University of Fort Hare
Together in Excellence

MAT 121F

Degree Examinations

November 2018

Subject: **Mathematics**

Time: **3 Hours** Marks: **100**

This question paper consists of 3 pages

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Instructions

Answer all questions

Symbols used have the usual meanings

Question 1

1.1 Find $\frac{dy}{dx}$ if

(a) $y = x^2 \sin x + 2\cos x - 2\sin x$ (3)

(b) $y = \ln \frac{x^4}{(3x-4)^2}$ (4)

(c) $y = \tan^2 (e^{3x})$ (4)

(d) $\cos 3y = \tan 2x$ (3)

1.2 Use logarithmic differentiation to compute $\frac{dy}{dx}$ if

$$y = \frac{x^5}{(1-10x)\sqrt{x^2+2}}$$ (6)

1.3 Let $y = \arcsin x$, where $\frac{-\pi}{2} < x < \frac{\pi}{2}$. Then, $\sin y = x$.

Use this definition of the arcsine to derive the formula for the derivative of arcsine with respect to x. (5)

[25]

Question 2

2.1 Air is being pumped into a spherical balloon at a rate of $5cm^3/\min$. Determine the rate at which the radius is increasing when the diameter is $20cm$ (12)

2.2 Water runs into a conical tank at a rate of $2 ft^3/\min$. The tank stands point down and has a height of $10 ft$ and a base radius of $5 ft$. How fast is the water rising when the water level is $6 ft$ deep. (13)

[25]

Question 3

3.1 Find all critical points of the function f defined below

$$f(x)=x^3+3x^2-24x$$ (5)

3.2 Locate the intervals of x-values on the X-axis where the function

$$f(x)=2x^3-3x^2+3$$

is increasing and decreasing. (10)

3.3 Find the absolute maximum and absolute minimum values of the function

$$f(x)=x^3-3x+2$$

on the interval $[0,2]$. (10)

[25]

Question 4

4.1 Use L'Hopita'ls rule to evaluate the indeterminate limits below:

(a) $\lim_{x\to 2}\frac{x^2-4}{x-2}$ (4)

(b) $\lim_{x\to 3}\frac{x^3-27}{x-3}$ (4)

(c) $\lim_{x\to 1}\frac{\ln x}{x-1}$ (4)

4.2 A farmer has 1200m of fencing and wants to fence off a rectangular field that borders a straight river. He needs no fence along the river. What are the dimensions of the field that give the largest area of the field? (13)

[25]