15.3?
- d) 15.4?

The only way to get better in mathematics is the same like cricket and rugby or skill, mathematics is a skill, the only way to get better is to practice. So in the beginning we will do thousands and thousands of examples, practically as much as teacher assisted as we can in class so that they can see how it is physically done so they can hang on to those concepts. But as I've said, practice makes perfect, mathematics you have to drill that skill, drill the methodology, the more examples you give the children the better they sort of become. So with those challenges, I will go through it with them, I will model a lot of techniques, I will model a lot of methodology and do thousands and thousands of examples with them until I'm comfortable with them acquiring the skill to go do a couple on their own.



17. What strategies could you use to address the specific challenges of teaching fraction operations identified in questions

- a) 15.1?
- b) 15.2?
- c) 15.3?
- d) 15.4?

At my specific school where, I'm at we've identified that there are certain challenges in mathematics and fractions would definitely be one of them and we have set up an academy which takes place on weekends and in holidays, holiday academy parents do pay for it then teachers would identify target areas and then give them extra tuition and smaller scenarios where they can get more help. We've previously have had family mathematics seminars where we've actually taught the parents so the parents

can help the child at home. Especially with our parents socio economic backgrounds at our school, the parents are unable to their child because their child actually has a higher educational background than them. That's obviously a problem and if we can help bridge that gap. I believe if a parent is sitting with their child every day helping them, the child soars.

18. Do you think there are enough practical exercises and time allocated for learners to understand the concept of fraction operations of addition, subtraction and multiplication? Please elaborate your answer with an example for validation.

I think there are plenty of practical exercises. I actually feel that the CAPS curriculum is far too condensed. I speak under correction, we've just covered fractions just after 2nd term had started. As a Grade 7 teacher, there are about 25 exercises with roughly 30 examples in each. So it's a lot of work to get through and the children have a lot of opportunity to practice the methodology.

Time allocated, I think we could have more time even for maths as it is we struggle to complete, the fact that exams are moved forward, obviously in a model C setting where we have to get marks out to the children before the holidays. We are looking at a 20 week term but we are writing exams after week 15. So the fact is that we don't completed everything till June. The CAPS curriculum dictates that we should write the exams during the holidays because it is a 20 week programme. Its actually a 40 weeks programme, but 20 weeks up till June exams. We are struggling because we have to write our exams earlier because of departmental deadlines. So we have to rush a lot of the work to get through but there is plenty of examples and enough for the children to know what they are doing.

19. Look at the following sum: $6\frac{2}{4} + 4\frac{4}{5} = ?$ How could you possibly teach this sum? If

you have more than one method please write them all done. Do you teach from the unknown to the known? Or when do you start?

There are two methods to teach this sum. See below;

$$6 \frac{2}{4} + 4 \frac{4}{5}$$

$$= 6 + 4$$

$$= 10$$

$$\frac{2 \times 5}{4 \times 5} + \frac{4 \times 4}{5 \times 4} = \frac{10 + 16}{20}$$

$$= \frac{26}{20}$$

$$= 1 \frac{6 \div 2}{20 \div 2}$$

$$= 1 \frac{3}{10}$$

$$10 + 1 \frac{3}{10} = 11 \frac{3}{10}$$



$$6 \frac{2}{4} + 4 \frac{4}{5}$$

$$= \frac{26 \times 5}{4 \times 5} + \frac{24 \times 4}{5 \times 4} = \frac{130 + 96}{20}$$

$$\begin{array}{r} 3 \\ 26 \\ \times 5 \\ \hline 130 \end{array} \quad \begin{array}{r} 1 \\ 24 \\ \times 4 \\ \hline 96 \end{array}$$

$$= 130$$

$$+ 96$$

$$\hline 226$$

$$= \frac{226}{20} = 11 \frac{6 \div 2}{20 \div 2}$$

$$= 11 \frac{3}{10} \rightarrow$$

The whole method approach is the one I prefer because there's not much calculating in it.

Teach from the known to the unknown, interesting aspect in schools, teachers should teach cross grades teachers teaching maths to Grade 6 and Gr 7 to specific classes so they know what they children are learning the year before and currently and they can consolidate. Teachers teaching in 1 Grade won't know what the base knowledge is and they can't elaborate on that because they don't know and they fall out of sink on what's going on. If I have that knowledge and I know what's going on then have a definite advantage when I teaching children because then I can always teach from the known to the unknown and build on it, because normally how it works, knowledge is always established and as the Grades go higher, the concepts just become more complex and more condensed.



