WAVE & OPTICS

(PHY 221)

DURATION: 3 Hours

INTERNAL EXAMINER

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INSTRUCTIONS

Answer any five (5) questions

MARKS: 100

MODERATOR

Dr. P. Mukumba
**Question 1**

(a) Derive the wave equation.

(b) Show that \( y = f(ct + x) \) satisfies the wave equation.

(c) Show that the wave profile \( y = f(ct - x) \) remains unchanged with time.

(d) Show that for a left-going wave, \( \frac{\partial y}{\partial t} = c \frac{\partial y}{\partial x} \).

(e) Derive an expression for the total energy in a wave.

(f) A wave of amplitude 0.1 m propagates along a string of linear mass density 0.01 kg/m, at a frequency of 5 Hz and speed of 9.55 m/s. Determine the rate at which energy is dissipated per cycle.

**Question 2**

(a) Consider two long strings of different densities, joined together at \( x = 0 \), and subject to the same tension \( T \). Assume the wave is incident from the negative \( x \) direction. At the junction the wave suffers partial reflection and partial transmission.

(i) State the boundary conditions. 2 x [3]

(ii) Determine the coefficients of reflection and transmission. [10]

(b) Show that \( u_g = v - \lambda \frac{\partial v}{\partial \lambda} \).

**Question 3**

(a) For standing waves on strings of finite length, show that the displacement of the \( n \)th harmonic is given by \( y_n = (A_n \cos \omega_n t + B_n \sin \omega_n t) \sin \frac{n\pi x}{L} \).

(b) Show that this displacement satisfies the time independent form of the wave equation:

\[ \frac{\partial^2 y}{\partial x^2} + k^2 y = 0 \]
(c) A string of length \( l \), mass per unit length \( \sigma \), and tension \( T \) is initially displaced a distance \( b \) (\( b \ll l \)) at the midpoint and is then released. Determine the Fourier coefficients for the subsequent motion. [6]

(d) Write the first four terms of the series. [4]

Question 4

(a) In Young’s double slit experiment, determine the positions from the central bright fringe, of the bright and the dark spots. [8]

(b) A viewing screen is separated from a double-slit source by 1.2 m. The distance between the two slits is 0.03 mm. The second – order bright fringe is 4.5 cm from the centre line.

(i) Determine the wavelength of the light. [2]

(ii) Calculate the distance between adjacent bright fringes. [4]

(iii) A light source emits visible light of two wavelengths: \( \lambda = 430 \text{ nm} \) and \( \lambda' = 520 \text{ nm} \). The source is used in a double-slit interference experiment in which \( L = 1.50 \text{ m} \) and \( d = 0.0250 \text{ mm} \). Find the separation distance between the third – order bright fringes. [6]

Question 5

(a) Calculate the minimum thickness of a soap-bubble film that results in constructive interference in the reflected light if the film is illuminated with light whose wavelength in free space is \( \lambda = 600 \text{ nm} \). [4]

(b) What if the film is twice as thick? Does this situation produce constructive interference? [2]

(c) Derive an expression between object and image distances for a spherical refracting surface. [8]

(d) A spherical convex mirror has a radius of curvature with a magnitude of 40.0 cm. Determine the position of the virtual image formed and magnification for object distances of 30.0 cm and 60.0 cm. [6]

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