UNIVERSITY OF FORT HARE
Department of Pure and Applied Mathematics

MNU 122

SUPPLEMENTARY EXAMINATIONS
JANUARY 2019

Time: 3 Hours

Subject: COMPUTATIONAL METHODS

Marks: 100

This question paper consists of 2 pages

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Instructions
Answer ALL Questions
Symbols have the usual meanings.
Question One

1.1 Consider the following matrices:

\[
A = \begin{bmatrix} 6 & 2 \\ -1 & 5 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & -1 \\ 11 & -3 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 3 & 2 \\ 2 & 8 & 3 \end{bmatrix}, \quad D = \begin{bmatrix} 10 & 0 & 11 \\ -2 & 3 & -1 \\ 5 & -25 & 4 \end{bmatrix}
\]

Perform the following matrix operations where possible. Give reasons why some operations may not be possible.

(i) \(3B - 2A\)
(ii) \(4C\)
(iii) \(2AB\)
(iv) \(DC\)

1.2 Solve the system of linear equations using the Gauss-Jordan Elimination Method.

\[
\begin{align*}
\begin{cases}
x + y + z &= 5 \\
2x + 3y + 5z &= 8 \\
4x + 5z &= 2
\end{cases}
\end{align*}
\]

Question Two

2.1 Without a detailed plotting of points, sketch the graphs of the following functions, showing relevant information on the graphs.

(i) \(5xy = 40\)
(ii) \(y = 4x - x^2\)

2.2 If \(R = a + \frac{b}{x^2}\), find the best values for \(a\) and \(b\) from the set of corresponding values below:

<table>
<thead>
<tr>
<th>(d)</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.5</th>
<th>0.8</th>
<th>1.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>(R)</td>
<td>5.78</td>
<td>2.26</td>
<td>1.6</td>
<td>1.27</td>
<td>1.53</td>
<td>1.1</td>
</tr>
</tbody>
</table>

2.3 If the function relating \(I\) and \(V\) is \(I = aV^n\), determine the values of the constants \(a\) and \(n\) that best fit the set of values recorded.

<table>
<thead>
<tr>
<th>(V)</th>
<th>8</th>
<th>12</th>
<th>15</th>
<th>20</th>
<th>28</th>
<th>36</th>
</tr>
</thead>
<tbody>
<tr>
<td>(I)</td>
<td>41.1</td>
<td>65.6</td>
<td>65.8</td>
<td>81.6</td>
<td>105</td>
<td>127</td>
</tr>
</tbody>
</table>

Question Three

3.1 Using the Bisection Method, estimate the positive root of the function \(f(x) = x^2 - \sin(x) - 0.5\) to 5 decimal places, with an accuracy of \(f(x) < 0.01\)

3.2 Use the Newton Method to find \(\sqrt{10}\) to 5 decimal places, with an accuracy of \(f(x) < 0.001\). Let \(x_0 = 3\)
Question Four

4.1 Use the Central Difference formula to approximate \( f'(x) \) of \( f(x) = \ln(x) \) at \( x = 1.8 \) for \( h = 0.1 \), \( h = 0.05 \) and \( h = 0.01 \). Determine the absolute error for these approximations.

(8)

4.2 Use Simpson’s Rule with \( n = 10 \) to approximate the integral \( \int_{1}^{3} \frac{1}{x} \, dx \). How large should \( n \) be to guarantee that this approximation is accurate to within 0.0001?

(10)