Appendix 19: Raw data interview – Kim Lee

Interview questions – Teacher

1. Biographical Information

Table 19

<u>Pseudo name:</u>	1.	2. Kim Lee	3.
<u>Years of teaching</u>		5	
<u>Position at your school</u>		Grade 6 teacher	
<u>Academic qualifications</u>		Honours in Educational Management	
<u>Age group</u>		Group 1	

Age Groups: Group 1 (23-30), Group 2 (31-40), Group 3 (41-50), Group 4 (51-65)

2. What strengths do learners present when calculating like-fractions, fractions with same denominators in



a) Addition?

b) Subtraction?

c) Multiplication?

They are able to do it when it is like-fractions when the denominators are the same. To add is easy. Your denominator is usually small so they are able to add a quarter plus two thirds so it's simple enough for them. And with subtraction, it's their basic understanding so they find it easy to do addition and subtraction. Once you get to multiplication it becomes a problem because they don't know their tables. When it is the 2, 3 or 5 times tables it's fine, but when it gets to the difficult ones, then no I don't think they have strengths. When it gets to 6 times table and up they struggle cause they don't know their times tables.

3. Have you identified strengths presented by your learners when doing proper fractions/unlike-fractions, fractions with different denominators, in

a) Addition?

b) Subtraction?

c) Multiplication?

I feel like when the denominators are different they don't know how to find a common denominator and they don't know how to find the lowest common denominator so even if they understand they need to make the denominators the same they don't identify that sometimes one of the existing denominators are already given and most times will be the lowest common denominator. What I do find, they add and subtract the denominators so I don't know if there is strengths when you have different denominators with in all operations because they don't know their times tables.

4. Can you mention the strengths learners present when calculating improper fractions, fractions where the numerator is bigger than the denominator, in

a) Addition?

b) Subtraction?

c) Multiplication?



No, I don't think they understand improper fractions, firstly they wouldn't know how to convert it back to a normal fraction or a mixed fraction, I think they would be able to add the improper fractions but when you ask them to convert it to a mixed fraction, I think using different calculations, they would find it difficult in order to do so. But with additions and subtractions, they can just add the numerators, I think they would do that with ease. I just feel like if you would tell them to put it back into a mixed fraction they wouldn't be able to, they would find great difficulty. So with like-fractions they have strength in that, but once you work with different types of fractions and different denominators, and improper fractions, I think that's where it loses them.

Multiplication isn't really taught in the lower grades. Its normally just in addition and subtractions, so come to multiplication in Grade 6 they would find it difficult, because they already don't know the basics of normal fractions and now you throw in multiplication where on its own they aren't really strong at and now they must do it with fractions which is also difficult concept to understand.

5. What strengths do your learners show when doing mixed fractions, where there are whole numbers and fractions, in

a) Addition?

b) Subtraction?

c) Multiplication?

With addition, I feel like they are okay with that only some learners will grasp the concept. There are different ways of doing it, so they will either add the whole numbers first or they will create it into an improper fraction then do it and convert it back. Addition I feel like they are fine.

With subtraction, it becomes a problem when your first mixed fraction is smaller than your second one and then they don't know how to borrow that's when subtraction becomes a problem, when they don't know how to borrow, so I feel like there is no strength with subtraction because they don't understand the concept of borrowing from your whole number and when your denominator is 4, it is $\frac{4}{4}$ they don't understand that concept. So with addition I feel like there is strength, with subtraction there is no strength because it is simple for them how to add, the add the whole numbers and then adding the numerators, basic concepts of adding since Grade 1, that's why I'm saying they will get it, but with subtraction not really.

With multiplication they have not had much practice so I don't think they have any strength with it.

6. What challenges do the learners experience when calculating like-fractions, fractions with the same denominators, in

a) Addition?

b) Subtraction?

c) Multiplication?

I don't think there is any challenge when it comes to addition because it is straight forward, they are able to add.

Subtraction same thing, it is the same thing because it's the basic concepts they've been doing since Grade 1. Additions and subtraction are fine.

When it comes to multiplication there's a challenge because they don't know their tables. And if you are speaking about 'of' sums, they don't know you must divide the bottom and then multiply by the top. Or if they turn the whole number and put it over 1, they don't understand that concept, because there's different ways to do this multiplication one. So the more advanced learners will probably put the whole number over one and multiply across. Whereas if they do an 'of' sum they don't know you must divide your denominator by your whole number then multiply that answer by your numerator. Some learners will get it, if the number can divide into it fully, but if there is a remainder they drop the remainder and just carry on. So if a number goes into a whole number and it's 33.3 times, they don't know to take 33.3 and multiply it by the numerator. They get confused with that part, I think simple when it goes in without remainders they are able to do it.

7. List the challenges your learners face when calculating unlike/proper fractions, fractions with different denominators, in



a) Addition.

b) Subtraction.

c) Multiplication.

