University of Fort Hare

AMB121F

Degree Examinations: November 2016

Subject: Mathematics
Paper: Business Mathematics

Time: 3 Hours       Marks: 100       Subminimum: 40

This question paper consists of 4 pages

Internal examiner(s)                  External examiner(s)
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Instructions
Answer FIVE(5) questions. Symbols used have the usual meanings.

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Question One

1.1 Trapezium ABCD has AE perpendicular to BC and $AD \parallel BC$, AE = 6 cm, AD = 7cm, BE = 3 cm and DC = 8 cm. Compute the area and the perimeter of the trapezium. (8)

1.2 A cube has a volume of 624 cm$^2$. Compute its surface area correct to 2 decimal places. (3)

1.3 A right circular cylinder has a height of 30 cm and its volume is 960 cm$^3$. Compute its surface area. (5)

1.4 A right circular cone of slant height 24 cm has base radius of 4 cm. Compute its volume and its surface area accurate to two decimal places. (6)

1.5 Determine the values of $x$ and $y$ if $(-3, y)$ is the midpoint of the line segment joining $(-5, 6)$ and $(x, -2)$. (3)

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Question Two

2.1 Find the value of $x$ for which $M(x, -2)$ and $N(5, 2)$ are equidistant from $P(-3, 6)$. (3)
2.2 Find the equation of the circle that passes through \((-2, -3)\) and \((4, 5)\) with its centre on the line \(x - y = 1\).

2.3 If the line \(x + y = 1\) cuts the circle \(x^2 + y^2 = 13\) at \(A\) and \(B\); (a) find the coordinates of \(A\) and \(B\); (b) find the length of the chord \(AB\); (c) find the midpoint \(M\) of the chord \(AB\); (d) show that \(OM \perp AB\) where \(O\) is the origin; (e) show that \(T(-3, -2)\) lies on the circle; (f) find the slope of \(OT\); (g) find the equation of the tangent to the circle at \(T\). Illustrate with a diagram.

2.4 If \(\sin 70^\circ = x\), evaluate \(-1 + \frac{\cos(-380^\circ)}{x}\) without using a calculator. Show all the details.

**Question Three**

3.1 If \(\tan x = -4/3\), \(180^\circ \leq x \leq 360^\circ\), find, without using a calculator
   (i) \(\sin 2x\)
   (ii) \(\cos 2x\)
   (iii) \(\tan 2x\).

3.2 Evaluate \(\cos(-15^\circ)\) without using a calculator.

3.3 Find a general solution of the equation \(3\cos^2 x + 5\sin x = 1\).

3.4 Find (a) \(\lim_{x \to 2} \frac{x-2}{x-2}\), (b) \(\lim_{x \to 5/2} \frac{6x^2-11x-10}{2x-5}\).

3.5 Find \(\lim_{x \to -2} \frac{x^3+8}{x+2}\).

**Question Four**

4.1 \(ABC\) is a triangle having \(AC = 10\) cm; \(BC = 7\) cm; and \(AB = 6\) cm. \(D\) is a point on \(AC\) so that \(\angle DBC = 50^\circ\). Calculate \(\angle C\) and side \(DC\).

4.2 Calculate the area of \(\triangle ABC\) if \(BC = 7\) m; \(AB = 6\) m and \(\angle A = 36^\circ\).
4.3 Find the equations of the tangent and the normal to the curve \( y = -5x^4 + 3/x^3 + 2 \) at \( x = -1 \).

4.4 Find the period and the amplitude of the function \( y = -2\sin(3x) \) and then draw the graph of this function.

**Question Five**

5.1 Find the derivative of \( y = \frac{3}{1-2x} \) from first principles.

5.2 Discuss and draw the graph of the equation \( y = -x^3 + 6x^2 - 9x \), showing the turning points, local maxima and minima, points of inflection, regions of decrease and increase.

5.3 A cylindrical can (with top and bottom) has fixed volume \( V \). Find the ratio of the height \( h \) to the radius \( r \) of the can with minimum external surface area.

5.4 Graph the region represented by the inequalities:
\[ x \geq 1; \ y \geq 1/2; \ x + y \leq 3; \ x + 2y \leq 4. \] Maximize \( u = 5x + 8y - 7 \) subject to the given constraints.

**Question Six**

6.1 Joe wants to set up a computer centre. He has R75 000 with which to purchase at most 15 computers. Two types are available viz. comp 1 and comp 2. He needs at least 5 of comp 1 and 3 of comp 2. Comp 1 costs R6250 and comp 2 costs R7500.

(a) Write a system of equations to represent the above information.
(b) Draw a graph to determine the feasible region.
(c) The rates for using the computers per hour are comp 1 @ R45 and comp 2 at R75. Draw the profit line on the graph in the optimum position (position of maximum profit).
(d) What is the profit per hour possible?
6.2 Solve the following system of equations using Row reduction (elementary row operations) on the augmented matrix:

\[
\begin{align*}
2x + y - z &= 2 \\
x - y + z &= 7 \\
2x + 2y + z &= 4
\end{align*}
\] (6)

6.3 Solve the following system of equations using Cramer's Rule:

\[
\begin{align*}
2x + y - z &= 2 \\
x - y + z &= 7 \\
2x + 2y + z &= 4
\end{align*}
\] (6)

6.4 Find the inverse of the matrix \( A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 1 \\ 1 & 2 & -1 \end{bmatrix} \). (5)

\[ \text{[27]} \]

END