So they are unable to know that they need to find the lowest common denominator, they are unable to identify that the existing denominators will be the lowest one like if it is 4 and 8, you can use 8 but they will go use 16 or 32 or another number. For instance if you look at $\frac{1}{4} + \frac{2}{3}$ now obviously none of the denominators are the LCD so

you will have to go find the denominator. Find the lowest common denominator is the difficulty and that is with subtraction and addition, they don't know how to do that. And like I said, what they would do is, like a child's mind they will just go and add the denominators so they will say $4 + 3 = 7$ and that will become the denominator, they won't know that you must go find a denominator which is 12, then ask 4 goes into 12 how many times and 3 goes into 12 how many times, they don't understand that

concept with both of them, so same with subtraction and addition they will say $4 - 3$ is 1.

There wouldn't be any challenges with 'of' sum because there will be no fraction on the other side of the 'of sum'. They will be saying $\frac{3}{4}$ of 200 oranges so then they are literally saying, 4 goes into 200 how many times, where as if there was another fraction on the other side of the 'of' sum then there is where the problem will come in.

8) What challenges do your learners present when calculating improper fractions, fractions where the numerator is bigger than the denominator, in

a) Addition?

b) Subtraction?

c) Multiplication?

Like I've said previously, they would be able to add the numerators but if you ask them to change it into a mixed they would have difficulty. If you talking about just improper fractions and you want your answer in improper. If the denominators are different they will have difficulty in finding the lowest common denominator because then they will just add the denominators and add the numerators. But if the denominators are the same and the numerator is bigger than the denominator then they would just add it and that's simple for them. But challenges come in when they have to convert it into a different fraction or if your denominators are different, this would be the same for subtraction, they will be able to subtract unless the first improper fraction is smaller than your 2nd one they wouldn't know how to borrow in that sense so that would be a problem.

In multiplication would be fine in 'of' sums by multiplying the bottoms and the tops. The only thing is if the number on top is big they would find it difficult to multiply because of their tables and then they would have to do another sum and that's where they might get lost if they don't know their times tables.

9) Please name the challenges your learners show when calculating mixed fractions, whole numbers and fractions, in:

a) Addition.

b) Subtraction.

c) Multiplication.

So they would be able to add their whole numbers fine and they would be able to add the numerators fine if the denominators are the same, so don't think there will be any challenges there. There would be a challenge with subtraction if they have to start borrowing that's where it becomes problematic because they are not used of that. So simple fractions they are able to do, but once they have to start converting into a different fraction or using a different denominator or borrowing that's where children start to have problems in fractions in general. So that's where subtraction would become a problem if they had to borrow, but if it was a straight forward sum they wouldn't have problems.

With multiplication of mixed fractions I don't think learners will know where to start with the whole multiplication just because they haven't had much practice with multiplication in fractions, they've done the basic 'of' fractions where it goes in fully, but when it comes to multiplication of fractions not much practice has been done because it's a new concept they only really start in the higher grades, it's not something that has been brought over from Grade 4, it's a new concept so it will take longer to grasp.

10. As a Mathematics teacher, what are your specific strengths in teaching the fraction operation of addition? Refer to Section A of the learners' questionnaire. What strategies would you use to teach those questions?

For question 3, I would make one big denominator and add whole numbers first, find the common denominator, then I first check if 4 can go into 3, no. Can 3 go into 4, no. When I ask can it go in, I mean can it go in fully without a remainder. Children will say 3 goes into 4 once, so it needs to go in fully. Then I need to ask what is the next lowest number that can go into both numbers. It is important for learners to identify that the steps are the same. Add whole numbers first, find the lowest common denominators, you could have 20 sums what they need to realise is that the steps are the same, it doesn't matter what fractions get thrown at you, the steps are the same. Once they have established the order of the steps then they are able to do their sums. A challenge would be here, for a child to remember all these steps, that's where they get lost along

the way, so they might do the first few steps that they remember, there after they will forget steps and that's where the problem occurs.

11. Name the various strengths you have when teaching the fraction operation of subtraction? Refer to Section B of the learners' questionnaire. How would you teach those questions?

The same things would apply as addition so I think, before you get into all this, you need to write down all of your steps like when you divide you have your daddy, mummy, sister, brother so they are able to tick off each and every single step so they what they are doing. Something with fractions you should list the steps and every time they are able to read and tick so they get it because it's the same routine the whole time so whether you are adding or subtracting you are doing the exact same steps. And so you need to make sure you write down the steps and the children are able to tick it off just as they do with daddy, mummy, sister, brother.

Some people convert it into an improper fraction then subtract and then convert back, the reason I do this is so that you keep the steps the same. Because if you teach them all these different ways I think they will  getting confused. My strength would be sticking to one method constantly so that learners can understand. Basically keep the methods the same not to confuse them as fractions is confusing already, so keep it simple and write down the steps so they are able to tick off along the way.

12. List specific strategies you use to teach the fraction operation of multiplication. Using the questions from Section C of the learners' questionnaire to help you, illustrate the strategies you use.

The learners just learned the concept of converting the remainders in division to a decimal. It becomes problematic when your denominator cannot divide fully into your whole number and you are left with a remainder. To be honest, most of the textbooks questions don't have this problem where you cannot easily divide most of them go in fully. You won't find this in the textbook, this will probably be in a problem/word sum so you won't have these straight forward one and unlikely to get a question like no. 2, this would be a higher order question. So when it comes to sums like this, I don't feel to confident in teaching it.

13. When teaching fraction operation of addition, what challenges do you find difficult to explain, and why?

My challenge would be in trying to get them to understand how all 3 numbers need to have a common denominator (question 5, section A), to get them to understand how to go from one point to another to find the next number because there's lots of fractions involved, so trying to get them to understand how to find the lowest common denominator that is a challenge.

14. Do you face challenges when teaching fraction operations of subtraction and multiplication? Please elaborate your answer using examples for each operation.

When it comes to subtraction and you have to start borrowing that becomes a challenge to make them understand this is what we are borrowing, so if your denominator is 10, when you borrow from the whole it becomes $\frac{10}{10}$ and then you add it to the first fraction before subtracting, to make them understand that borrowing, that's an issue but if it straight forward subtraction, the issue would come in when you have to explain to them how to borrow. It becomes difficult as the teacher, you know the answers in your head, you know how to work it out, you can do it, but then to try teach it to children who have no clue what's happening, to teach it from scratch that becomes the issue.

So to teach multiplications when you are left with remainders, becomes difficult because you must convert to decimals and I didn't know that till recently.

15.1 In your opinion, please elaborate your thoughts on whether or not you think learners struggle to do like-fractions in

- a) Addition.
- b) Subtraction.
- c) Multiplication.

Learners don't struggle to add or subtract like-fractions because it is basics, something they have been doing.

Multiplication, if they don't know how to multiply or do their sum they will get lost, because lots of them, along the working out there's mistakes, within the adding, within the multiplication, or they forget those magic zeros to add in, that's where the problems might come in.

15.2 In your opinion, please elaborate your thoughts on whether or not you think learners struggle to do proper fractions in

- a) Addition.
- b) Subtraction.
- c) Multiplication.

Yes some of them will struggle, your stronger learners will know okay find your common denominator, some will add the denominators. So the stronger ones would find the lowest common denominator whereas, your weaker ones will just add the denominator and the numerators. Same with subtraction, stronger ones will get the concept, the weaker ones won't.



They don't know their tables so they will struggle with multiplication.

15.3 In your opinion, please elaborate your thoughts on whether or not you think learners struggle to do improper fractions in

- a) Addition.
- b) Subtraction.
- c) Multiplication.

They would just add or subtract the numerators and denominators in mixed and improper fractions. with multiplication they also struggle, because it is more advanced, more steps involved. So your clever children would grasp the concept, where your weaker ones will switch off immediately when they see all the steps.

15.4 In your opinion, please elaborate your thoughts on whether or not you think learners struggle to do mixed fractions in

- a) Addition.

b) Subtraction.

c) Multiplication.

They would just add or subtract the numerators and denominators in mixed and improper fractions. with multiplication they also struggle, because it is more advanced, more steps involved. So your clever children would grasp the concept, where your weaker ones will switch off immediately when they see all the steps.

16. How do you address the challenges learners struggle with when it comes to learning fraction operations, referring to questions

a) 15.1?

b) 15.2?

c) 15.3?

d) 15.4?

So what I would do is first teach it, do a few examples on the board give them the work, in maths I allow them to work with a partner because children are able to explain better than a teacher would, two children speaking the same language. What I normally do, if a child comes to me with a problem, I stop and go straight to the board and I explain, often than not you will have 2 or 3 children coming to ask the same question then you keep explaining. So I re-explain it on the board to everyone. If another child comes to me because they don't understand, I go back to the board, so it's a constant showing them the steps, refreshing and allowing them to work with a peer so they are able to learn from one another.



17. What strategies could you use to address the specific challenges of teaching fraction operations identified in questions

a) 15.1?

b) 15.2?

c) 15.3?

d) 15.4?

So like I said, keep your steps the same, it would be good to write down the steps so they can refer to it like the daddy, mummy, sister, brother, that's in their head that they would automatically remember. So it is repetition, same steps used, it's something you need to get into the children's heads when they do fractions they have the steps already and they are able to tick off.

18. Do you think there are enough practical exercises and time allocated for learners to understand the concept of fraction operations of addition, subtraction and multiplication? Please elaborate your answer with an example for validation.

I don't think there's enough time. I feel like there is so many concepts that need to be covered in a term, for instance you got 8 concepts, so you basically only have 1 week per concept because that's 8 weeks then term is almost at an end. So you only really have a week to get through the work because obviously we need to complete the syllabus, we tend to not exceed that time, so if a child does battle you need to move on otherwise you are going to fall behind and so does that learner. So I don't feel like there is enough time to spend on concepts. I do feel like the CAPS document needs to be restructured so when it comes to these more difficult concepts which learners don't understand, I feel more time should be allocated to that and then when it comes to your place values etcetera, less time should be allocated to that and less activities for that as well, there shouldn't be so many much repetition of that every term because if you look at the textbook there will be place values, addition and subtraction in all four terms. Whereas, with fractions will only pop in in term 1 and term 3 by that time you will have to reteach it. and practical examples, there's not much, like between 5-10 question in a limited number of exercises and that's it. everything else you must go find your own resources if you are going to add, and like I've said there's no time to still give them more. It's difficult to give them homework because more than often than not parents won't be able to help and then they get it wrong and come back to school and say they didn't understand it. So there is definitely not enough time for such a difficult concept and it's not frequent enough for them to actually practice.

19. Look at the following sum: $6\frac{2}{4} + 4\frac{4}{5} = ?$ How could you possibly teach this sum? If you have more than one method please write them all done. Do you teach from the unknown to the known? Or when do you start?

①. $6\frac{2}{4} + 4\frac{4}{5}$

$10 \frac{10 + 16}{20} = 10 \frac{26}{20}$

$11 \frac{26}{20} - \frac{20}{20} = 11 \frac{6}{20} \div 2 = 11 \frac{3}{10}$

$6 \div 2 = 3$
 $20 \div 2 = 10$

$11 \frac{3}{10}$

$6 \div 4 = 1 \text{ of } 2$

I use this one method because I try to keep the method the same because if you can see there's already so many steps involved because you have to first find a common denominator, then it's an improper fraction then you must convert to a mixed number then you must simplify, so there is 4 different things they are asking you to do in this, so this is really confusing for the children. And now to throw in different types of methods would confuse them even more. But what I do is, if parents show them a different method and it gets the right answer I say they can use that method if they understand it. For example; I teach the divide and multiply of the whole number, whereas a child's father showed the converting the whole number to a fraction method and I told the child he must use that method if he understands it, I never tell the children they can't use other methods, because I always tell them there is more than one way to get to the answer. When I teach it, I try to keep the method the same so that it doesn't confuse them because there's lots of steps involved.

Appendix 20: Raw data interview – Maverick

Interview questions – Teacher

1. Biographical Information

Table 20

<u>Pseudo name:</u>	1.	2.	3. Maverick
<u>Years of teaching</u>			3
<u>Position at your school</u>			Grade 5
<u>Academic qualifications</u>			Bachelor of Education Degree
<u>Age group</u>			Group 1

Age Groups: Group 1 (23-30), Group 2 (31-40), Group 3 (41-50), Group 4 (51-65)

2. What strengths do learners present when calculating like-fractions, fractions with same denominators in



a) Addition?

b) Subtraction?

c) Multiplication?

It's definitely easier for the learners when the denominators are the same, in Grade 5 the challenge comes in when the denominators are different.

Its basically addition and subtraction in Grade 5, once they get to Gr6 then they multiply and divide.

With multiplying you have to find the lowest common denominator and they have to know the difference between the different types of fractions. They start with that in Grade 5, so Gr4 they have the basic concepts of fractions with common fractions and in Gr5 we start building onto that foundation.

3. Have you identified strengths presented by your learners when doing proper fractions/unlike-fractions, fractions with different denominators, in

a) Addition?

b) Subtraction?

c) Multiplication?

See now that's where the problem arises with many of the learners, some children do understand the concept of finding the LCD, other children find that to be a challenge so our mission in Gr 5, personally, is to get them to understand how we find the lowest common denominator. So that in actuality that is one of the problems we face. The strengths will depend on the learners as some learners have more strengths than the weaker ones. But with your stronger children they grasp the concept a lot quicker, but with your weaker children find it as a challenge and it is something we need to focus on.

4. Can you mention the strengths learners present when calculating improper fractions, fractions where the numerator is bigger than the denominator, in



a) Addition?

b) Subtraction?

c) Multiplication?

In Gr 5 when we deal with improper fractions, what I like to do is relate to the learners, so I like to use physical, tangible things in the classroom, such as; using a pizza and then get them to understand a part of a whole. So when we show them an improper fraction it's a whole pizza with extra pieces so because they are dealing with things they can relate with and they can see it, it is a visual stimuli in the classroom that helps them understand. If I don't bring in any visual stimuli then learners looked at me gazed not sure what they are doing, but the minute I bring in examples they can relate to then the penny drops.

5. What strengths do your learners show when doing mixed fractions, where there are whole numbers and fractions, in

- a) Addition?
- b) Subtraction?
- c) Multiplication?

It comes down to whether or not learners know their basic times tables and if they understand their basic times tables it is easier for them. If they are weaker in their times tables then they will definitely find it more difficult.

6. What challenges do the learners experience when calculating like-fractions, fractions with the same denominators, in

- a) Addition?
- b) Subtraction?
- c) Multiplication?

In my personal classroom, when the denominators are the same they find no challenge because they simply add the numerators together and if the numerator becomes bigger than the denominator then they understand that it is now an improper fraction and they understand to change the improper to a mixed fraction. So if the denominators are the same they are very comfortable with the sum, as soon it's different that's when the children start getting a bit hesitant.



With multiplication because they cross multiply I don't think they find any challenges, its mainly addition and subtraction where denominators must be the same.

7. List the challenges your learners face when calculating unlike/proper fractions, fractions with different denominators, in

- a) Addition.
- b) Subtraction.
- c) Multiplication.

Feel that with multiplication there is no problem because learners must cross multiply.

Learners struggle with addition and subtraction when denominators are not the same because then they don't know what to do.

8) What challenges do your learners present when calculating improper fractions, fractions where the numerator is bigger than the denominator, in

- a) Addition?
- b) Subtraction?
- c) Multiplication?

The main thing they must recognise the numerator is bigger than the denominator and if they can add the numerators together, the denominators must still stay the same, then it is still an improper fraction and if they must convert back to a mixed fraction, so I don't think they struggle with that in Grade 5 with all three operations. However, if it is converting improper fractions to mixed fractions some learners find it a challenge how to do that.

9) Please name the challenges your learners show when calculating mixed fractions, whole numbers and fractions, in



- a) Addition.
- b) Subtraction.
- c) Multiplication.

Now that they have to deal with whole numbers and parts of a whole you put them together, in actuality it is a simple thing, but your learners in Gr 5 become overwhelmed by it. There are different methods we can use with improper and proper fractions. So there's the easier method where they add the whole numbers then deal with the proper fractions together. Or, they just have to convert the mixed fractions to improper fractions and if they understand the steps in order to complete the steps they will be fine. It depends on whether or not the teachers have taught them the various methods because there is more than one method to teach this to the learners. I feel, if a teacher can touch on these steps in the beginning then the learners can sort it out.

10. As a Mathematics teacher, what are your specific strengths in teaching the fraction operation of addition? Refer to Section A of the learners' questionnaire. What strategies would you use to teach those questions?

My strengths as a teacher I am able to relate the content to the learners with knowledge they already know, for example; I can bring the visual stimuli into the classroom so the learners can physically see for themselves what is happening so it is not just black ink on paper, so they can physically see what is happening.

Section A, question 1. The first thing I tell the children is to see that the denominators are the same, so if I had to cut an apple into quarters because you are dealing with quarters in the sum, then I'll take one piece out, then I take two pieces out, then I'll say if I put the one piece and two pieces together, how much will I have? The learners will answer and say I have 3 pieces because they can do the basic bonds, then I will say 3 over the whole which is 4 so the fraction becomes $\frac{3}{4}$.

Question 3. The first thing I would do with them is check if the denominators are the same, if they are different I will change  into improper fractions. I will show them how to convert the mixed number into improper fractions then once we have done that they can see the denominators are different and we have to find the LCD. Basically we have to find the LCD where there is no remainder, once the learners have the knowledge and steps to find the LCD then they can do it step by step.

How to find the LCD, So basically I usually get them to do is to times the denominators together because in that case you usually find that they can go into each other without a remainder. In some cases it can go lower then we go lower.

11. Name the various strengths you have when teaching the fraction operation of subtraction? Refer to Section B of the learners' questionnaire. How would you teach those questions?

The first thing I do is look at the denominators and they are the same, it becomes a basic bonds sum, what is $3 - 2 = 1$.

In question 3, in this case you can't minus 4 from a 2, so we need to teach our learners how to change the mixed fractions into improper fractions. Learners can't take 4 away from 2, so they need to understand how to borrow from a whole number. And how they

borrow they change into an improper fraction so times denominator by the whole number then add the numerator to the answer. We need to get learners to understand the concept of borrowing from the whole with subtraction sums.

12. List specific strategies you use to teach the fraction operation of multiplication. Using the questions from Section C of the learners' questionnaire to help you, illustrate the strategies you use.

So the first question, I get the learners to ask themselves what they think half of 30 is, before using the method. They can understand the concept of halving a number and in their minds they are able to half 30 which is 15. Once they get the idea, I get them to work out the sum using the method and they can see their understanding is in line with the method. So basically 2 goes into 30, 15 times the learners will then see yes, half of 30 is 15 and the method to work it out will give us the same answer.

Now the 2nd questions gets more challenging because we are going away from the half which learners are familiar with and the numerator is increasing as well, so doing it straight, the 4 can't go into 25 straight so learners need to learn how to work with decimals. This is the method I would use to teach a sum like this.



13. When teaching fraction operation of addition, what challenges do you find difficult to explain, and why?

So obviously with fractions you want to get learners to grasp the concept of fractions being a part of a whole, sometimes the challenges learners faces is why can sometimes the denominators can mean the same thing for example $\frac{2}{4}$ is equivalent to $\frac{4}{8}$ so those simple things are a challenge in my classroom, learners also battle with simplifying fractions in my classroom. The main thing which helps me, as I've echoed before is to bring physical, tangible, visual stimuli things to class so they can physically see what is happening, because without that it does take time, especially Gr 5s, because fractions is a fairly new concept which they've slightly touched on in Grade 4 and if we can get more visual stimuli in the classrooms it will be easier for the Grade 5s to understand.

14. Do you face challenges when teaching fraction operations of subtraction and multiplication? Please elaborate your answer using examples for each operation.

With some learners it is difficult to get them to understand how to borrow from the whole and they will minus the whole number first then they get stuck, for instance you have to minus three quarters from a quarter, they can't do this so they leave the sum out because they don't know how to take away so it becomes a negative number. So we have to get them to be fluently and become comfortable with changing to improper from mixed fractions and verse vice. So the main thing here will be for teachers to do here is to ensure the learners know how to change mixed numbers into improper fractions, especially when there's challenges with subtracting fractions.

Multiplication of fractions is not something we do in Grade 5. I think learners will get confused between the methodologies between addition and subtraction and the methodology used for multiplication and division, so they understand the method of adding and subtracting and all of a sudden they move to the two new operations which they need to use the new methodology. So I think that could be an issue they are facing. So the learners need to be aware of the inverse problems being addition and subtraction go hand in hand and multiplication and division go hand in hand. So I think that would be problem, getting confused between the two methods and secondly, I think another problem where the learners will face if they are not proficient in their times tables and they don't know their bonds that well, this will definitely implicate their ability to solve the problem.

15.1 In your opinion, please elaborate your thoughts on whether or not you think learners struggle to do like-fractions in

- a) Addition.
- b) Subtraction.
- c) Multiplication.

When denominators are the same learners are fine subtracting or adding. If learners don't know their tables and basic bonds they could struggle with multiplication.

15.2 In your opinion, please elaborate your thoughts on whether or not you think learners struggle to do proper fractions in

- a) Addition.

b) Subtraction.

c) Multiplication.

In addition, with the various fractions, the main thing they have to understand is they have to work with the same denominators, now when they aren't the same, depending on the learner, they tend to get stuck. So the main issue would be the learners must know how to use the LCD to work out the additions. Once they are the same, it is simple step by step just adding the numerators that is basic bonds.

With multiplication it is cross multiplying so with multiplication I think it is simpler compared to addition and subtraction, in my opinion. They must just be strong in their times tables.

****Subtraction example;** it is easier for learners to minus the whole numbers then they get stuck when the first fraction numerator is smaller than the 2nd. In their minds they don't know how to take 3 away from 1, so we have to encourage them to change the mixed number into an improper fraction as I've echoed before. So they need to know how to change to an improper then  back to a mixed number again. So it's about them needing to know when to apply changing the mixed fraction to an improper, this is not always the case because sometimes they can simply minus the 1 from the 3. So they just need to know when to use what method.

15.3 In your opinion, please elaborate your thoughts on whether or not you think learners struggle to do improper fractions in

a) Addition.

b) Subtraction.

c) Multiplication.

As echoed above.

15.4 In your opinion, please elaborate your thoughts on whether or not you think learners struggle to do mixed fractions in

a) Addition.

b) Subtraction.

c) Multiplication.

As I've said in the previous 15.1 and 15.2.

16. How do you address the challenges learners struggle with when it comes to learning fraction operations, referring to questions

a) 15.1?

b) 15.2?

c) 15.3?

d) 15.4?

So the first thing I do before I address the challenges, I must make sure their foundation is set, so if they are comfortable and proficient in the basics of fractions and they understand it fully then we are able to move on to the problems and address them individually. For example, if the learners struggle to change the mixed fractions to improper fractions then I know as a teacher to slow down when I get to this section and spend more time on this. So basically it comes down to them knowing the basic operations. So basically building their foundation before moving onto more complex challenges. So I'm not a fan of rote learning, repeat, repeat, repeat. But in mathematics, the more you repeat the better you become, so I really, and truly believe that teachers need to practise and practise maths and not just use examples from the textbook but stray away and use their own or even the learners examples. Then they can apply the formula to any situation and not only the examples in the textbook. So it is about using the methods and their knowledge. So I find some teacher tend to use examples just from the textbook which doesn't truly help learners the learners apply that knowledge in their everyday lives.

17. What strategies could you use to address the specific challenges of teaching fraction operations identified in questions

a) 15.1?

b) 15.2?

c) 15.3?

d) 15.4?

So the main theme of my questionnaire, which I'm going to echo today, is to bring visual stimuli into the classroom. If the learners can physically see and touch the physical things, they are able to see for themselves what is happening, working with tangible, like sweets, they can physically see what is happening. It is a challenge to some learners who find it difficult to just work from a textbook and copy and paste into their book. I believe we need to bring the fun back into maths, it is important. So that the main thing I can say, is to bring visual stimuli back into the classroom.

18. Do you think there are enough practical exercises and time allocated for learners to understand the concept of fraction operations of addition, subtraction and multiplication? Please elaborate your answer with an example for validation.

It comes down to the syllabus, we need to cover the syllabus and have set time for each topic we dealing with. You know children work at different levels and we've got inclusive education in our schools, we have strong learners and weak learners and we have to try and cater for both needs and you know. Some learners will pick up and grasp the concept quicker than others and where we are slowing down and helping the learners who find it more difficult they are becoming stagnant and bored and we have to keep them occupied with other worksheets before we can continue with the curriculum. So I find that it depends on how large your classroom is that there is enough time for fractions in the curriculum, but sometimes there could be too much time for sum learners and not enough for others, so it is a varied issue depends on the class sizes you have. If you have classes aimed at higher Grade learners I think the teachers would have suitable time for fractions and to go ahead. We are talking here which is frowned upon in schools but I personally believe streaming can aid learners. streaming will be putting learners of the same strengths in the same classrooms, it's all pro with inclusive education with all learners with different backgrounds are in one classroom, but with maths it will help putting the same strength learners together so it can help them fly, and for those who need extra help the teachers can slow down and work slower with the learners who need it. So it is difficult for one teacher with a class of 35 – 40 learners in a class and we have to address all their needs.

19. Look at the following sum: $6\frac{2}{4} + 4\frac{4}{5} = ?$ How could you possibly teach this sum? If you have more than one method please write them all done. Do you teach from the unknown to the known? Or when do you start?

I have briefly said before that there are different methods to work out sums like this.

Handwritten work showing two methods for adding mixed fractions:

Method 1 (Left):

$$6\frac{2}{4} + 4\frac{4}{5}$$

$$10\frac{10}{20} + 16\frac{16}{20}$$

$$= 26\frac{26}{20}$$

$$= 11\frac{6}{20}$$

$$= 11\frac{3}{10}$$

Method 2 (Right):

$$6\frac{2}{4} + 4\frac{4}{5}$$

$$= \frac{26 \times 5}{4 \times 5} + \frac{24 \times 4}{5 \times 4}$$

$$= \frac{130}{20} + \frac{96}{20}$$

$$= \frac{226}{20}$$

$$= 11\frac{6}{20}$$

$$= 11\frac{3}{10}$$

The University of Fort Hare logo is visible in the center of the work.

The first method will be when you add whole numbers together then work out the fraction. The denominators are not the same so we have to find the LCD. I timed the denominators by each other and then what you do to the bottom you do to the top. Then the learners need to see that it is in an improper fraction and need to convert it back to a mixed fractions, and how do they do that, they divide the denominator into the numerator and it only goes in once and that whole must be added to the other whole, the remainder fraction can be simplified so the learners must be able to identify that. I would use this method to teach my children.

The other method I know of would be to change it into improper fraction first then by multiplying our whole number by our denominator then adding that answer to our numerator and that answer becomes the new numerator over the original denominator. Find the LCD, what you do to the bottom you must do to the top. So now

you get an improper fraction then you must convert back into a mixed fraction and simplify the answer.

I prefer using the first method I've explained as there are less steps involved and I think once there are more steps involved it gets complicated.



Appendix 21: Language Certificate

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R Thomas
08 October 2018

TO WHOM IT MAY CONCERN

CERTIFICATE OF LANGUAGE EDITING

TITLE OF THESIS: ENHANCING THE LEARNING AND TEACHING OF FRACTION OPERATIONS IN
GRADE 6 – A CASE STUDY IN A SOUTH AFRICAN PRIMARY SCHOOL

Student : ASHNEE CHETTY

Student number : 201001275

Supervisor : Mammen KJ

Degree : Master of Education, Faculty of Education, University of Fort Hare

I confirm that the above thesis was checked for language and the suggestions for
improvement have been implemented.



Rothman D

R Thomas
(MA- English)